KYBERNETIKA — VOLUME *50* (2014), NUMBER 4, PAGES 580–595

NON-FRAGILE SAMPLED DATA H_∞ FILTERING OF GENERAL CONTINUOUS MARKOV JUMP LINEAR SYSTEMS

Mouquan Shen, Guangming Zhang, Yuhao Yuan and Lei Mei

This paper is concerned with the non-fragile sampled data H_∞ filtering problem for continuous Markov jump linear system with partly known transition probabilities (TPs). The filter gain is assumed to have additive variations and TPs are assumed to be known, uncertain with known bounds and completely unknown. The aim is to design a non-fragile H_∞ filter to ensure both the robust stochastic stability and a prescribed level of H_∞ performance for the filtering error dynamics. Sufficient conditions for the existence of such a filter are established in terms of linear matrix inequalities (LMIs). An example is provided to demonstrate the effectiveness of the proposed approach.

Keywords: Markov jump linear system, sampled data H_∞ filtering, linear matrix equality

Classification: 93E12, 62A10

1. INTRODUCTION

Markov jump linear systems (MJLSs), whose state space consists of a continuous part, the kinematics, and a discrete part, the mode which determines the dynamics in place, are suitable to model dynamical systems with variable structures caused by random failures or repairs, modification of the operating components, changes in the interconnections of subsystems, sudden environment changes, etc. During the past decades, many issues on MJLSs such as stability and stabilization, H_∞ control and filtering, optimal tracking, have been well investigated [6, 8, 9, 14, 16] and the references therein.

On the other hand, in most of the digital implemented filtering and control schemes, the measurements are obtained at discrete sample points rather than continuously. Therefore, much attention has been given to study how a continuous-time system can be controlled using measurements obtained only at discrete sample points [3, 4, 12]. For stochastic systems with Markov switching under sampled measurements, very few results are available in the literature [5, 15]. Specially, sufficient conditions for stochastic stability or exponential mean square stability of sampled-data systems with Markov jump parameters are established in [5]. Based on the LMI technique, the H_∞ filtering problem of MJLSs under sampled measurements are discussed in [15]. Note that the above

DOI: 10.14736/kyb-2014-4-0580

mentioned works on the control or filtering problem of MJLSs are based on an implicit assumption that the controller or filter should be implemented exactly. However, in practice, controllers or filters do have a certain degree of errors due to finite word length in any digital systems, the imprecision inherent in analog systems and additional tuning of parameters in the final controller implementation. Therefore, a significant issue is how to design a filter or controller for a given plant such that the filter or controller is insensitive to some amount of errors with respect to its gain, i. e., the designed filter or controller is resilient or non-fragile [2, 7, 11, 17]. On the other hand, for controller or filter design for MJLSs, an important assumption is made that TPs are completely known. In fact, not all the probabilities of the jumps are easy to be measured, and even part of the elements in the desired transition rate matrix is not available. Therefore, it is necessary to study more general MJLSs with partly known TPs [10], namely, TPs are allowed to be known, unknown with known lower and upper bounds and completely unknown. So far, the non-fragile H_∞ filtering problem for continuous MJLSs with partly known TPs under sampled measurements has not yet been fully investigated, which is the motivation of this study.

The non-fragile sampled data H_∞ filtering problem for continuous MJLSs with partly known TPs is investigated in this paper. The filter gain has additive variations and partly known TPs cover the cases that some elements are known, some are uncertain with known lower and upper bounds and some are completely unknown. Attention is focused on designing a non-fragile filter to ensure both the robust stochastic stability and a prescribed level of H_∞ performance for the filtering error dynamics. Combing the TP matrix property with the parameter dependent Lyapunov function approach, sufficient conditions for designing the non-fragile H_∞ filter are established in terms of linear matrix inequalities (LMIs). An example is provided to demonstrate the effectiveness of the proposed approach.

Notation: Throughout this paper, M^T represents the transpose of matrix M. The notation $X \leq Y$ $(X < Y)$ where X and Y are symmetric matrices, means that $X - Y$ is negative semi-definite (negative definite), respectively. I and 0 represent identity matrix and zero matrix, respectively. $\mathcal{L}_2$ denotes the space of square integrable vector functions of a given dimension over $[0, \infty)$, with norm $||x||_2 = \left\{\int_0^\infty x(t)^T x(t)\,\mathrm{d}t\right\}^{\frac{1}{2}}$. $||\ ||_{\mathcal{L}_2}$ stands for $\mathcal{L}_2[0\ \ \infty)$ norm over $[0\ \ \infty)$ and $||\ ||_{\mathcal{E}_2}$ represents the norm in the space $\mathcal{L}_2((\Omega, F, P), [0\ \ \infty))$, $*$ denotes the entries of matrices implied by symmetry. Matrices, if not explicitly stated, are assumed to have appropriate dimensions. Finally, the symbol $He(X)$ is used to represent $(X + X^T)$ and $\mathbb{E}\{\Delta\}$ denotes the expectation operator with respect to some probability measure P;.

2. SYSTEM DESCRIPTION

Consider the following continuous-time MJLSs with sample data measurement

$$\begin{cases} \dot{x}(t) & = A_1(r(t))x(t) + B_1(r(t))w(t), \quad (t \neq kh) \\ x(kh) & = A_2(r(t))x(kh^-) + B_2(r(t))\eta(kh), \quad (k = 0, 1, 2, \ldots) \\ z(t) & = L(r(t))x(t) \end{cases} \tag{1}$$

where $x(t)$ is the state vector, $w(t)$ is the disturbance input which belongs to $L_2[0\ \ \infty)$ and the $z(t)$ is the controlled output. The stochastic process $\{r(t)\}, t \geq 0$ is a continuous time, discrete-state homogenous Markov process, which takes values in a finite set $\mathcal{I} \triangleq \{1, \ldots, N\}$ and has the following mode TPs.

$$\begin{aligned} & Pr\{r(t+\mathrm{d}t) = j | r(t) = i\} \\ & = \begin{cases} \pi_{ij}\mathrm{d}t + o\,(\mathrm{d}t) & i \neq j \\ 1 + \pi_{ii}\mathrm{d}t + o\,(\mathrm{d}t) & i = j \end{cases} \end{aligned} \tag{2}$$

where $\mathrm{d}t > 0$, $\lim_{\mathrm{d}t \to 0} \frac{o(\mathrm{d}t)}{\mathrm{d}t} = 0$. π_{ij} is the jump rate from mode i to mode j that satisfies the following relations:

$$\begin{cases} \pi_{ij} \geq 0 & \forall i \neq j \in \mathcal{I} \\ \sum_{j=1, i \neq j}^{N} \pi_{ij} = -\pi_{ii} & i = (1, \ldots, N). \end{cases} \tag{3}$$

Additionally, the TPs of the jumping process $\{r(k), k \geq 0\}$ in this paper are assumed to be partly unknown, namely, some uncertain elements have known lower and upper bounds, or some have no information available. For instance, for system (1) with four operation modes, the TP matrix may be as:

$$\begin{bmatrix} \pi_{11} & ? & \pi_{13} & ? \\ ? & \pi_{22} & ? & \pi_{24} \\ \alpha & ? & \pi_{33} & ? \\ ? & ? & \beta & ? \end{bmatrix} \tag{4}$$

where "?" represents the unknown elements with no bounds information available and α, β represent the unknown elements with known lower and upper bounds ($\underline{\alpha} \leq \alpha \leq \bar{\alpha}$ and $\underline{\beta} \leq \beta \leq \bar{\beta}$). For the above example, the information of the elements in the TP matrix is composed of the following three sets.

$$\begin{aligned} & \mathcal{R}_k^i \triangleq \{j : \pi_{ij} \text{ is known}\}, \\ & \mathcal{R}_{uk1}^i \triangleq \{j : \text{ the bounds of } \pi_{ij} \text{ is known}\}, \\ & \mathcal{R}_{uk2}^i \triangleq \{j : \text{ there is no information for } \pi_{ij}\}. \end{aligned} \tag{5}$$

Therefore, $\forall i \in \mathcal{I}$, we have $\mathcal{I}^i = \mathcal{R}_k^i \cup \mathcal{R}_{uk1}^i \cup \mathcal{R}_{uk2}^i$. The partly known TPs are further given as

$$\mathcal{I}_k^i = \mathcal{R}_k^i \cup \mathcal{R}_{uk1}^i, \tag{6}$$

$$\mathcal{I}_{uk}^i = \mathcal{R}_{uk2}^i. \tag{7}$$

Now suppose we have

$$y(kh) = C(r(t))x(kh) + D(r(t))\eta(kh) \tag{8}$$

where $y(kh)$ is the sampled measurements, $\eta(kh)$ is the discrete measurement noise which belongs to $\mathcal{L}_2[0\ \ \infty)$, $C(r(t))$ and $D(r(t))$ are known constant matrices.

In this paper, the following mode-dependent full-order non-fragile filter is employed to estimate $z(t)$

$$\begin{cases} \dot{x}_f(t) & =(A_f(r(t))+\triangle A_f(r(t)))x(t) \\ x_f(kh) & =(B_f(r(t))+\triangle B_f(r(t)))x_f(kh^-)+(C_f(r(t))+\triangle C_f(r(t)))y(kh) \\ z_f(t) & =(L_f(r(t))+\triangle L_f(r(t)))x_f(t) \end{cases} \tag{9}$$

where $x_f(t)$ and $z_f(t)$ are the state and output of the filter, respectively. $A_f(r(t))$, $B_f(r(t)), C_f(r(t)) \in R^{n\times q}$ and $L_f(r(t))$ are filter parameters to be determined. $\triangle A_f(r(t))$, $\triangle B_f(r(t))$, $\triangle C_f(r(t))$ and $\triangle L_f(r(t))$ are uncertainties defined as:

$$\begin{cases} \triangle A_f(r(t)) = H_{A_f(r(t))}\triangle_{A_f(r(t))}E_{A_f(r(t))} \\ \triangle B_f(r(t)) = H_{B_f(r(t))}\triangle_{B_f(r(t))}E_{B_f(r(t))} \\ \triangle C_f(r(t)) = H_{C_f(r(t))}\triangle_{C_f(r(t))}E_{C_f(r(t))} \\ \triangle L_f(r(t)) = H_{L_f(r(t))}\triangle_{L_f(r(t))}E_{L_f(r(t))} \end{cases} \tag{10}$$

where $H_{\beta(r(t))}$, $E_{\beta(r(t))}$ $(\beta = A, B, C, L)$ constant matrices with appropriate dimensions and $\triangle_{\beta(r(t))}$ are uncertain matrices bounded such as $\triangle^T_{\beta(r(t))}\triangle_{\beta(r(t))} \leq I$.

For simplicity, we denote the matrices associated with $r(t) = i$ by $A_1(r(t)) = A_{1i}$, $B_1(r(t)) = B_{1i}$, $A_2(r(t)) = A_{2i}$, $B_2(r(t)) = B_{2i}$, $C(r(t)) = C_i$, $D(r(t)) = D_i$, $A_f(r(t)) = A_{fi}$ $B_f(r(t)) = B_{fi}$, $C_f(r(t)) = C_{fi}$, $L_f(r(t)) = L_{fi}$, $\triangle A_f(r(t)) = H_{A_{fi}}\triangle_{A_{fi}}E_{A_{fi}}$, $\triangle B_f(r(t)) = H_{B_{fi}}\triangle_{B_{fi}}E_{B_{fi}}$, $\triangle C_f(r(t)) = H_{C_{fi}}\triangle_{C_{fi}}E_{C_{fi}}$, $\triangle L_f(r(t)) = H_{L_{fi}}\triangle_{L_{fi}}E_{L_{fi}}$.

Defining the augmented state vector $\zeta(t) = [x^T(t) \quad x_f^T(t)]^T$ and $e(t) = z(t) - z_f(t)$, the following filtering error system is obtained.

$$\begin{cases} \dot{\zeta}(t) = \hat{A}_{1i}\zeta(t) + \bar{B}_i w(t) \\ \zeta(kh) = \hat{A}_{2i}\zeta(kh^-) + \hat{B}_{2i}\eta(kh) \\ e(t) = \hat{L}_i\zeta(t) \end{cases} \tag{11}$$

where

$$\hat{A}_{1i} = \bar{A}_{1i} + H_{\bar{A}_{1i}}\Delta_{\bar{A}_{1i}}E_{\bar{A}_{1i}}, \qquad \hat{A}_{2i} = \bar{A}_{2i} + H_{\bar{A}_{2i}}\Delta_{\bar{A}_{2i}}E_{\bar{A}_{2i}},$$
$$\hat{B}_{2i} = \bar{B}_{2i} + H_{\bar{B}_{2i}}\Delta_{\bar{B}_{2i}}E_{\bar{B}_{2i}}, \qquad \hat{L}_i = \bar{L}_i + H_{L_i}\Delta_{L_i}E_{L_i}$$

and

$$\bar{A}_{1i} = \begin{bmatrix} A_i & 0 \\ 0 & A_{fi} \end{bmatrix}, H_{\bar{A}_{1i}} = \begin{bmatrix} 0 \\ H_{A_{fi}} \end{bmatrix}, \Delta_{\bar{A}_{1i}} = \Delta_{A_{fi}}, E_{\bar{A}_{1i}} = \begin{bmatrix} 0 & E_{A_{fi}} \end{bmatrix}, \Delta_{\bar{B}_{2i}} = \Delta_{C_{fi}}$$

$$\bar{A}_{2i} = \begin{bmatrix} A_{2i} & 0 \\ C_{fi}C_i & B_{fi} \end{bmatrix}, H_{\bar{A}_{2i}} = \begin{bmatrix} 0 & 0 \\ H_{C_{fi}} & H_{B_{fi}} \end{bmatrix}, \Delta_{\bar{A}_{2i}} = \begin{bmatrix} \Delta_{C_{fi}} & 0 \\ 0 & \Delta_{B_{fi}} \end{bmatrix}, E_{\bar{B}_{2i}} = E_{C_{fi}}D_i$$

$$E_{\bar{A}_{2i}} = \begin{bmatrix} E_{C_{fi}}C_i & 0 \\ 0 & E_{B_{fi}} \end{bmatrix}, \bar{B}_i = \begin{bmatrix} B_{1i} \\ 0 \end{bmatrix}, H_{\bar{B}_{2i}} = \begin{bmatrix} 0 \\ H_{C_{fi}} \end{bmatrix}, \bar{L}_i = \begin{bmatrix} L_i^T \\ -L_{fi}^T \end{bmatrix}^T,$$

$$H_{\bar{L}_i} = -H_{L_{fi}}, \Delta_{\bar{L}_{fi}} = \Delta_{L_{fi}}, E_{\bar{L}_{fi}} = \begin{bmatrix} 0 & E_{C_{fi}} \end{bmatrix}, \bar{B}_{2i} = \begin{bmatrix} B_{2i} \\ C_{fi}D_i \end{bmatrix}$$

Before formulating the considered problem, the following notations are needed.

Definition 1. (Ji and Chizeck [6]) The nominal system (1) with $w(t) = 0$ and $\eta(kh) = 0$ is said to be stochastically stable (SS) if there exists a finite positive constant $T(x_0, r_0)$, such that the following holds for any initial conditions (x_0, r_0):

$$\mathbb{E}\left\{\int_0^\infty ||x(t)||^2 \,\mathrm{d}t|x_0, r_0\right\} < T(x_0, r_0). \tag{12}$$

The main purpose of the paper is to design H_∞ filter (9) such that the filter error system (11) is stochastically stable and has a prescribed H_∞ noise attenuation level γ, namely, $e(t)$ satisfies

$$||e(t)||_2 \le \gamma \left(||w(t)||^2_{\mathcal{L}_2} + ||\eta(kh)||^2_{\mathcal{L}_2}\right)^{\frac{1}{2}} \tag{13}$$

under zero-initial conditions for non-zero $(w(t), \eta(kh))$, where $\gamma > 0$ is a given scalar.

In order to get the main result of this paper, some useful lemmas are introduced firstly.

Lemma 2.1. (Chang and Yang [2]) From (14), one has (15)

$$\begin{bmatrix} T + He(MA) & * \\ P^T - M^T + GA & He(-G) \end{bmatrix} < 0 \tag{14}$$

$$T + He(PA) < 0. \tag{15}$$

Lemma 2.2. (Cao and Frank [1]) Let V, H, E, Q and $\triangle$ be real matrices with appropriate dimensions and $\triangle^T\triangle \le I$. Then, for any scalar $\epsilon > 0$,

$$(V + H\triangle E)^T Q(V + H\triangle E) \le V^T(Q^{-1} - \epsilon^{-1}HH^T)^{-1}V + \epsilon EE^T. \tag{16}$$

Lemma 2.3. (Shi et al. [19]) Let X, Y and F be real matrices with appropriate dimensions and $F^TF \le I$. Then, for any scalar $\sigma > 0$

$$XFY + Y^TF^TX^T \le \sigma^{-1}XX^T + \sigma Y^TY. \tag{17}$$

3. MAIN RESULTS

In this section, a solution to the filter design problem formulated in the previous section is established by using a LMI approach.

For later discussion, we denote

$$\pi_k^i = -\sum_{j\in\mathcal{I}_k^i, j\neq i} \pi_{ij}, \quad \delta_k^i = -\pi_{ii} - \sum_{j\in\mathcal{I}_k^i, j\neq i} \pi_{ij}, \quad \mathcal{P}_k^i = \sum_{j\in\mathcal{I}_k^i, j\neq i} \pi_{ij}P_j$$

$$\underline{\pi}_k^i = -\sum_{j\in\mathcal{I}_k^i, j\neq i} \underline{\pi}_{ij}, \quad \underline{\delta}_k^i = -\underline{\pi}_{ii} - \sum_{j\in\mathcal{I}_k^i, j\neq i} \underline{\pi}_{ij}, \quad \bar{\mathcal{P}}_k^i = \sum_{j\in\mathcal{I}_k^i, j\neq i} \bar{\pi}_{ij}P_j.$$

The following theorem presents a solution to the non-fragile H_∞ filtering problem for the MJLSs with partly known TPs.

Theorem 3.1. For a positive scalar γ, the filtering error system (11) is stochastic stable with the H_∞ performance index γ, if there exist matrices $G_i = \begin{bmatrix} G_{i1} & G_{i2} & G_{i3} \\ G_{i4} & G_{i2} & G_{i6} \\ G_{i7} & 0 & G_{i8} \end{bmatrix}$, $N_i = \begin{bmatrix} N_{i1} & N_{i2} \\ N_{i3} & N_{i2} \end{bmatrix}$, $P_i = \begin{bmatrix} P_{i11} & P_{i12} \\ * & P_{i22} \end{bmatrix}$, H_{i11}, H_{i12}, H_{i2}, M_{i1}, M_{i3}, M_{i4}, M_{i6}, M_{i7}, M_{i8}, a_{fi}, b_{fi}, c_{fi}, l_{fi}, scalars $\sigma_i > 0$, $\epsilon_i > 0$ $(i \in \mathcal{I})$ such that the following inequalities are satisfied

For $i \in \mathcal{I}_k^i$

$$\begin{bmatrix} \Sigma_{i11}^k & * & * & * & * & * & * \\ \Sigma_{i21} & \Sigma_{i22} & * & * & * & * & \\ \Sigma_{i31} & 0 & -\sigma_i I & * & * & * & * \\ \sigma_i \Sigma_{i41} & 0 & 0 & -\sigma_i I & * & * & * \\ \Sigma_{i51} & 0 & 0 & 0 & -I & * & * \\ \epsilon_i \Sigma_{i61} & 0 & 0 & 0 & 0 & -\epsilon_i I & * \\ 0 & 0 & 0 & 0 & \Sigma_{i76} & 0 & -\epsilon_i I \end{bmatrix} < 0 (l \in \mathcal{I}_{uk}^i). \tag{18}$$

For $i \in \mathcal{I}_{uk}^i$

$$\begin{cases} \begin{bmatrix} \Sigma_{i11}^k & * & * & * & * & * & * \\ \Sigma_{i21} & \Sigma_{i22} & * & * & * & * & \\ \Sigma_{i31} & 0 & -\sigma_i I & * & * & * & * \\ \sigma_i \Sigma_{i41} & 0 & 0 & -\sigma_i I & * & * & * \\ \Sigma_{i51} & 0 & 0 & 0 & -I & * & * \\ \epsilon_i \Sigma_{i61} & 0 & 0 & 0 & 0 & -\epsilon_i I & * \\ 0 & 0 & 0 & 0 & \Sigma_{i76} & 0 & -\epsilon_i I \end{bmatrix} < 0 \\ P_l \le P_i \end{cases} (l \in \mathcal{I}_{uk}^i) \tag{19}$$

$$\begin{bmatrix} -P_i & * & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * & * \\ \Phi_{i1} & \Phi_{i2} & \Phi_{i3} & * & * & * \\ 0 & 0 & \Phi_{i4}^T & -\xi_i I & * & * \\ 0 & 0 & \Phi_{i5}^T & 0 & -\xi_i I & * \\ \xi_i \Phi_{i6} & 0 & 0 & 0 & 0 & -\xi_i I \end{bmatrix} < 0 \tag{20}$$

$$P_i = \begin{bmatrix} P_{i11} & * \\ P_{i12}^T & P_{i22} \end{bmatrix} > 0 \tag{21}$$

where

$$\Sigma_{i11}^k = \begin{bmatrix} \bar{\Sigma}_{i11}^k & * & * \\ \bar{\Sigma}_{i21}^k & \bar{\Sigma}_{i22}^k & * \\ \bar{\Sigma}_{i31}^k & \bar{\Sigma}_{i32}^k & \bar{\Sigma}_{i33}^k \end{bmatrix}, \quad \Sigma_{i21} = \begin{bmatrix} \Lambda_{i11} & \Lambda_{i12} & \Lambda_{i13} \\ \Lambda_{i21} & \Lambda_{i22} & \Lambda_{i23} \\ \Lambda_{i31} & \Lambda_{i32} & \Lambda_{i33} \end{bmatrix}, \Sigma_{i22} = \begin{bmatrix} \bar{\Lambda}_{i11} & * & * \\ \bar{\Lambda}_{i21} & \bar{\Lambda}_{i22} & * \\ \bar{\Lambda}_{i31} & \bar{\Lambda}_{i32} & \bar{\Lambda}_{i33} \end{bmatrix}$$

$$\bar{\Sigma}_{i11}^k = \begin{cases} He\left(M_{i1}A_i\right) + \bar{\mathcal{P}}_{i11}^k + \bar{\pi}_{ii}P_{i11} + \underline{\delta}_k^i P_{l11}(i \in \mathcal{I}_k^i) \\ He\left(M_{i1}A_i\right) + \bar{\mathcal{P}}_{i11}^k + \underline{\pi}_k^i P_{i11}(i \in \mathcal{I}_{uk}^i) \end{cases}$$

$$\bar{\Sigma}_{i21}^k = \begin{cases} M_{i4}A_i + a_{fi}^T + (\bar{\mathcal{P}}_{i12}^k)^T + \bar{\pi}_{ii}P_{i12}^T + \underline{\delta}_k^i P_{l12}^T(i \in \mathcal{I}_k^i) \\ M_{i4}A_i + a_{fi}^T + (\bar{\mathcal{P}}_{i12}^k)^T + \underline{\pi}_k^i P_{i12}^T(i \in \mathcal{I}_{uk}^i) \end{cases}$$

$$\bar{\Sigma}_{i22}^k = \begin{cases} He(a_{fi}) + \bar{\mathcal{P}}_{i22}^k + \bar{\pi}_{ii}P_{i22} + \underline{\delta}_k^i P_{l22}(i \in \mathcal{I}_k^i) \\ He(a_{fi}) + \bar{\mathcal{P}}_{i22}^k + \underline{\pi}_k^i P_{i22}(i \in \mathcal{I}_{uk}^i) \end{cases},$$

$$\bar{\Sigma}_{i31}^k = M_{i7}A_i + M_{i1}B_{1i}, \bar{\Sigma}_{i32}^k = \left(M_{i4}B_{1i}\right)^T, \bar{\Sigma}_{i33}^k = He\left(M_{i7}B_{1i}\right) - \gamma^2 I,$$

$$\Lambda_{i11} = P_{i11} - M_{i1}^T + G_{i1}A_i, \Lambda_{i12} = P_{i12} - M_{i4}^T + a_{fi}, \quad \Lambda_{i13} = -M_{i7}^T + G_{i1}B_{1i}$$

$$\Lambda_{i21} = P_{i12}^T - G_{i2}^T + G_{i4}A_i, \quad \Lambda_{i22} = P_{i22} - G_{i2}^T + a_{fi}, \Lambda_{i23} = G_{i4}B_i,$$

$$\Lambda_{i31} = H_{i11}^T - M_{i3}^T + G_{i7}A_i, \Lambda_{i32} = H_{i12}^T - M_{i6}^T, \quad \Lambda_{i33} = H_{i2}^T - M_{i8}^T + G_{i7}B_{1i}$$

$$\bar{\Lambda}_{i11} = He(-G_{i1}), \bar{\Lambda}_{i21} = -G_{i4} - G_{i2}^T, \bar{\Lambda}_{i22} = He(-G_{i2}), \bar{\Lambda}_{i31} = -G_{i7} - G_{i3}^T,$$

$$\bar{\Lambda}_{i32} = -G_{i6}^T, \bar{\Lambda}_{i33} = He(-G_{i8}), \Sigma_{i31} = \begin{bmatrix} (P_{i12}H_{A_{fi}})^T & (P_{i22}H_{A_{fi}})^T & 0 \end{bmatrix},$$

$$\Sigma_{i41} = \begin{bmatrix} 0 & E_{A_{fi}} & 0 \end{bmatrix}, \Sigma_{i51} = \begin{bmatrix} L_i & -L_{fi} & 0 \end{bmatrix}, \Sigma_{i61} = \epsilon_i \begin{bmatrix} 0 & E_{L_{fi}} & 0 \end{bmatrix}, \Sigma_{i76} = -H_{L_{fi}}^T$$

$$\Phi_{i1} = \begin{bmatrix} N_{i1}A_{2i} + c_{fi}C_i & b_{fi} \\ N_{i3}A_{2i} + c_{fi}C_i & b_{fi} \end{bmatrix}, \Phi_{i2} = \begin{bmatrix} N_{i1}B_{2i} + c_{fi}D_i \\ N_{i3}B_{2i} + c_{fi}D_i \end{bmatrix}, \Phi_{i5} = \begin{bmatrix} N_{i2}H_{C_{fi}} \\ N_{i2}H_{C_{fi}} \end{bmatrix}$$

$$\Phi_{i4} = \begin{bmatrix} N_{i2}H_{C_{fi}} & N_{i2}H_{B_{fi}} \\ N_{i2}H_{C_{fi}} & N_{i2}H_{B_{fi}} \end{bmatrix}, \Phi_{i3} = \begin{bmatrix} He(-N_{i1}) + P_{i11} & * \\ -N_{i3} - N_{i2}^T + P_{i12}^T & He(-N_{i2}) + P_{i22} \end{bmatrix}$$

$$\Phi_{i6} = \begin{bmatrix} E_{\bar{A}_{2i}} & 0 & 0 \\ 0 & E_{\bar{B}_{2i}} & 0 \end{bmatrix}.$$

Moreover, the desired H_∞ filter is given in the form of (9) with parameters as follows:

$$A_{fi} = G_{i2}^{-1}a_{fi}, B_{fi} = N_{i2}^{-1}b_{fi}, C_{fi} = N_{i2}^{-1}c_{fi}, L_{fi} = l_{fi}. \tag{22}$$

Consider the following Lyapunov function:

$$V(\zeta(t), i) = \zeta(t)^T P_i \zeta(t). \tag{23}$$

Then,

$$\mathbb{E}\left(\dot{V}(\zeta(t),i)\right) = \dot{\zeta}(t)^T P_i \zeta(t) + \zeta(t)^T P_i \dot{\zeta}(t) + \zeta(t)^T \sum_{j=1}^{N} \pi_{ij} P_j \zeta(t). \tag{24}$$

Moreover

$$\begin{aligned} &\mathbb{E}\left(\dot{V}(\zeta(t),i) + e^T(t)e(t) - \gamma^2 w^T(t)w(t)\right) \\ =&(\hat{A}_{1i}\zeta(t) + \hat{B}_i w(t))^T P_i \zeta(t) + \zeta(t)^T P_i (\hat{A}_i \zeta(t) + \hat{B}_i w(t)) \\ &+\zeta(t)^T \sum_{j=1}^{N} \pi_{ij} P_j \zeta(t) + (\hat{L}_i \zeta(t))^T (\hat{L}_i \zeta(t)) - \gamma^2 w^T(t) w(t). \end{aligned} \tag{25}$$

Taking $\xi(t) = \begin{bmatrix}\zeta(t)^T & w(t)^T\end{bmatrix}^T$, then (25) is rewritten as

$$\begin{aligned} &\mathbb{E}\left(\dot{V}(\zeta(t),i) + e^T(t)e(t) - \gamma^2 w^T(t)w(t)\right) \\ &= \xi^T(t)\left\{He\left(\begin{bmatrix}P_i & H_{i1}\\ 0 & H_{i2}\end{bmatrix}\begin{bmatrix}\hat{A}_{1i} & \bar{B}_i\\ 0 & 0\end{bmatrix}\right) + \begin{bmatrix}\sum\limits_{j=1}^{N}\pi_{ij}P_j & 0\\ 0 & -\gamma^2 I\end{bmatrix} + \begin{bmatrix}\hat{L}_i^T\\ 0\end{bmatrix}\begin{bmatrix}\hat{L}_i^T\\ 0\end{bmatrix}^T\right\}\xi(t) \\ &= \xi^T(t)\left\{He\left(\mathscr{P}_i\left(\mathscr{A}_i + H_{\mathscr{A}_i}\triangle_{\mathscr{A}_i}E_{\mathscr{A}_i}\right)\right) + \Xi_i + \left(\mathscr{C}_i + H_{\mathscr{C}_i}\triangle_{\mathscr{C}_i}E_{\mathscr{C}_i}\right)^T\left(\mathscr{C}_i + H_{\mathscr{C}_i}\triangle_{\mathscr{C}_i}E_{\mathscr{C}_i}\right)\right\}\xi(t) \end{aligned} \tag{26}$$

where

$$\begin{aligned} &\mathcal{P}_i = \begin{bmatrix}P_i & H_{i1}\\ 0 & H_{i2}\end{bmatrix}, \mathscr{A}_i = \begin{bmatrix}\bar{A}_{1i} & \bar{B}_i\\ 0 & 0\end{bmatrix}, \Xi_i = \begin{bmatrix}\sum\limits_{j=1}^{N}\pi_{ij}P_j & 0\\ 0 & -\gamma^2 I\end{bmatrix}, H_{\mathscr{A}_i} = \begin{bmatrix}H_{\bar{A}_{1i}}\\ 0\end{bmatrix}, \\ &\triangle_{\mathscr{A}_i} = \triangle_{\bar{A}_i}, E_{\mathscr{A}_i} = \begin{bmatrix}E_{\bar{A}_i} & 0\end{bmatrix}, \mathscr{L}_i = \begin{bmatrix}\bar{L}_i & 0\end{bmatrix}, H_{\mathscr{L}_i} = H_{\bar{D}_i}, \\ &\triangle_{\mathscr{L}_i} = \begin{bmatrix}\triangle_{\bar{L}_i} & 0\end{bmatrix}, E_{\mathscr{L}_i} = \begin{bmatrix}E_{\bar{C}_i} & 0\end{bmatrix}. \end{aligned}$$

Therefore, the condition for $E\left(\dot{V}(\zeta(t),i) + e^T(t)e(t) - \gamma^2 w^T(t)w(t)\right) < 0$ can be tested by the following inequality

$$\Pi_i = He\left(\mathcal{P}_i\left(\mathscr{A}_i + H_{\mathscr{A}_i}\triangle_{\mathscr{A}_i}E_{\mathscr{A}_i}\right)\right) + \Xi_i + \left(\mathscr{C}_i + H_{\mathscr{C}_i}\triangle_{\mathscr{C}_i}E_{\mathscr{C}_i}\right)^T\left(\mathscr{C}_i + H_{\mathscr{C}_i}\triangle_{\mathscr{C}_i}E_{\mathscr{C}_i}\right) < 0 \tag{27}$$

By Lemma 2.3 and Lemma 2.2,

$$\Pi_i \leq He(\mathcal{P}_i\mathscr{A}_i) + \Xi_i + \sigma_i^{-1}\mathcal{P}_iH_{\mathscr{A}_i}(P_iH_{\mathscr{A}_i})^T + \sigma_iE_{\mathscr{A}_i}^TE_{\mathscr{A}_i} \\ +\mathscr{C}_i^T(I-\epsilon_i^{-1}H_{\mathscr{C}_i}H_{\mathscr{C}_i}^T)^{-1}\mathscr{C}_i + \epsilon_iE_{\mathscr{C}_i}E_{\mathscr{C}_i}^T. \tag{28}$$

The condition for $E\left(\dot{V}(\zeta(t),i) + e^T(t)e(t) - \gamma^2w^T(t)w(t)\right) < 0$ is further converted to

$$He(\mathcal{P}_i\mathscr{A}_i) + \Xi_i + \sigma_i^{-1}\mathcal{P}_iH_{\mathscr{A}_i}(P_iH_{\mathscr{A}_i})^T + \sigma_iE_{\mathscr{A}_i}^TE_{\mathscr{A}_i} \\ +\mathscr{C}_i^T(I-\epsilon_i^{-1}H_{\mathscr{C}_i}H_{\mathscr{C}_i}^T)^{-1}\mathscr{C}_i + \epsilon_iE_{\mathscr{C}_i}E_{\mathscr{C}_i}^T \leq 0. \tag{29}$$

Because the TPs are partly known, Ξ_i is further rewritten as

$$\Xi_i = \Xi_i^k + \pi_{ii}\Xi_i^i + \sum_{l\in\mathcal{I}_{uk}^i}\pi_{il}\Xi_i^{uk} \tag{30}$$

where

$$\Xi_i^k = \begin{bmatrix} \sum\limits_{j\in\mathcal{I}_k^i, j\neq i}\pi_{ij}P_j & * \\ 0 & -\gamma^2I \end{bmatrix}, \Xi_i^i = \begin{bmatrix} P_i & 0 \\ 0 & 0 \end{bmatrix}, \Xi_i^{uk} = \begin{bmatrix} P_l & 0 \\ 0 & 0 \end{bmatrix}. \tag{31}$$

Recalling the fact that $\pi_{ii} \leq 0$, two cases given below are considered.

Case I, $i \in \mathcal{I}_k^i$, that is, π_{ii} is known.

From $\pi_{ii} + \sum_{j\in\mathcal{I}_k^i, j\neq i}\pi_{ij} + \sum_{l\in\mathcal{I}_{uk}^i, l\neq i}\pi_{il} = 0$, one has $\theta = \frac{\sum_{l\in\mathcal{I}_{uk}^i, l\neq i}\pi_{il}}{-\pi_{ii}-\sum_{j\in\mathcal{I}_k^i, j\neq i}\pi_{ij}} = 1$. By taking $1 = \theta$ into (29), it is equivalent to

$$\begin{aligned} &He(\mathcal{P}_i\mathscr{A}_i) + \Xi_i + \sigma_i^{-1}\mathcal{P}_iH_{\mathscr{A}_i}(P_iH_{\mathscr{A}_i})^T + \sigma_iE_{\mathscr{A}_i}E_{\mathscr{A}_i}^T + \mathscr{C}_i^T(I-\epsilon_i^{-1}H_{\mathscr{C}_i}H_{\mathscr{C}_i}^T)^{-1}\mathscr{C}_i + \epsilon_iE_{\mathscr{C}_i}E_{\mathscr{C}_i}^T \\ =&\theta(He(\mathcal{P}_i\mathscr{A}_i) + \Xi_i^k + \pi_{ii}\Xi_i^i + \sum_{l\in\mathcal{I}_{uk}^i}\pi_{il}\Xi_i^{uk} + \sigma_i^{-1}\mathcal{P}_iH_{\mathscr{A}_i}(P_iH_{\mathscr{A}_i})^T + \sigma_iE_{\mathscr{A}_i}E_{\mathscr{A}_i}^T \\ &+\mathscr{C}_i^T(I-\epsilon_i^{-1}H_{\mathscr{C}_i}H_{\mathscr{C}_i}^T)^{-1}\mathscr{C}_i + \epsilon_iE_{\mathscr{C}_i}E_{\mathscr{C}_i}^T) \\ =&\frac{\sum\limits_{l\in\mathcal{I}_{uk}^i, l\neq i}\pi_{il}}{-\pi_{ii}-\sum\limits_{j\in\mathcal{I}_k^i, j\neq i}\pi_{ij}}(He(\mathcal{P}_i\mathscr{A}_i)+\Xi_i^k+\pi_{ii}\Xi_i^i+(-\pi_{ii}-\sum_{j\in\mathcal{I}_k^i, j\neq i}\pi_{ij})\Xi_i^{uk}+\sigma_i^{-1}\mathcal{P}_iH_{\mathscr{A}_i}(P_iH_{\mathscr{A}_i})^T \\ &+\sigma_iE_{\mathscr{A}_i}E_{\mathscr{A}_i}^T + \mathscr{C}_i^T(I-\epsilon_i^{-1}H_{\mathscr{C}_i}H_{\mathscr{C}_i}^T)^{-1}\mathscr{C}_i + \epsilon_iE_{\mathscr{C}_i}E_{\mathscr{C}_i}^T). \end{aligned} \tag{32}$$

By Lemma 2.1, (29) holds if the following inequality is establised

$$\begin{bmatrix} \Upsilon_i & * \\ \mathcal{P}_i - \mathcal{M}_i^T + \mathcal{G}_i\mathscr{A}_i & He(-\mathcal{G}_i) \end{bmatrix} < 0 \tag{33}$$

where

$$\begin{aligned} \Upsilon_i =& He(\mathcal{M}_i\mathscr{A}_i) + \Xi_i^k + \pi_{ii}\Xi_i^i + (-\pi_{ii} - \sum_{j\in\mathcal{I}_k^i}\pi_{ij})\Xi_i^{uk} + \sigma_i^{-1}\mathcal{P}_iH_{\mathscr{A}_i}(P_iH_{\mathscr{A}_i})^T \\ &+ \sigma_iE_{\mathscr{A}_i}^TE_{\mathscr{A}_i} + \mathscr{C}_i^T(I-\epsilon_i^{-1}H_{\mathscr{C}_i}H_{\mathscr{C}_i}^T)^{-1}\mathscr{C}_i + \epsilon_iE_{\mathscr{C}_i}E_{\mathscr{C}_i}^T. \end{aligned}$$

By Schur complement, one has

$$\begin{bmatrix} \psi & * & * & * & * & * & * \\ \mathcal{P}_i^T - \mathcal{M}_i^T + \mathcal{G}_i\mathscr{A}_i & He(-\mathcal{G}_i) & * & * & * & * & * \\ (\mathcal{P}_iH_{\mathscr{A}_i})^T & 0 & -\sigma_iI & * & * & * & * \\ \sigma_iE_{\mathscr{A}_i} & 0 & 0 & -\sigma_iI & * & * & * \\ \mathscr{C}_i & 0 & 0 & 0 & -I & * & * \\ \epsilon_iE_{\mathscr{C}_i}^T & 0 & 0 & 0 & 0 & -\epsilon_iI & * \\ 0 & 0 & 0 & 0 & 0 & H_{\mathscr{C}_i}^T & -\epsilon_iI \end{bmatrix} < 0 \tag{34}$$

where

$$\psi = He(\mathcal{M}_i\mathscr{A}_i) + \Xi_i^k + \pi_{ii}\Xi_i^i + (-\pi_{ii} - \sum_{j\in\mathcal{I}_k^i}\pi_{ij})\Xi_i^{uk}.$$

Case II, $i \in \mathcal{I}_{uk}^i$, that is, π_{ii} is completely unknown.

Considering $\pi_{ii} = -\sum_{j\in\mathcal{I}_k^i}\pi_{ij} - \sum_{j\in\mathcal{I}_{uk}^i}\pi_{ij}$, then (29) is rewritten as

$$\begin{aligned} &He(\mathcal{P}_i\mathscr{A}_i) + \Xi_i + \sigma_i^{-1}\mathcal{P}_iH_{\mathscr{A}_i}(P_iH_{\mathscr{A}_i})^T + \sigma_iE_{\mathscr{A}_i}E_{\mathscr{A}_i}^T + \mathscr{C}_i^T(I-\epsilon_i^{-1}H_{\mathscr{C}_i}H_{\mathscr{C}_i}^T)^{-1}\mathscr{C}_i + \epsilon_iE_{\mathscr{C}_i}E_{\mathscr{C}_i}^T \\ =& He(\mathcal{P}_i\mathscr{A}_i) + \Xi_i^k - \sum_{j\in\mathcal{I}_k^i, j\neq i}\pi_{ij}\Xi_i^i + \sigma_i^{-1}\mathcal{P}_iH_{\mathscr{A}_i}(P_iH_{\mathscr{A}_i})^T + \sigma_iE_{\mathscr{A}_i}E_{\mathscr{A}_i}^T \\ &+ \mathscr{C}_i^T(I-\epsilon_i^{-1}H_{\mathscr{C}_i}H_{\mathscr{C}_i}^T)^{-1}\mathscr{C}_i + \epsilon_iE_{\mathscr{C}_i}E_{\mathscr{C}_i}^T + \sum_{l\in\mathcal{I}_{uk}^i}\pi_{il}(\Xi_i^{uk} - \Xi_i^i). \end{aligned} \tag{35}$$

Taking the similar lines as case I, (29) holds if the following inequalities are satisfied

$$\begin{cases} \begin{bmatrix} He(\mathcal{M}_i\mathscr{A}_i)+\Xi_i^k-\sum\limits_{j\in\mathcal{I}_k^i,j\neq i}\pi_{ij}\Xi_i^i & * & * & * & * & * & * \\ \mathcal{P}_i-\mathcal{M}_i^T+\mathcal{G}_i\mathscr{A}_i & He(-\mathcal{G}_i) & * & * & * & * & * \\ (\mathcal{P}_iH_{\mathscr{A}_i})^T & 0 & -\sigma_iI & * & * & * & * \\ \sigma_iE_{\mathscr{A}_i}^T & 0 & 0 & -\sigma_iI & * & * & * \\ \mathscr{C}_i & 0 & 0 & 0 & -I & * & * \\ \epsilon_iE_{\mathscr{C}_i}^T & 0 & 0 & 0 & 0 & -\epsilon_iI & * \\ 0 & 0 & 0 & 0 & 0 & H_{\mathscr{C}_i}^T & -\epsilon_iI \end{bmatrix}<0 \\ P_l=\begin{bmatrix} P_{l11} & P_{l12} \\ * & P_{l22}\end{bmatrix}\leq P_i=\begin{bmatrix} P_{i11} & P_{i12} \\ * & P_{i22}\end{bmatrix}. \end{cases} \tag{36}$$

By taking P_i, $\mathcal{P}_i$, $\mathcal{M}_i$ and $\mathcal{G}_i$ as the following structure,

$$P_i=\begin{bmatrix} P_{i11} & P_{i12} \\ * & P_{i22}\end{bmatrix}>0, \mathcal{P}_i=\begin{bmatrix} P_{i11} & P_{i12} & H_{i11} \\ * & P_{i22} & H_{i12} \\ 0 & 0 & H_{i2}\end{bmatrix}$$
$$\mathcal{M}_i=\begin{bmatrix} M_{i1} & G_{i2} & M_{i3} \\ M_{i4} & G_{i2} & M_{i6} \\ M_{i7} & 0 & M_{i8}\end{bmatrix}, \mathcal{G}_i=\begin{bmatrix} G_{i1} & G_{i2} & G_{i3} \\ G_{i4} & G_{i2} & G_{i6} \\ G_{i7} & 0 & G_{i8}\end{bmatrix}. \tag{37}$$

From (37), we obtain (18) and (19) with $a_{fi}=G_{i2}A_{f_i}$, $b_{fi}=G_{i2}B_{f_i}$, $c_{fi}=C_{f_i}$, $d_{fi}=D_{f_i}$. Then we get

$$\mathbb{E}\left\{\dot{V}(\zeta(t),i)+e^T(t)e(t)-\gamma^2w^T(t)w(t)\right\}<0. \tag{38}$$

Integrating both sides of (38) from kh to t ($t\in[ih\ \ (i+1)h)$), it follows

$$\mathbb{E}\left\{\int_{kh}^t\left(e^T(s)e(s)-\gamma^2w(s)w(s)\right)\mathrm{d}s\right\}<\mathbb{E}\left\{V(\zeta(kh),i)\right\}. \tag{39}$$

On the other hand, applying the Schur complement to (20) and considering the second equation in (1), we get

$$-\gamma^2\eta^T(kh)\eta(kh)\leq\mathbb{E}\left\{V(\zeta(kh^-),i)\right\}-\mathbb{E}\left\{V(\zeta(kh),i)\right\}. \tag{40}$$

Noting the zero initial conditions and combing (39) and (40), for all $kh\in[0,\ \ t]$, the following formulation is obtained.

$$\mathbb{E}\left\{\int_0^t e^T(s)e(s)\,\mathrm{d}s - \gamma^2\int_0^t w(s)w(s)\,\mathrm{d}s\right\} - \gamma^2 \sum_{kh\in(0,t)} \eta^T(kh)\eta(kh) < 0 \tag{41}$$

which implies $||e(t)||_{E_2} \le \gamma\,(||w(t)||_{\mathcal{L}_2} + ||\eta(kh)||_{\mathcal{L}_2})^{\frac{1}{2}}$.

Remark 3.2. According to Theorem 3.1, if the filter variations satisfy the assumption, the designed filter can guarantee the stochastical stability of the filtering error system and the prescribed H_∞ performance level. In this case, the solution is easy to work out. The reason is that the conditions proposed in Theorem 1 are expressed in terms of linear matrix inequalities (LMIs), which can be effectively solved by Matlab LMI toolbox, although there are several scalars and matrices to be determined.

Remark 3.3. It is noted that the conditions in Theorem 3.1 are in the framework of LMIs in (P_i, H_{i11}, H_{i12}, H_{i2}, M_{i1}, M_{i3}, M_{i4}, M_{i6}, M_{i7}, M_{i8}, G_i, N_i, a_{fi}, b_{fi}, c_{fi}, d_{fi}, scalars $\sigma_i > 0$, $\epsilon_i > 0$ and γ^2), hence, a minimum γ^2 using convex optimization algorithms to obtain the minimum noise-attention level bound can be obtained. Then, the problem of optimal non-fragile H_∞ filter design is converted to the following optimization problem:

$$\min_{P_i,H_{i11},H_{i12},H_{i2},M_{i1},M_{i3},M_{i4},M_{i6},M_{i7},M_{i8},G_i,N_i,a_{fi},b_{fi},c_{fi},d_{fi},\sigma_i,\epsilon_i} \delta \quad \text{s.t. } (18) -- (21) \tag{42}$$

where γ^2 is replaced with δ. The minimum noise-attention level bound is given by $\gamma = \sqrt{\delta^*}$, where δ^* is the optimal value of δ. Meanwhile, the optimal filter parameters are given by (22).

4. NUMERICAL EXAMPLE

In this section, a numerical example is given to show the effectiveness of the proposed method given in Theorem 3.1.

Consider the MJLS (1) with four operating modes described by

$$A_{11} = \begin{bmatrix} -0.5 & 0.25 \\ 0.2 & -0.5 \end{bmatrix}, A_{12} = \begin{bmatrix} -0.4 & -0.2 \\ -0.7 & -0.8 \end{bmatrix}, A_{13} = \begin{bmatrix} -0.4 & 0.2 \\ -0.2 & -0.5 \end{bmatrix}, A_{14} = \begin{bmatrix} 0 & 0.2 \\ -0.2 & -0.5 \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} 0.15 & 0 \\ 0 & 0.15 \end{bmatrix}, A_{22} = \begin{bmatrix} 0.12 & 0 \\ 0 & 0.12 \end{bmatrix}, A_{23} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, A_{24} = \begin{bmatrix} 0.11 & 0 \\ 0 & 0.11 \end{bmatrix},$$
$$B_{11} = \begin{bmatrix} 0 & 0.5 \end{bmatrix}^T, B_{12} = \begin{bmatrix} 0 & 0.3 \end{bmatrix}^T, B_{13} = \begin{bmatrix} -0.2 & -0.5 \end{bmatrix}^T, B_{14} = \begin{bmatrix} 0.6 & -0.3 \end{bmatrix}^T,$$
$$B_{21} = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix}^T, B_{22} = \begin{bmatrix} 0.3 & 0.3 \end{bmatrix}^T, B_{23} = \begin{bmatrix} 0.4 & 0.4 \end{bmatrix}^T, B_{24} = \begin{bmatrix} 0.25 & 0.25 \end{bmatrix}^T,$$
$$L_1 = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, L_2 = \begin{bmatrix} 0.1 & 0.5 \end{bmatrix}, L_3 = \begin{bmatrix} 0.2 & 0 \end{bmatrix}, L_4 = \begin{bmatrix} -0.8 & -0.2 \end{bmatrix},$$

$$C_1 = \begin{bmatrix}0.1 & -0.1\end{bmatrix}, C_2 = \begin{bmatrix}0.1 & -0.4\end{bmatrix}, C_3 = \begin{bmatrix}0.1 & 0\end{bmatrix}, C_4 = \begin{bmatrix}0.1 & 0.3\end{bmatrix},$$
$$H_{A_{f1}} = H_{A_{f2}} = H_{A_{f3}} = H_{A_{f4}} = H_{B_{f1}} = H_{B_{f2}} = H_{B_{f3}} = H_{B_{f4}} = \begin{bmatrix}-0.2 & 0.3\end{bmatrix}^T,$$
$$E_{A_{f1}} = E_{A_{f2}} = E_{A_{f3}} = E_{A_{f4}} = E_{B_{f1}} = E_{B_{f2}} = E_{B_{f3}} = H_{B_{f4}} = \begin{bmatrix}-0.2 & 0.3\end{bmatrix}^T,$$
$$H_{C_{f1}} = H_{C_{f2}} = H_{C_{f3}} = H_{C_{f4}} = \begin{bmatrix}0.3 & -0.3\end{bmatrix}^T, E_{C_{f1}} = E_{C_{f2}} = E_{C_{f3}} = E_{C_{f4}} = 0.1,$$
$$H_{L_{f1}} = H_{L_{f2}} = H_{L_{f3}} = H_{L_{f4}} = 0.1, E_{L_{f1}} = E_{L_{f2}} = E_{L_{f3}} = E_{L_{f4}} = 0.5.$$
$$D_1 = D_2 = D_3 = D_4 = 0.1.$$

Our purpose here is to design a mode-dependent full-order non-fragile H_∞ filter in the form of (9) such that the resulting filtering error system (11) is stochastic stable and has a guaranteed H_∞ performance. The partly known transition matrix is given as follows.

$$\begin{bmatrix}-1.4 & 0.2 & ? & ? \\ ? & ? & 0.3 & 0.3 \\ 0.2 & ? & -0.8 & ? \\ ? & ? & ? & -0.8\end{bmatrix} \tag{43}$$

By solving the optimal problem (42), the minimum H_∞ performance index $\gamma = 1.6287$ is obtained, that is, when the filter has gain variations, the H_∞ performance $\gamma = 1.6287$ is always guaranteed for any uncertainties satisfying $\triangle^T_{\beta(r(t))}\triangle_{\beta(r(t))} \le I$. Moreover, the filter parameters are given as follows

$$A_{f1} = \begin{bmatrix}-0.5591 & 0.4964 \\ 0.3005 & -1.7639\end{bmatrix} \quad B_{f1} = \begin{bmatrix}0.0266 & 0.0167 \\ 0.0291 & 0.0161\end{bmatrix} \quad C_{f1} = \begin{bmatrix}-0.6369 \\ 0.6297\end{bmatrix}$$
$$A_{f2} = \begin{bmatrix}-0.4722 & 0.7967 \\ -0.6076 & -1.9095\end{bmatrix} \quad B_{f2} = \begin{bmatrix}0.0333 & 0.0211 \\ 0.0145 & 0.0067\end{bmatrix}^T \quad C_{f2} = \begin{bmatrix}-0.1834 \\ 0.1851\end{bmatrix}$$
$$A_{f3} = \begin{bmatrix}-0.4997 & 0.6202 \\ 0.1246 & -2.0370\end{bmatrix} \quad B_{f3} = \begin{bmatrix}-0.0159 & -0.0218 \\ 0.0018 & -0.0195\end{bmatrix} \quad C_{f3} = \begin{bmatrix}-1.7840 \\ -0.5655\end{bmatrix}$$
$$A_{f4} = \begin{bmatrix}-1.0193 & 0.6845 \\ 1.0055 & -1.2341\end{bmatrix} \quad B_{f4} = \begin{bmatrix}0.0086 & 0.0162 \\ -0.0106 & -0.0058\end{bmatrix} \quad C_{f4} = \begin{bmatrix}0.0295 \\ -0.4615\end{bmatrix}$$
$$L_{f1} = \begin{bmatrix}-0.1083 & -0.1892\end{bmatrix} \quad L_{f2} = \begin{bmatrix}0.1707 & 0.1043\end{bmatrix}$$
$$L_{f3} = \begin{bmatrix}-0.1059 & -0.1871\end{bmatrix} \quad L_{f4} = \begin{bmatrix}-0.1060 & -0.1887\end{bmatrix}.$$

Based on the obtained filter parameters, with one possible system mode (Figure 1), the simulation curves of the filter error ($||e(t)||_2$) and disturbances ($d(t) = \left(||w(t)||^2_{\mathcal{L}_2} + ||\eta(kh)||^2_{\mathcal{L}_2}\right)^{\frac{1}{2}}$) are given in Figure 2, under the initial condition $x(0) = x_f(0) = \begin{bmatrix}0 & 0\end{bmatrix}^T$.

According to Figure 2, it is shown that the H_∞ norm is indeed lower than γ.

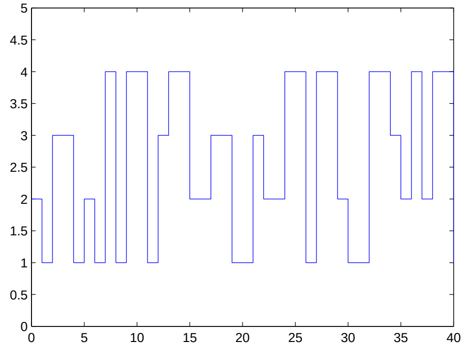

Fig. 1. One possible system mode.

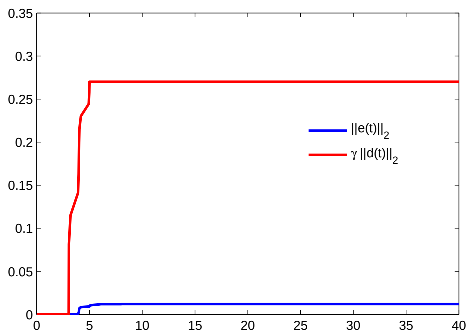

Fig. 2. Curves of $||e(t)||_2$ and $\gamma||d(t)||_2$.

5. CONCLUSION

The non-fragile sampled data H_∞ filtering problem for continuous MJLSs with partly known TPs is discussed in this paper. The filter gain to be designed is assumed to have additive gain variations and the TPs are assumed to be known, uncertain with known bounds and completely unknown. By using the parameter dependent Lyapunov function approach, sufficient conditions for the desired filter design are established in terms of solutions to a set of LMIs, which guarantees the filtering error system to be stochastic stable and has a prescribed H_∞ disturbance attenuation performance. A numerical example has been given to show the effectiveness of the proposed method.

ACKNOWLEDGEMENT

The authors would like to thank the Editor, the Associate Editor and the Reviewers for their valuable comments and suggestions which help to significantly improve the quality and presentation of this paper. This work was supported by the Doctoral Fundation of Ministry of Education of China (No. 20133221120012), the Natural Science Foundation of Jiangsu Province of China (No. BK20130949), the Natural Science Foundation of Jiangsu Provincial Universities of China (No.13KJB120004), 2012 Qing Lan Project of Jiangsu Province, the National Natural Science Foundation of China (Nos. 51205185, 51307080, 61273155, 61322312, 61403189), New Century Excellent Talents in University (No. NCET-11-0083), and Jiangsu Postdoctoral Science Foundation (1401015B).

(Received April 21, 2013)

REFERENCES

[1] Y. Cao and M. Frank: Robust H_∞ disturbance attenuation for a class of uncertain discrete-time fuzzy systems. IEEE Trans. Fuzzy Systems *8* (2000), 406–415.

[2] X. Chang and G. Yang: Nonfragile H_∞ filtering of continuous-time fuzzy systems. IEEE Trans. Signal Process. *59* (2011), 1528–1538.

[3] T. Chen and B. Francis: Optimal Sampled-Data Control Systems. Springer, London 1995.

[4] H. Gao, W. Sun, and P. Shi: Robust sampled-data control for vehicle active suspension systems. IEEE Trans. Control Systems Technol. *18* (2010), 238-245.

[5] L. Hu, P. Shi, and B. Huang: Stochastic stability and robust control for sampled-data systems with Markovian jump parameters. J. Math. Anal. Appl. *313* (2006), 504–517.

[6] Y. Ji and H. J. Chizeck: Jump linear quadratic Gaussian control: Steady-state solution and testable conditions. Control Theory Advanced Technol. *6* (1990), 289–319.

[7] L. H. Keel and S. P. Bhattacharyya: Robust, fragile, or optimal? IEEE Trans. Automat. Control *42* (1997), 1098–1105.

[8] J. Lam, Z. Shu, S. Xu, and E. K. Boukas: Robust control of descriptor discrete-time Markovian jump systems. Int. J. Control *80* (2007), 374–385.

[9] X. Mao: Stability of stochastic differential equations with Markovian switching. Stochastic Process. Appl. *79* (1999), 45–67.

[10] M. Shen and G. Yang: H_2 filter design for discrete Markov jump linear system with partly unknown transition probabilities. Optimal Control Appl. Methods *33* (2012), 318–337.

[11] B. Song, S. Xu, and Y. Zou: Non-fragile H_∞ filtering for uncertain stochastic time-delay systems. Int. J. Innovative Comput. Inform. and Control *5* (2009), 2257–2266.

[12] W. Sun, K. M. Nagpal, and P. P. Khargonekar: H_∞ control and filtering for sampled-data systems. IEEE Trans. Automat. Control *38* (1993), 1162–1175.

[13] V. Suplin, E. Fridman, and U. Shaked: Sampled-data H_∞ control and filtering: Nonuniform uncertain sampling. Automatica *43* (2007), 1072–1083

[14] J. Xiong, J. Lam, H. Gao, and D. W. C. Ho: On robust stabilization of Markovian jump systems with uncertain switching probabilities. Automatica *41* (2005), 897–903.

[15] S. Xu and T. Chen: Robust H_∞ filtering for uncertain impulsive stochastic systems under sampled measurements. Automatica *39* (2003), 509–516.

[16] S. Xu and T. Chen: Robust control for uncertain discrete-time stochastic bilinear systems with Markovian switching. Int. J. Robust Nonlinear *15* (2005), 201–217.

[17] G. Yang and W. Che: Non-fragile H_∞ filter design for linear continuous-time systems. Automatica *44* (2008), 2849–2856.

[18] P. Shi: Filtering on sampled-data systems with parametric uncertainty. IEEE Trans. Automat. Control *43* (1998), 1022–1027.

[19] P. Shi, E. K. Boukas, and R. K. Agarwal: Kalman filtering for continuous-time uncertain systems with Markovian jumping parameters. IEEE Trans. Automat. Control *44* (1999), 1592–1597.

Mouquan Shen, College of Automation and Electrical Engineering , Nanjing Technology University, Nanjing, 211816. P. R. China.
e-mail: mouquanshen@gmail.com

Guangming Zhang, College of Automation and Electrical Engineering , Nanjing Technology University, Nanjing, 211816. P. R. China.
e-mail: zgmchina@163.com

Yuhao Yuan, College of Automation and Electrical Engineering , Nanjing Technology University, Nanjing, 211816.P. R. China. Corresponding Author.
e-mail: yyhmds@sohu.com

Lei Mei, College of Automation and Electrical Engineering , Nanjing Technology University, Nanjing, 211816. P. R. China.
e-mail: meilei@njtech.edu.cn