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MODELING OF PERMANENT MAGNET LINEAR GENERATOR AND STATE ESTIMATION BASED ON SLIDING MODE OBSERVER: A WAVE ENERGY SYSTEM APPLICATION

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This paper synopsis a new solution for Permanent Magnets Linear Generator (PMLG) state estimation subject to bounded uncertainty. Therefore, a PMLG modeling method is presented based on an equivalent circuit, wherein a mathematical model of the generator adapted to wave energy conversion is established. Then, using the Linear Matrix Inequality (LMI) optimization and a Lyapunov function, this system's Sliding Mode Observer (SMO) design method is developed. Consequently, the proposed observer can give a robust state estimation. At last, numerical examples with and without uncertainty are included to exemplify the effectiveness and applicability of the suggested approaches.

Keywords: wave energy, modeling, permanent magnet linear generator (PMLG), state estimation, sliding mode observer (SMO), linear matrix inequality (LMI)

Classification: 93B07, 49M30

1. INTRODUCTION

There are renewable energy resources (RERs) that can adequately satisfy our electricity needs. The coverage of electric power generated from their resources is progressively increasing in power systems [16]. Recently, many researchers have been devoted to exploiting Ocean Wave Energy (OWE) which is regarded as an important source of renewable energy and has developed rapidly [7, 8],

In addition, Wave Energy Conversion (WEC) systems conceivably have the highest power density among all renewable energy systems. Generally, these systems are composed of a mechanical converter, an electrical generator, that converts sea waves into electricity, and a power electronics section dedicated to the conversion, transmission, and storage of the generated electrical power [20]. Also, the WEC system must resist severe weather conditions and maintains high reliability, high robustness, and low cost, in terms of investment and maintenance cost. In fact, according to the principle of hydro-mechanical conversion by Falcao et al. [6], WEC systems are classified into three

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families: overtopping devices (fixed or floating), Oscillating Water Columns (OWC), and point absorbers (floating or submerged) [2, 5]. Every WEC system's structure includes an electrical generator. Permanent magnet linear generator PMLG is becoming prominent thanks to their benefit to connect with direct drive systems as they are distinguished by a high force and a low speed [19]. Moreover, this device develops high mechanical torques considering the large number of poles it contains [15]. Likewise, it enables performance with a high-power factor and satisfactory efficiency, which makes it applicable to electrical power generation systems [1].

However, due to their location underwater, which is considered unfavorable and severe conditions, then, the system state estimation by measuring devices is usually difficult for technical or economic reasons of construction, positioning, and cost of the sensors which requires, in most cases, the intervention of an expert swimmer.

Once the system is observed, we can estimate the state of our machine through a mathematical model using an observer to control the generator. Researchers have gone far in the field of robust state estimation. The general strategy is to devise an observer that uses the state and receives some basic measurements from the system to estimate all variables of this system. In [3, 10], relying on the Luenberger method [12] and using the pole placement technique, researchers suggested a solution for state estimation of linear electrical systems without uncertainty. Likewise, a lot of studies have focused on Unknown Input Observer (UIO) [4, 14, 17]. However, these approaches have proven to be unsuitable with uncertain systems, which mean that using UIO for state estimation has extra restrictions on system disturbance description. In recent years, the sliding mode SMO approach has earned great consideration for WEC systems [24, 25, 26].

It is an efficient estimator considering its exceptional advantage of strong robustness against external factors and model uncertainties.

In the literature, SMO-based state estimation for a PMLG has not been explored. Wherefore, this paper presents a novel sliding mode observer that observes the generator velocity and currents. The novelty of this approach is that only the inputs and outputs of the system are needed to obtain a robust estimation. Moreover, the control output is not required in the proposed observer. Compared with the existing results, our study offers the advantage of the ability to create a sliding motion on the output error ensuring that a sliding mode observer generates a set of state estimates that are highly accurate.

Impressed by the above summary, this paper included two main goals; the first concerns the modeling of the basic element of WEC systems. The second goal is dedicated to the SMO design using the Linear Matrix Inequality (LMI) approach, and a Lyapunov function. It is worthy of notice that no work on this topic has ever been investigated yet, which encourages our approach. Also, a comparative study, with [21] design method, is established using the root mean square error (RMSE) to measure the performances of the proposed SMO. The rest of this paper is organized in the following way: in section 2, the matrix portrayal of a PMLG is determined using Park Transformation. In section 3, an SMO design is demonstrated for the state estimation of a generator dedicated to wave energy. Section 4 shows the simulation results. A comparative study is developed in section 5. The conclusion of this paper is given in section 6.

2. LINEAR GENERATOR MODELING

The modeling of the linear synchronous machine follows several assumptions, such as magnetic hysteresis, Foucault currents, and saturation effects are negligible [23]. Also, the resistances are constant, the stator windings are positioned sinusoidal and the stator slots are not considered [11, 18]. The dynamic model of PMLG is obtained after the transformation three-phase system to the d-q frame using Park transformation which is defined in [13]. The voltages equations along d and q axes are given by:

$$V_{sd} = R_s \cdot I_{sd} + \frac{d\phi_{sd}}{dt} - \omega \cdot \phi_{sq} \tag{1}$$

$$V_{sq} = R_s \cdot I_{sq} + \frac{d\phi_{sq}}{dt} + \omega \cdot \phi_{sd}. \tag{2}$$

Where R_s is the resistance of the stator, I_{sd} and I_{sq} are the stator currents and ω represents the angular electrical speed.

Flux linkages along d and q are expressed by:

$$\phi_{sd} = L_{sd} \cdot I_{sd} + \phi_a \tag{3}$$

$$\phi_{sq} = L_{sq} \cdot I_{sq}. \tag{4}$$

Where L_{sd} and L_{sq} are the inductances along d and q axes respectively. ϕ_a is the permanent magnet flux linkage. The current equations became:

$$\frac{dI_{sd}}{dt} = \frac{1}{L_{sd}} \left(-R_s \cdot I_{sd} + \omega \cdot L_{sq} \cdot I_{sq} + V_{sd}\right) \tag{5}$$

$$\frac{dI_{sq}}{dt} = \frac{1}{L_{sq}} \left(-R_s \cdot I_{sq} - \omega \left(L_{sd} \cdot I_{sd} + \phi_a\right) + V_{sd}\right). \tag{6}$$

The mechanical equation, which is needed to complete the generator model, is defined by:

$$\frac{J}{P} \cdot \frac{d\omega}{dt} = C_{em} - C_r - \frac{f}{P}\omega. \tag{7}$$

Where C_{em} and C_r are respectively the electromagnetic torque and the load torque, f represents the coefficient of friction, J is the moment of inertia and P is the number of generator pole pairs. Furthermore, by taking into account that $C_{em} = P \cdot I_{sq}\phi_a$ and multiplying the equation (7) by $\frac{P}{J}$, we get:

$$\frac{d\omega}{dt} = \frac{P^2}{J} \cdot I_{sq}.\phi_a - \frac{P}{J}C_r - \frac{f}{J}\omega. \tag{8}$$

Since the relation between electrical rotating speed and mechanical speed is given by:

$$\omega = \frac{\pi}{\tau} \cdot \omega_g. \tag{9}$$

Equation (8) will be:

$$\frac{d\omega_g}{dt} = \frac{\tau \cdot P^2}{\pi . J} I_{sq} . \phi_a - \frac{\tau \cdot P}{\pi . J} . C_r - \frac{f}{J}\omega_g. \tag{10}$$

Where ω_g represents the linear speed. In practical applications, the generator dynamical model inevitably introduces modeling uncertainties due essentially to variation of the resistance Rs. Therefore, a 10% model uncertainty is supposed in this study. So, the linear obtained model has the following structure:

$$\begin{gathered} \dot{x}(t) = Ax(t) + Bu(t) + F\xi(x(t), t) \\ y(t) = Cx(t) \end{gathered} \tag{11}$$

where $x = \begin{bmatrix} I_{sd} & I_{sq} & \omega_q \end{bmatrix}^T$ and $x = \begin{bmatrix} V_{sd} & V_{sq} & C_r \end{bmatrix}^T$. Where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ and $F \in \mathbb{R}^{n\times q}$ with $p \geq q$.

The matrices A, B, C, and F are assumed to be constants. As well, we assumed that C and F are full ranks, and the function $\xi : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^q$ is unknown but bounded as follows:

$$\xi(x(t), t) \leq r_1 u + \alpha(t, y). \tag{12}$$

Where r_1 is an unknown positive scalar and $\alpha : \mathbb{R}_+ \times \mathbb{R}^p \to \mathbb{R}_+$ is an unknown function.

In this section, the proposed methodology consists to obtain the expression of the observer gains. Thus, based on the Lyapunov approach to ensure the asymptotic convergence of the error state estimation and solving the inequalities of this approach leads to resolving an LMI problem optimization.

3. SLIDING MODE OBSERVER DESIGN

Consider an SMO of the system (11) described by:

$$\begin{gathered} \dot{\widehat{x}}(t) = A\widehat{x}(t) + Bu(t) - G_1 e_y + G_2 v(t) \\ \widehat{y}(t) = C\widehat{x}(t). \end{gathered} \tag{13}$$

Where $G_1 \in \Re^{n\times p}$ and $G_2 \in \Re^{n\times p}$ are respectively the linear and nonlinear gains. The discontinuous vector $v(t)$ is defined by:

$$V(t) = \left\{ \begin{matrix} -\rho(t, y, u) \left\| f_2 \right\| \frac{P_0 e_y(t)}{\left\| P_2 . e_y(t) \right\|} & \text{if } e_y(t) \neq 0 \\ 0 & \text{otherwise} \end{matrix} \right\}. \tag{14}$$

Where $e_y = \widehat{y}(t) - y(t)$ and $P_2 \in \mathbb{R}^{p\times p}$ is a symmetric positive definite matrix will be determined later. The scalar function $\rho : \mathbb{R}_+ \times \mathbb{R}^p \times \mathbb{R}^m \to \mathbb{R}_+$ satisfied:

$$\rho(t, y, u) \geq r_1 u + \alpha(t, y) + \gamma_0. \tag{15}$$

With γ_0 is a positive scalar.

The dynamics of state estimation error $e(t) = \widehat{x}(t) - x(t)$ generated by the observer (13) and the system (11) is governed by the following equation:

$$\dot{e}(t) = A_0 e(t) + G_2 v(t) - F\xi(x(t), t) \tag{16}$$

where $A_0 = A - G_1 C$ is a Hurwitz matrix. Edwards and Tan in [25] show that there exists a stable sliding motion on the sliding surface $S = \{e \in \mathbb{R}^n : e_y = 0\}$ only if the following two conditions are satisfied:

- Assumption A1: $Rank(CF) = q$;
- Assumption A2: Invariant zeros of (A, F, C) lie in the open LHP.

Remark 1. The assumption A1 is verified directly through the calculation of the rank, and it enables the decoupling of the uncertainty from the sliding mode dynamics. The assumption A2 is verified if the pair (A, C) is detectable.

Verifying these conditions makes it possible to move to a new base using coordinates change T_0. The system can be defined, in the new space coordinates, by:

$$\begin{aligned} \dot{\overline{x}}(t) &= \overline{A}\overline{x}(t) + \overline{B}u(t) + \overline{F}\xi(x(t), t) \\ \overline{y}(t) &= \overline{C}\overline{x}(t)). \end{aligned} \tag{17}$$

Where $(\overline{A}, \overline{F}, \overline{C})$ has the following structure:

$$\overline{A} = \begin{bmatrix} \overline{A}_{11} & \overline{A}_{12} \\ \overline{A_{21}} = \begin{bmatrix} \overline{A}_{211} \\ \overline{A}_{212} \end{bmatrix} & \overline{A}_{22} \end{bmatrix} \tag{18}$$

$$\overline{F} = \begin{bmatrix} 0 \\ F_2 \end{bmatrix} \tag{19}$$

$$\overline{C} = \begin{bmatrix} 0 & T \end{bmatrix}. \tag{20}$$

Where $\overline{A}_{11} \in \mathbb{R}^{(n-p)\times(n-p)}$ and $\overline{A}_{211} \in \mathbb{R}^{(p-q)\times(n-p)}$ defined by:

$$\overline{A}_{11} = \begin{bmatrix} \overline{A}^0_{11} & \overline{A}^0_{12} \\ 0 & \overline{A}^0_{22} \end{bmatrix} \tag{21}$$

$$\overline{A}_{211} = \begin{bmatrix} 0 & \overline{A}^0_{21} \end{bmatrix} \tag{22}$$

with $\overline{A}^0_{11} \in \mathbb{R}^{r\times r}$ and $\overline{A}^0_{21} \in \mathbb{R}^{(p-q)\times(n-p-r)}$ for $r \geq 0$ and the pair $(\overline{A}^0_{22}, \overline{A}^0_{21}$ is completely observable. Moreover, the eigenvalues of $\overline{A}^0_{11}$ are the invariant zeros of (A, F, C).
Where $F_2 = \begin{bmatrix} 0 & \overline{F}_2 \end{bmatrix}^T \in \mathbb{R}^{p\times q}, \overline{F}_2 \in \mathbb{R}^{q\times q}$ is not singular matrix and $T \in \mathbb{R}^{p\times p}$ is an orthogonal matrix.

We define $\overline{A}_0 = \overline{A} - \overline{G}_1\overline{C}$ the gain $\overline{G_1}$ will be determined later and the $\overline{G_2}$ is given by:

$$\overline{G}_2 = \begin{bmatrix} -\overline{L}^T T \\ T^T \end{bmatrix}. \tag{23}$$

Where $\overline{L} \in \mathbb{R}^{(n-p)\times p}$ and $\overline{L} = \begin{bmatrix} L & 0 \end{bmatrix}$, with Ł $\in \mathbb{R}^{(n-p)\times(p-q)}$ and the orthogonal matrix T is defined in (20).

Proposition. If there exists a positive definite Lyapunov matrix $\overline{P}$ satisfies $\overline{PA_0} + \overline{A}_0^T\overline{P} < 0$, with the structure:

$$\overline{P} = \begin{bmatrix} \overline{P}_1 & \overline{P}_1\overline{L} \\ \overline{L}^T\overline{P}_1 & \overline{P}_2 + \overline{L}^T\overline{P}_1\overline{L} \end{bmatrix} > 0. \tag{24}$$

Where $\overline{P}_1 \in \mathbb{R}^{(n-p)\times(n-p)}$ and $\overline{P}_2 \in \mathbb{R}^{p\times p}$, then the error system is quadratically stable.

Proof. Consider the following Lyapunov function:

$$V(\overline{e}) = \overline{e}^T\overline{P}\overline{e} \tag{25}$$

where $\overline{e} = T_0 e$. Notice that, from the special form of $\overline{P}$ (23), if $\overline{P}_1, \overline{P}_2 \succ 0$ then $\overline{P} > 0$.
The derivative of $V(\overline{e})$ along the trajectory of (16) is given by:

$$\dot{V} = \overline{e}^T(\overline{A}_0^T\overline{P} + \overline{PA}_0)\overline{e} + 2\overline{e}^T\overline{PG}_2V - 2\overline{e}^T\overline{PF\xi}. \tag{26}$$

From equations (23) and (24), we can obtain the following:

$$\overline{PG}_2 = \begin{bmatrix} 0 \\ \overline{P}_2T^T \end{bmatrix} = \overline{C}^T P_2 \tag{27}$$

where $P_2 = T\overline{P}_2T^T$. So, using the structures of $\overline{L}$ and F_2 we can find $\overline{L}F_2 = 0$ and:

$$\overline{PF} = \begin{bmatrix} 0 \\ \overline{P}_2F2 \end{bmatrix} = \overline{C}^T P_2 f_2. \tag{28}$$

Where $f_2 = TF_2$ which implies $\|f_2\| = \|F_2\|$. Therefore, using (27) and (28), equation (26) becomes:

$$\dot{V} = \overline{e}^T(\overline{A}_0^T\overline{P} + \overline{PA}_0)\overline{e} + 2e_y^TP_2V - 2e_y^TP_2f_2\xi \tag{29}$$

$$\dot{V} \leq \overline{e}^T(\overline{A}_0^T\overline{P} + \overline{PA}_0)\overline{e} - 2\rho\,\|f_2\| - 2e_y^TP_2f_2\xi. \tag{30}$$

From equations (12) and (15), we get:

$$\dot{V} \leq \overline{e}^T(\overline{A}_0^T\overline{P} + \overline{PA}_0)\overline{e} - 2\rho\,\|f_2\|\,\|P_2e_y\| + 2\,\|f_2\|\,[r_1\,\|u\| + \alpha(y)]\,\|P_2e_y\| \tag{31}$$

$$\dot{V} \leq \overline{e}^T(\overline{A}_0^T\overline{P} + \overline{PA}_0)\overline{e} - 2\gamma_0\,\|f_2\|\,\|P_2e_y\|\,. \tag{32}$$

Since $(\overline{A}_0^T\overline{P} + \overline{PA}_0) < 0$, so $\dot{V} < 0$ $(\overline{e} \neq 0)$. □

Corollary. An ideal sliding motion takes place on S in finite time. Therefore, the sliding dynamics is governed by $\overline{A}_{11} + L\overline{A}_{211}$. Define a transformation:

$$T_2 = \begin{bmatrix} I_{n-p} & \overline{L} \\ 0 & T \end{bmatrix}. \tag{33}$$

This transformation is applied to the triplet $(\overline{A} \quad \overline{F} \quad \overline{C})$ and to the Lyapunov matrix $\overline{P}$ which will be, respectively:

$$\widetilde{A} = \begin{bmatrix} \widetilde{A}_{11} & \widetilde{A}_{12} \\ \widetilde{A}_{21} & \widetilde{A}_{22} \end{bmatrix}, \widetilde{F} = \begin{bmatrix} 0 \\ \widetilde{f}_2 \end{bmatrix}, \widetilde{C} = \begin{bmatrix} 0 & I_p \end{bmatrix} \tag{34}$$

with $\widetilde{A}_{11} = \overline{A}_{11} + L\overline{A}_{211}$ and $\widetilde{f}_2 = T\overline{F}_2$.

$$\widetilde{P} = (T_2^{-1})\widetilde{P}(T_2^{-1}) = \begin{bmatrix} \overline{P}_1 & 0 \\ 0 & T\overline{P}_2 T \end{bmatrix}. \tag{35}$$

One notice, $\widetilde{P}$ is a diagonal and Lyapunov matrix of $\widetilde{A}_0 = \widetilde{A} - \widetilde{G}_1\widetilde{C}$, then $\widetilde{A}_{11}$ is stable. So, this means that the sliding motion is also stable. Also, the linear and the nonlinear gains are given by:

$$\widetilde{G}_1 = T_2\overline{G}_1 = \begin{bmatrix} \widetilde{G}_{11} \\ \widetilde{G}_{12} \end{bmatrix} \tag{36}$$

and

$$\widetilde{G}_2 = T_2\overline{G}_2 = \begin{bmatrix} 0 \\ I_p \end{bmatrix}. \tag{37}$$

In the following, we are interested in the synthesis problem under LMIs. Wherein, involving the LQR technique, the choice of $\overline{P}$ and $\overline{G}_1$ must verify the following matrix inequality:

$$\overline{A}_0^T\overline{P} + \overline{PA}_0 < -\overline{P}W\overline{P} - \overline{PG}_1 M\overline{G}_1^T\overline{P}. \tag{38}$$

Where W and M are two positive definite symmetric weighting matrices and the matrix $\overline{P}$ is given by (24). Thus, it is clear that the inequality (38) guarantees $\overline{A}_0^T\overline{P} + \overline{PA}_0 < 0$ Assume that $\overline{Y}^T = \overline{PG}_1$ and replacing $\overline{A}_0$ by their expression, (38) becomes:

$$\overline{A}^T\overline{P} + \overline{PA} - (\overline{YC})^T + \overline{YC} + \overline{P}W\overline{P} + \overline{Y}M\overline{Y}^T < 0. \tag{39}$$

After the development of inequality (39), we get:

$$\overline{PA} + \overline{A}^T\overline{P} + (\overline{Y}^T - M^{-1}\overline{C})^T M(\overline{Y}^T - M^{-1}\overline{C}) - \overline{C}^T M^{-1}\overline{C} + \overline{P}W\overline{P} < 0. \tag{40}$$

So, choosing $\overline{Y}^T = M^{-1}\overline{C}$, this leads to:

$$\overline{PA} + \overline{A}^T\overline{P} + \overline{P}W\overline{P} - \overline{C}^T M^{-1}\overline{C} < 0. \tag{41}$$

Thus, the gain $\overline{G}_1$ can be expressed by:

$$\overline{G}_1 = \overline{P}^{-1}\overline{C}^T M^{-1}. \tag{42}$$

Now, to improve the obtained results, we will integrate the trace minimization technique. The considered problem amounts consist of minimizing the trace of ($\overline{P}^{-1}$) such that $\overline{P}$ satisfies the inequality (41).

Then, using the Schur complement, the matrix inequality (41) is equivalent to:

$$\begin{bmatrix} \overline{PA} + \overline{A}^T\overline{P} - \overline{C}^T M^{-1}\overline{C} & \overline{P} \\ \overline{P} & -W^{-1} \end{bmatrix} < 0. \tag{43}$$

If $\overline{X} \in \mathbb{R}^{n\times n}$ is symmetric positive definite matrix, we get:

$$\begin{bmatrix} -\overline{P} & I \\ I & -\overline{X} \end{bmatrix} < 0. \tag{44}$$

According to Schur complement, one more time, the inequality (44) is equivalent to $\overline{X} > \overline{P}^{-1}$. Thus, the minimization of the trace of $\overline{P}^{-1}$ is equivalent to the minimization of the trace of $\overline{X}$. Hence, after the resolution of the LMIs (43) and (44), the Lyapunov matrix $\overline{P}$ becomes:

$$\overline{P} = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} > 0 \tag{45}$$

where $P_{11} \in \mathbb{R}^{(n-p)\times(n-p)}$, $P_{22} \in \mathbb{R}^{q\times q}$ and $P_{12} = \begin{bmatrix} P_{121} & 0 \end{bmatrix}$ with $P_{121} \in \mathbb{R}^{(n-p)\times(p-q)}$.

$$P_{11} = \overline{P}_1 \tag{46}$$

$$\overline{L} = P_{11}^{-1}P_{12} \tag{47}$$

$$\overline{P}_2 = P_{22} - P_{12}^T P_{11}^{-1} P_{12}. \tag{48}$$

Thus, after obtaining $\overline{L}$, we can find directly $\overline{G}_2$ from equation (23). Hence, the gains G_1 and G_2 in the original coordinate system are:

$$G_1 = T_0^{-1}T_2^{-1}\widetilde{G}_1 \tag{49}$$

$$G_2 = T_0^{-1}T_2^{-1}\widetilde{G}_2. \tag{50}$$

This means that the proposed PMLG SMO design method is achieved. This observer is able to give a robust state estimation, which is equivalent to resolve (35), (43), and (44) with respect to the variables $\overline{P}$ and $\overline{X}$. Moreover, to solve this convex optimization problem, software like MATLAB LMI Control Toolbox [9] is used to find G_1, G_2, $\overline{P}$ and L. After this, we will present the PMLG numerical model, developed in section 2, to demonstrate the effectiveness of the design method proposed in this paper.

4. SIMULATION RESULTS

The system matrices are given by:

$$A = \begin{bmatrix} -11.2093 & 0 & 0 \\ 0 & -11.2093 & -5.1408 \\ 0 & 0.2464 & -0.0091 \end{bmatrix}, \; B = \begin{bmatrix} 11.6279 & 0 & 0 \\ 0 & 11.6279 & 0 \\ 0 & 0 & -0.1369 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } F = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

The coordinate transformation matrices are: $T_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $T_2 = \begin{bmatrix} 1 & 2.0322 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

The evolution of the real and observed states of the system are illustrated in figures 1,2 and 3 with an uncertainty $\xi(x(t),t) = 0$ and initial conditions $\widehat{x}(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ and $x(0) = \begin{bmatrix} 100 & 10 & 6 \end{bmatrix}^T$. In order to validate the robustness of the designed SMO, Figure 4, 5 and 6 shows the state system and the state estimation in the presence of an uncertainty $\xi(x(t),t) \neq 0$ and different initial conditions.The simulation is executed on MATLAB (2018a MathWorks).

The observer states are given by a discontinued line $(\cdots)$ and the real state by the solid lines $(-)$. It is noticeable that tracking is almost perfect in all states. So, the above simulations demonstrate that the proposed method is able to give a robust observation of the PMLG system in finite time, despite the presence of uncertainty. This improves the efficiency of the proposed method of SMO design and the problem of robust estimation for this type of machine is solved.

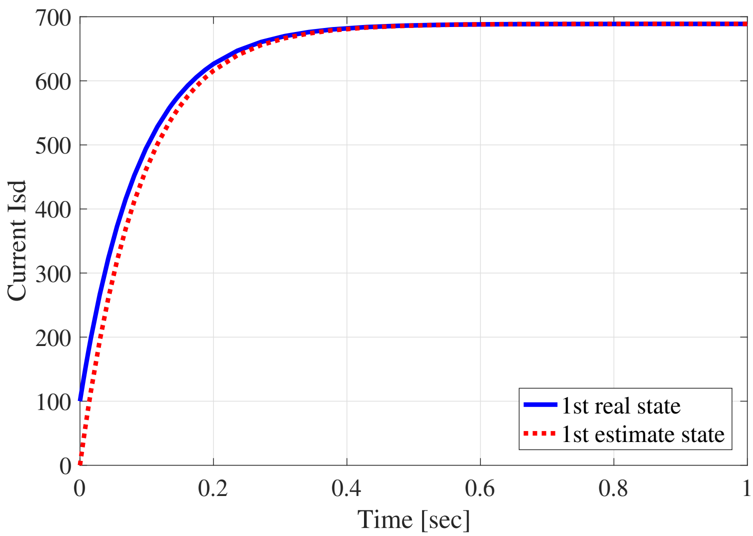

Fig. 1. x_1 and its estimation $\widehat{x}_1$.

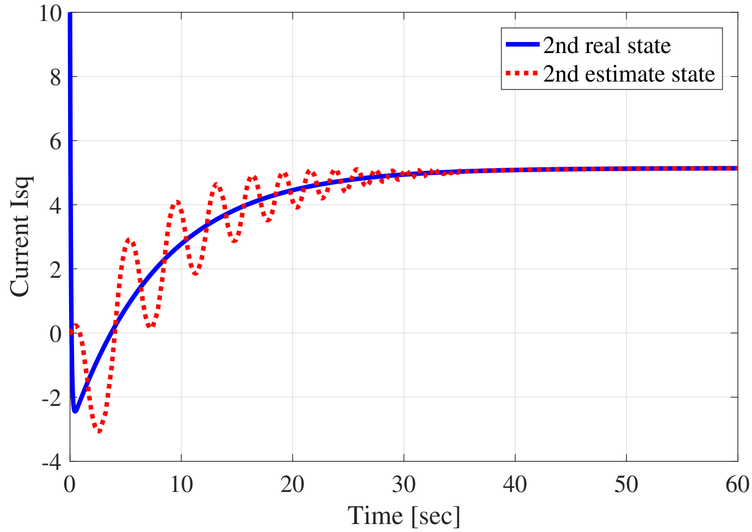

Fig. 2. x_2 and its estimation $\widehat{x}_2$.

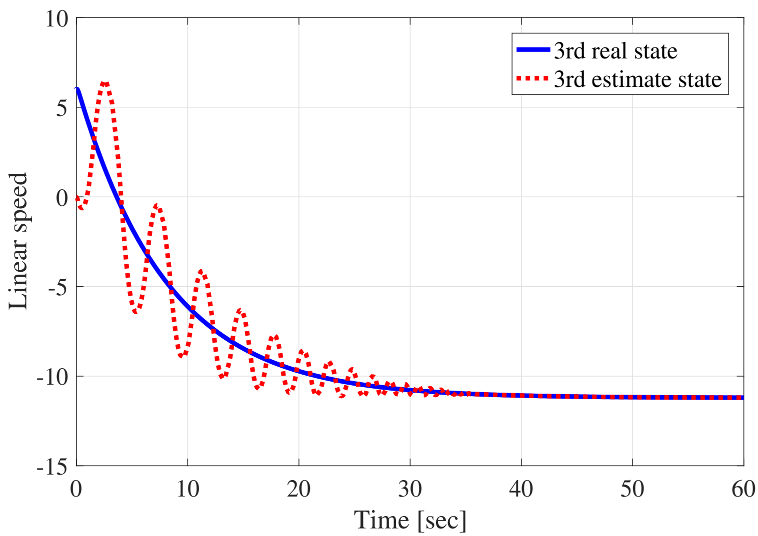

Fig. 3. x_3 and its estimation $\widehat{x}_3$.

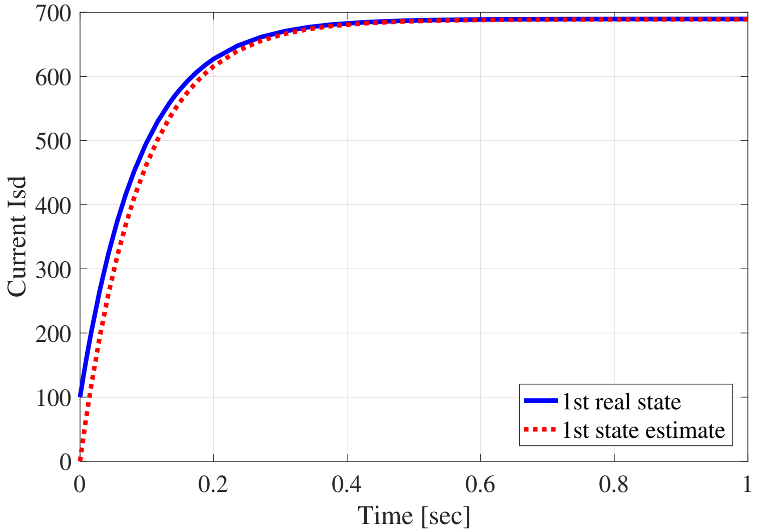

Fig. 4. x_1 and its estimation $\widehat{x}_1$.

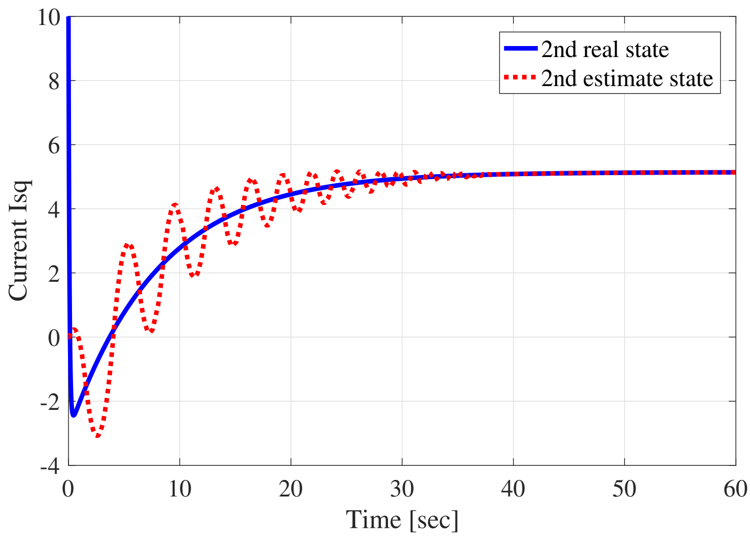

Fig. 5. x_2 and its estimation $\widehat{x}_2$.

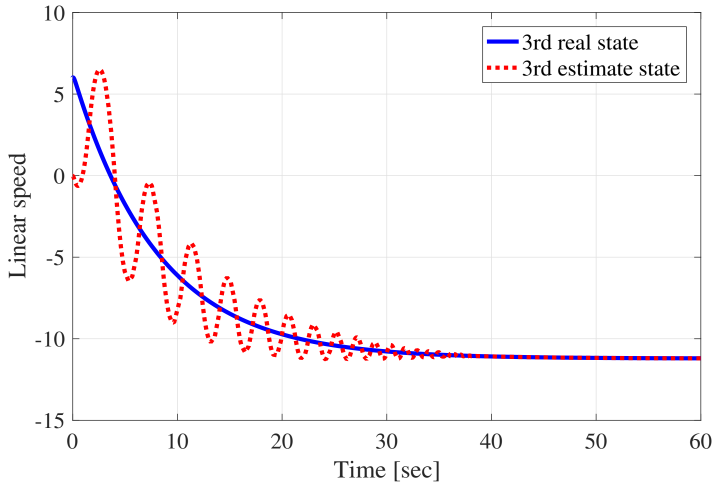

Fig. 6. x_3 and its estimation $\widehat{x}_3$.

5. COMPARATIVE STUDY

In this comparative study, comparing our proposed SMO with an inspired LMI design Luenberger observer method [21], where this observer is described by:

$$\dot{\widehat{x}}(t) = A\widehat{x}(t) + Bu(t) + LC(x(t) - \widehat{x}(t)) \tag{51}$$

$$\widehat{y}(t) = C\widehat{x}(t) \tag{52}$$

where L is the Luenberger gain. In Table 1 we compare our results of the designed SMO with those of the constructed Luenberger observer [21], using the RMSE of states in the presence of uncertainty for three cases.

Cases	**States**	**Luenberger-based method**	**SMO-based method**
1st case ξ=[0 0]y	x1 (t)	0.63	0.48
	x2 (t)	2.12	2.02
	x3 (t)	3.73	3.27
2nd case ξ=[0.8 1]y	x1 (t)	1.39	0.57
	x2 (t)	3.90	2.39
	x3 (t)	5.88	4.47
3rd case ξ=[1.7 2.2]y	x1 (t)	∞	1.02
	x2 (t)	4.98	2.85
	x3 (t)	7.84	4.87

Tab. 1. RMSE of outputs.

As illustrated in Table 1, the comparison results exhibit that our proposed methods achieve the best RMSE.

6. CONCLUSION

This research has presented a state estimation for a PMLG. The primary objective is to model the generator in a reference (d, q) using Park transformation. Then, a sliding mode observer design method was established to estimate the generator states. To ensure the stability dynamics of the estimation error and obtain its winnings, adequate conditions using the linear matrix inequality (LMI) approach and a Lyapunov function are derived for this observer. Simulation results were enclosed to illustrate the robustness of the estimation approach despite the presence of uncertainty. Finally, based on the RMSE technique, a comparative study is established to valorize the constructed SMO.

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REFERENCES

[1] T. Ackermann: Wind Power in Power Systems. John Wiley and Sons, 2012.

[2] J. Brooke: Wave Energy Conversion. Elsevier, 2003.

[3] D. Calabrese, G. Tricarico, E. Brescia, G. L. Cascella, V. G. Monopoli, and F. Cupertino: Variable structure control of a small ducted wind turbine in the whole wind speed range using a Luenberger observer. MDPI, Energies *13* (2020), 4647. DOI:10.3390/en13184647

[4] W. Chen and M. Saif: Unknown input observer design for a class of nonlinear systems: an LMI approach. Amer. Control Confer. (2006), 5. DOI:10.1109/ACC.2006.1655461

[5] A. Clément, P. McCullen, A. Falcão, A. Fiorentino, F. Gardner, K. Hammarlund, G. Lemonis, T. Lewis, K. Nielsen, S. Petroncini, and others: Wave energy in Europe: current status and perspectives. Renewable Sustainable Energy Rev. *6* (2002), 405–431. DOI:10.1016/S1364-0321(02)00009-6

[6] A. F. de Falcao: Wave energy utilization: A review of the technologies. Elsevier *14* (2010), 899–918. DOI:10.1016/j.rser.2009.11.003

[7] O. Farrok, Md R. Islam, Md R. I. Sheikh, Y. Guo, J. Zhu, Jianguo, and G. Lei: Oceanic wave energy conversion by a novel permanent magnet linear generator capable of preventing demagnetization. IEEE Trans. Industry Appl. *54* (2018), 6005–6014. DOI:10.1109/tia.2018.2863661

[8] S. Foteinis: Wave energy converters in low energy seas: Current state and opportunities. Elsevier *162* (2022), 112448. DOI:10.1016/j.rser.2022.112448

[9] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali: LMI Control Toolbox, the MathWorks Inc, Natick. MA 1995.

[10] Z. Gao and X. Liu: An overview on fault diagnosis, prognosis and resilient control for wind turbine systems. MDPI J., Processes *9* (2021), 300. DOI:10.3390/pr9020300

[11] N. S. Jayalakshmi, D. N. Gaonkar, and K. S. K. Kumar: Dynamic modeling and performance analysis of grid connected PMSG based variable speed wind turbines with simple power conditioning system. IEEE Int. Confer. Power Electron. Drives Energy Systems (PEDES) (2012), 1–5. DOI:10.1109/PEDES.2012.6484474

[12] D. Luenberger: An introduction to observers. IEEE Trans. Automat. Control *16* (1971), 596–602. DOI:10.1109/TAC.1971.1099826

[13] E. Mouni, S. Tnani, and G. Champenois: Synchronous generator modelling and parameters estimation using least squares method. Simul- Modell. Practice Theory *16* (2008), 678–689. DOI:10.1016/j.simpat.2008.04.005

[14] P. F. Odgaard and J. Stoustrup: Unknown input observer based detection of sensor faults in a wind turbine. IEEE Int. Confer. Control Appl. (2010), 310–315. DOI:10.1109/CCA.2010.5611266

[15] H. Polinder, M. A. Mueller, M. Scuotto, and M. G. de Sousa Prado: Linear generator systems for wave energy conversion. In: Proc. 7th European Wave and Tidal Energy Conference, Porto 2007, pp. 11–14.

[16] D. Remon, A. M. Cantarellas, and P. Rodriguez: Equivalent model of large-scale synchronous photovoltaic power plants. IEEE Trans. Industry Appl. *52* (2016), 5029–5040. DOI:10.1109/tia.2016.2598718

[17] O. Sename: New trends in design of observers for time-delay systems. Kybernetika *37* (2001), 427–458.

[18] M. G. Simões and F. A. Farret: Modeling and Analysis with Induction Generators. CRC Press 2014.

[19] B. Tagliafierro, I. Martínez-Estévez, J. Domínguez, A. J. C. Crespo, M. Göteman, J. Engström, and M. Gómez-Gesteira: A numerical study of a taut-moored point-absorber wave energy converter with a linear power take-off system under extreme wave conditions. Appl. Energy, Elsevier *311* (2022), 118629. DOI:10.1016/j.apenergy.2022.118629

[20] M. Trapanese,V. Boscaino, G. Cipriani, D. Curto, V. Di Dio, and V. Franzitta: A permanent magnet linear generator for the enhancement of the reliability of a wave energy conversion system. IEEE Trans. Industrial Electron. *66* (2018), 4934–4944. DOI:10.1109/TIE.2018.2838076

[21] Z. Wang, Y. Shen, X. Zhang, and Q. Wang: Observer design for discrete-time descriptor systems: An LMI approach. Systems Control Lett. *61* (2012), 683–687. DOI:10.1016/j.sysconle.2012.03.006

[22] J. Wang, F. Wang, X. Wang, and L. Yu: Disturbance observer based integral terminal sliding mode control for permanent magnet synchronous motor system. Kybernetika *55* (2019), 586–603. DOI:10.14736/kyb-2019-3-0586

[23] J. Wang, F. Wang, X. Wang, and L. Yu: Disturbance observer based integral terminal sliding mode control for permanent magnet synchronous motor system. Kybernetika *55* (2019), 586–603. DOI:10.14736/kyb-2019-3-0586

[24] Y. Zhang and G. Li: Non-causal linear optimal control of wave energy converters with enhanced robustness by sliding mode control. IEEE Trans. Sustainable Energy *11* (2019), 2201–2209. DOI:10.1109/TSTE.2019.2952200

[25] Y. Zhang, G. Li, and T. Zeng: Wave excitation force estimation for wave energy converters using adaptive sliding mode observer. IEEE Amer. Control Confer. (ACC) (2019), 4803–4808. DOI:10.23919/acc.2019.8815196

[26] Y. Zhang, P. Stansby, and G. Li: Non-causal linear optimal control with adaptive sliding mode observer for multi-body wave energy converters. IEEE Trans. Sustainable Energy *12* (2020), 568–577. DOI:10.1109/TSTE.2020.3012412

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