

On Testing of General Random Closed Set Model Hypothesis

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Abstract: A new method of testing the random closed set model hypothesis (for example: the Boolean model hypothesis) for a stationary random closed set $\Xi \subseteq \mathbb{R}^d$ with values in the extended convex ring is introduced. The method is based on the summary statistics – normalized intrinsic volumes densities of the δ -parallel sets to Ξ . The estimated summary statistics are compared with their envelopes produced from simulations of the model given by the tested hypothesis. The p-level of the test is then computed via approximation of the summary statistics by multinormal distribution which mean and the correlation matrix is computed via given simulations. A new estimator of the intrinsic volumes densities from [6] is used, which is especially suitable for estimation of the intrinsic volumes densities of δ -parallel sets. The power of this test is estimated for planar Boolean model hypothesis and two different alternatives and the resulted powers are compared to the powers of known Boolean model tests. The method is applied on the real data set of a heather incidence.

Keywords: Boolean model; Boolean model hypothesis; contact distribution function; Euler–Poincaré characteristic; Intrinsic volumes; Laslett’s transform;

AMS Subject Classification: 60D05; 62G05;

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