IMPULSIVE PRACTICAL SYNCHRONIZATION OF N-DIMENSIONAL NONAUTONOMOUS SYSTEMS WITH PARAMETER MISMATCH

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This paper is concerned with impulsive practical synchronization in a class of n-dimensional nonautonomous dynamical systems with parameter mismatch. Some simple yet general algebraic synchronization criteria are derived based on the developed practical stability theory on impulsive dynamical systems. A distinctive feature of this work is that the impulsive control strategy is used to make n-dimensional nonautonomous dynamical systems with parameter mismatch achieve practical synchronization, where the parameter mismatch likewise exist in both system parameters and external excitation ones, and the synchronization error bound can be estimated by an analytical expression. Subsequently, the obtained results are applied to a typical gyrostat system, and numerical simulations demonstrate the effectiveness of the criteria and the robustness of the control technique.

Keywords: practical synchronization, impulsive control, n-dimensional nonautonomous systems, parameter mismatch, gyrostat system

Classification: 74H65, 70K40

1. INTRODUCTION

Since Pecora and Carroll [17] published their pioneering work on chaos synchronization, synchronization of chaotic systems has received a great deal of interest among scientists from various research fields [4, 12 – 14, 24]. Due to various environmental factors as well as a variety of inevitable external disturbances, the synchronization of coupled dynamical systems with parameter mismatch has become more and more significant topic. In general, parameter mismatch is considered to have a detrimental effect on the quality of synchronization between coupled identical chaotic systems. In some cases it even results in the loss of synchronization [1, 11]. Therefore, the synchronization of coupled dynamical systems with parameter mismatch has been an important topic in theoretical research and practical applications [2, 3, 5, 10, 15, 16, 18 – 20, 22].

However, it is worth noting that most of existing synchronization schemes with parameter mismatch are predominantly concentrated on autonomous dynamical systems [3, 10, 19, 22], or second-order nonautonomous systems [15, 16, 18, 20], due to the difficulties in convergence analysis of invariant set for nonautonomous dynamical systems and more complex computation for high-dimensional systems. The synchronization schemes with parameter mismatch for n-dimensional systems are studied in Refs. [2, 5]. Unfortunately, only partial parameters mismatched are considered in Refs. [2, 5]. It is obvious that such synchronization schemes previously proposed are quiet limited. In addition, it is believed that many chaotic models developed in physics, mechanics, chemistry, and biology are formulated in terms of n-dimensional nonautonomous dynamical systems [6 − 8, 23]. Therefore, new techniques and methods should be explored and developed to investigate synchronization for more general n-dimensional nonautonomous chaotic systems with more parameters mismatched. On the other hand, from Ref. [21], we know that the impulsive control is effective and easily realized, since it only needs to use small impulses and it can endure continuous disturbance. By the impulsive control, the slave system receives information from the master one only at discrete time instants, which drastically reduces the amount of synchronization information transmitted from the master system to the slave one, and makes this method more efficient in a great number of practical applications. As far as we know, the impulsive control was not used to investigate nonautonomous chaotic systems with parameter mismatch.

Motivated by the aforementioned comments, the impulsive control technique is used to investigate practical synchronization for n-dimensional nonautonomous chaotic systems with parameter mismatch. More generality for parameter mismatch here, the parameters both in the system and the external excitation can be mismatched. Some simple yet general algebraic criteria are derived and an analytical expression is obtained to estimate the synchronization error bound. It also can be shown that the synchronization error can be controlled as small as possible by choosing control gain and impulse interval properly. The obtained results are applied to a typical electromechanical gyrostat system, and simulations are provided to demonstrate the theoretical results.

The rest of this paper is organized as follows. In Section 2, model description and some preliminaries are presented. Section 3 gives the main results of this paper. Application example and simulations are presented in Section 4. Finally, conclusive remarks are given in Section 5.

2. PROBLEM FORMULATIONS AND PRELIMINARIES

Consider a n-dimensional nonautonomous chaotic system described by

$$\begin{cases} \dot{x} = A(t)x + f(x) + m(t), \\ x(t_0) = x_0, \end{cases} \tag{1}$$

where $x = (x_1, x_2, \ldots, x_n)^T \in R^n$ is the state vector, $A(t) \in R^{n\times n}$, $f(x) \in R^n$ is a continuous nonlinear function, and $m(t)$ is the external excitation which is independent of the system. We call system (1) as master system.

Remark 2.1. It should be noted that Eq. (1), as a representative n-dimensional nonautonomous dynamical system, can well formulate practical architectures of many chaotic models developed in physics, mechanics, chemistry, and biology, et al., such as horizontal platform system [8], FHN neuron oscillator [23], loudspeaker system [6], electromechanical gyrostat system [7] and so on.

It is known to all, parameters both in the system and the external excitation may be disturbed. Thus, a system with parameter mismatch and a sequence of impulses at time instants $\{t_k\}$ is constructed as

$$\begin{cases} \dot{y} = \tilde{A}(t)y + \tilde{f}(y) + \tilde{m}(t), \ t \neq t_k, \\ \Delta y|_{t=t_k} = -B_k(x(t_k) - y(t_k)), \ t = t_k, \\ y(t_0^+) = y_0, \ t_0 \geq 0, k = 1, 2, \ldots, \end{cases} \tag{2}$$

where $y = (y_1, y_2, \ldots, y_n)^T \in R^n$ is the state vector, $B_k \in R^{n\times n}$ denotes the control gain to be designed later, and $\Delta y|_{t=t_k} = y(t_k^+) - y(t_k)$ is the change of the state vector at the instant t_k, in which $y(t_k^+) = \lim_{t\to t_k^+} y(t)$. The set of discrete instants satisfies $0 < t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots$, $t_k \to \infty$ as $k \to \infty$. Due to parameter perturbation, the parameters in the functions $\tilde{A}(t)$, $\tilde{f}(y)$ and $\tilde{m}(t)$ can be slightly different from $A(t)$, $f(y)$ and $m(t)$ respectively. It implies that the parameters both in the system and the external excitation can be mismatched. System (2) is often called as slave system. To this end, we need two hypothesises.

Hypothesis 2.2. The parameter mismatch does not destroy the chaotic behavior of the chaotic system (1). Thus, due to the bounds of the chaotic signals, there exist positive constants M_j such that $|x_j| \leq M_j$, $|y_j| \leq M_j$, $j = 1, 2, \ldots, n$.

Hypothesis 2.3. There exists a bounded matrix $M(x, y)$ such that

$$\tilde{f}(x) - \tilde{f}(y) = M(x, y)(x - y), \tag{3}$$

where the elements of $M(x, y)$ are dependent on x and y.

Remark 2.4. Fortunately, due to the bounds of the chaotic signals, such a matrix $M(x, y)$ exists in many practical chaotic systems, such as systems mentioned in Remark 2.1.

Let $\Delta A(t) = A(t) - \tilde{A}(t)$, $\Delta f(x) = f(x) - \tilde{f}(x)$, and $\Delta m(t) = m(t) - \tilde{m}(t)$. Defining an error variable $e = x - y$, for $t \in (t_k^+, t_{k+1}]$, the synchronization error system can be obtained from systems (1) and (2)

$$\begin{cases} \dot{e} = (\tilde{A}(t) + M(x, y))e + \Delta A(t)x + \Delta f(x) + \Delta m(t), \ t \neq t_k, \\ \Delta e|_{t=t_k} = B_k e(t_k), \ t = t_k, \\ e(t_0^+) = x_0 - y_0, \ t_0 \geq 0, k = 1, 2, \ldots. \end{cases} \tag{4}$$

Before proceeding, we give some necessary preliminaries in the following.

Let an impulsively controlled nonautonomous chaotic system be

$$\begin{cases} \dot{z} = F(t, z, u(t)), \ t \neq t_k, \\ \Delta z|_{t=t_k} = U(k, z), \ t = t_k, \\ z(t_0^+) = z_0, \ t_0 \geq 0, \ k = 1, 2, \ldots, \end{cases} \tag{5}$$

where $z \in R^n$ is the state vector, $F : R^+ \times R^n \times R^m \to R^n$ is continuous function, R^+ denotes $[0, +\infty)$, $u : R^+ \to R^m$ is the external excitation, and $U(k, z) = z(t_k^+) - z(t_k)$ is the change of the state vector at the instant t_k. For system (5), we have the following definition.

Definition 2.5. (see Yang [21]) Comparison system. Let $V \in \nu_0$ and assume that

$$\begin{cases} D^+V(t,z) \le g(t,V(t,z),v(t)), t \neq t_k, \\ V(t,z+U(k,z)) \le \Psi_k(V(t,z)), t = t_k, \end{cases} \tag{6}$$

where $g : R^+ \times R^+ \times R^+ \to R$ is continuous and $\Psi_k : R^+ \to R^+$ is nondecreasing. Then the system

$$\begin{cases} \dot{\omega} = g(t,\omega,v(t)),\ t \neq t_k, \\ \omega(t_k^+) = \Psi_k(\omega(t_k)), t = t_k, \\ \omega(t_0^+) = \omega_0 \ge 0, \end{cases} \tag{7}$$

is the comparison system of (5). The definitions of ν_0 and $D^+V(t,z)$ can be seen in Ref.[21].

Letting the set Ω be

$$\Omega = \{u(t) \in R^m | \Gamma(t,u(t)) \le \Upsilon(t), t \ge t_0\},$$

where $\Gamma \in C[R^+ \times R^m, R^+]$ and $\Upsilon(t)$ is the maximal solution of the comparison system (7), the definition of practical stability of system (5) is presented as follows.

Definition 2.6. (see Yang [21]) Practical stability. System (5) is said to be practically stable with respect to (ξ,ε) if, given $\xi > 0$, and $\varepsilon > 0$, we have that $\|z_0\| < \xi$ implies that $\|z(t)\| < \varepsilon$, $t \ge t_0$, for some $t_0 \in R^+$ and every $u \in \Omega$, where $\|\cdot\|$ refers to the Euclidean vector norm.

Usually, the master-slave system (1) − (2) is difficult to achieve complete synchronization because of parameter mismatch. Therefore, based on the concept of practical stability mentioned above, a concept of practical synchronization is introduced as below.

Definition 2.7. The synchronization scheme (1) − (2) is said to achieve practical synchronization if, for given any initial values of system (4), there exist constants $\varepsilon > 0$ and $t_0 \in R^+$ such that the state error

$$e \in B_\varepsilon \overset{\text{def}}{=} \{e \in R^n \,|\, \|e\| < \varepsilon\}, \tag{8}$$

for all $t > t_0$.

In the above definition, the synchronization scheme (1) − (2) achieves practical synchronization, meaning that error system (4) achieves practical stability. Accordingly, ε is often referred to as desired synchronization error bound.

To study the practical stability of error system (4), the following necessary lemma is given, which will play an important role in the proof of the main results.

Lemma 2.8. Let the comparison system (7) be described by

$$\begin{cases} g(t,\omega,v(t)) = \beta\omega + \alpha^2, \beta > 0, \alpha > 0, t \neq t_k, \\ \Psi_k(\omega(t_k)) = \gamma^2\omega, \gamma > 0, t = t_k, \\ \omega(t_0^+) = \omega_0 \ge 0, k = 1,2,\dots. \end{cases} \tag{9}$$

For given (ξ,ε), $\xi>0$, and $\varepsilon>0$, if

$$\ln(\gamma^2)+\delta\beta<0, \tag{10}$$

and

$$\frac{\alpha^2(e^{\beta\delta}-1)}{\beta(1-\gamma^2e^{\beta\delta})}<\varepsilon, \tag{11}$$

where $\delta=t_{k+1}-t_k$, $0<\delta<+\infty$, then system (5) is practically stable with respect to (ξ,ε) for any $\xi<+\infty$.

P r o o f. Taking $\beta=\phi$, $\alpha^2=\theta$, $\gamma^2=d$, the comparison system (9) can be rewritten as

$$\begin{cases} \dot\omega=\phi\omega+\theta, t\neq t_k,\\ \omega(t_k^+)=d\omega, t=t_k,\\ \omega(t_0^+)=\omega_0\geq 0, k=1,2,\ldots. \end{cases} \tag{12}$$

Since the comparison system (12) has the same form as that of mentioned in Theorem 6.7.1 [21], the rest of the proof is similar to that of Theorem 6.7.1. Hence it is omitted here. The proof of Lemma 2.8 is completed. □

3. SYNCHRONIZATION CRITERIA AND ERROR ANALYSIS

Based on Lemma 2.8, some criteria are derived to make the synchronization scheme (1)−(2) achieve practical synchronization with desired error bound ε.

Theorem 3.1. Let $B_k=diag\{\gamma-1,\gamma-1,\ldots,\gamma-1\}\in R^{n\times n}$ with $\gamma>0$, and the sequence of impulses be equidistant and separated by an interval δ. Namely, $t_{k+1}-t_k=\delta$. Given (ξ,ε), $\xi>0$, and $\varepsilon>0$, $\|e(t_0^+)\|<\xi$, if there exist positive scalars β and α, such that the following conditions hold

(i) $Q(t)=\tilde A(t)+M(x,y)+(\tilde A(t)+M(x,y))^T+I-\beta I\leq 0$; (13)

(ii) $\|\Delta A(t)x+\Delta f(x)+\Delta m(t)\|\leq\alpha$; (14)

(iii) both inequalities (10) and (11) are satisfied;

then the master-drive systems (1)−(2) can achieve practical synchronization with desired error bound ε, for any $\xi<+\infty$.

P r o o f. Choose a quadratic Lyapunov function

$$V(t,e)=e^Te. \tag{15}$$

Obviously, $V(t,e)$ in Eq. (15) belongs to class ν_0. The derivative of $V(t,e)$ with respect to time $t\in(t_k^+,t_{k+1}]$ along the solution of error system (4) is

$$\begin{aligned} \dot V(t,e) &= e^T[\tilde A(t)+M(x,y)+(\tilde A(t)+M(x,y))^T]e+2e^T[\Delta A(t)x+\Delta f(x)+\Delta m(t)]\\ &\leq e^T[\tilde A(t)+M(x,y)+(\tilde A(t)+M(x,y))^T]e+e^Te\\ &\quad+\|\Delta A(t)x+\Delta f(x)+\Delta m(t)\|^2\\ &\leq e^T[\tilde A(t)+M(x,y)+(\tilde A(t)+M(x,y))^T+I-\beta I]e+\beta V(t,e)\\ &\quad+\|\Delta A(t)x+\Delta f(x)+\Delta m(t)\|^2. \end{aligned} \tag{16}$$

From conditions (i) and (ii), one can obtain

$$\dot{V}(t,e) \le \beta V(t,e) + \alpha^2. \tag{17}$$

At the impulse points, we have

$$V(t_k^+, e+\Delta e) = V(t_k, (I+B_k)e(t_k)) = \gamma^2 V(t_k, e(t_k)). \tag{18}$$

Hence, according to Definition 2.5, the comparison system of error system (4) is

$$\begin{cases} \dot{\omega} = \beta\omega + \alpha^2, t \neq t_k, \\ \omega(t_k^+) = \gamma^2\omega, t = t_k, \\ \omega(t_0^+) = \omega_0 \ge 0, k = 1, 2, \dots. \end{cases} \tag{19}$$

Based on the result of Lemma 2.8, if condition (iii) holds, then the synchronization error system (4) is practically stable with respect to (ξ, ε) for any $\xi < +\infty$. Thus, the synchronization scheme (1) − (2) achieves practical synchronization with desired error bound ε. This completes the proof of the theorem. □

Remark 3.2. Usually, α and β in Theorem 3.1 are related to the values of parameter mismatch. More reasonably, the values of parameter mismatch can be uncertain. It only needs to know their bounds. It can be seen clearly from example for more details in Section 4.

From inequality (10), we can get

$$0 < \gamma^2 < 1,\ \delta < -\frac{1}{\beta}\ln(\gamma^2). \tag{20}$$

Besides, for given a desired synchronization error bound ε, δ can be further obtained from inequality (11) as

$$\delta < \frac{1}{\beta}\ln\Big(\frac{\alpha^2 + \varepsilon\beta}{\alpha^2 + \gamma^2\varepsilon\beta}\Big). \tag{21}$$

Thus, given $0 < \gamma^2 < 1$, if δ is chosen to satisfy

$$\delta < \min\Big\{\frac{1}{\beta}\ln\Big(\frac{\alpha^2 + \varepsilon\beta}{\alpha^2 + \gamma^2\varepsilon\beta}\Big),\ -\frac{1}{\beta}\ln(\gamma^2)\Big\}, \tag{22}$$

then the real error $\|e\|$ is smaller than or equal to the desired synchronization error bound ε.

In the following, we will give an analytical expression to estimate synchronization error bound. If the synchronization scheme (1) − (2) achieves practical synchronization with error bound ε, then there exist constants $\varepsilon > 0$ and $t_0 > 0$ such that the state error $\|e\| < \varepsilon$ for all $t > t_0$. Thus, the estimated synchronization error bound can be taken as $\sigma = \varepsilon_{\min}$, where $\varepsilon_{\min}$ denotes the minimum value of ε. The minimum value of ε can be clearly seen from inequality (11). Hence, the estimated synchronization error bound equals

$$\sigma = \frac{\alpha^2(e^{\beta\delta} - 1)}{\beta(1 - \gamma^2 e^{\beta\delta})}. \tag{23}$$

It means that we can estimate the synchronization error bound by analytical expression (23) for given $\alpha,\ \beta,\ \delta,\ \gamma^2$.

Due to $B_k = diag\{\gamma-1, \gamma-1, \ldots, \gamma-1\} \in R^{n\times n}$, we can call $k = \gamma - 1$ as the control gain. Therefore, analytical expression (23) can be rewritten as

$$\sigma = \frac{\alpha^2(e^{\beta\delta}-1)}{\beta[1-(k+1)^2 e^{\beta\delta}]}, \tag{24}$$

with $k \in (-2,\ -1) \cup (-1,\ 0)$ since $0 < \gamma^2 < 1$. It can be seen from analytical expression (24) that the estimated synchronization error bound σ is monotonic increasing function with respect to α and the impulse interval δ, respectively. It also can be shown that the estimated error bound σ is monotonic decreasing function in the interval $(-2,\ -1)$, and monotonic increasing function in the interval $(-1,\ 0)$, with respect to the control gain k. Thus, the synchronization error can be controlled as small as possible by choosing the proper values of the impulse interval δ and the control gain k, respectively.

Remark 3.3. It seems that the estimated synchronization error bound σ can reach the minimum value when $k \to -1$. However, if we take $k = -1$, then $y(t_k^+)$ must be taken as $y(t_k^+) = x(t_k)$ in slave system (2). Obviously, it is hard to be implemented in practice.

Remark 3.4. If the synchronization scheme without parameter mismatch is considered, then $\sigma = 0$ will be obtained from Eq. (23) or (24), which is obviously originated from $\alpha = 0$. In this case, complete synchronization of n-dimensional nonautonomous systems without parameter mismatch is recovered.

4. APPLICATION EXAMPLE AND SIMULATION RESULTS

As an application of the above-derived theoretical criteria, the practical synchronization problem of gyrostat system as a representative example of chaotic systems is worked out in this section. The electromechanical gyrostat is one of the most interesting and everlasting dynamic systems. The advantages of the electromechanical gyrostat systems are that they can make the traditional mechanical system easier to be controlled and used [7].

The gyrostat system is given by [7]

$$\begin{cases} \dot{x_1} = -x_2x_3 - 0.5(1+6.5\cos t)x_2 + 0.4x_3 - 0.002(x_1 + x_1^3), \\ \dot{x_2} = x_1x_3 - 0.4x_3 + 0.5(1+6.5\cos t)x_1 - 0.002(x_2 + x_2^3), \\ \dot{x_3} = -0.2x_1 + 0.2x_2 - 0.2x_3 - 0.002(x_3 + x_3^3) + 1.625\sin t. \end{cases} \tag{25}$$

The gyrostat system exhibits chaos behavior, as shown in Figure 1.

Comparing with system (1), we have

$$A(t) = \begin{pmatrix} -0.002 & -0.5(1+6.5\cos t) & 0.4 \\ 0.5(1+6.5\cos t) & -0.002 & -0.4 \\ -0.2 & 0.2 & -0.202 \end{pmatrix},$$

$$f(x) = \begin{pmatrix} -x_2x_3 - 0.002x_1^3 \\ x_1x_3 - 0.002x_2^3 \\ -0.002x_3^3 \end{pmatrix},\ m(t) = \begin{pmatrix} 0 \\ 0 \\ 1.625\sin t \end{pmatrix}.$$

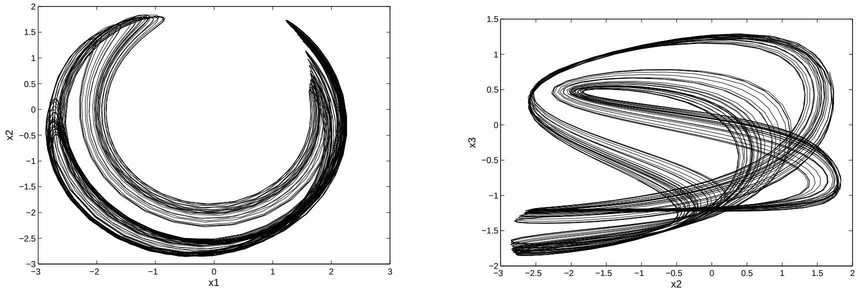

Fig. 1. Chaotic attractor of gyrostat system (25).

For simplicity, we consider some of the representative parameters mismatched, which are both in the system and the external excitation. The slave system for the gyrostat system is constructed as

$$\begin{cases} \dot{y}_1 = (-1+\Delta_1)y_2y_3 + (-0.5+\Delta_2)(1+6.5\cos t)y_2 + 0.4y_3 - 0.002(y_1+y_1^3), & t \neq t_k, \\ \dot{y}_2 = y_1y_3 - 0.4y_3 + 0.5(1+6.5\cos t)y_1 + (-0.002+\Delta_3)(y_2+y_2^3), & t \neq t_k, \\ \dot{y}_3 = -0.2y_1 + (0.2+\Delta_4)y_2 - 0.2y_3 - 0.002(y_3+y_3^3) + (1.625+\Delta_5)\sin(t+\Delta_6), & t \neq t_k, \\ \Delta y_1 = -(\gamma-1)(x_1(t_k)-y_1(t_k)),\ t = t_k, \\ \Delta y_2 = -(\gamma-1)(x_2(t_k)-y_2(t_k)),\ t = t_k, \\ \Delta y_3 = -(\gamma-1)(x_3(t_k)-y_3(t_k)),\ t = t_k. \end{cases} \tag{26}$$

Corresponding to system (2), one can get

$$\tilde{A}(t) = \begin{pmatrix} -0.002 & (-0.5+\Delta_2)(1+6.5\cos t) & 0.4 \\ 0.5(1+6.5\cos t) & -0.002+\Delta_3 & -0.4 \\ -0.2 & 0.2+\Delta_4 & -0.202 \end{pmatrix},$$

$$\tilde{f}(y) = \begin{pmatrix} (-1+\Delta_1)y_2y_3 - 0.002y_1^3 \\ y_1y_3 + (-0.002+\Delta_3)y_2^3 \\ -0.002y_3^3 \end{pmatrix},\ \tilde{m}(t) = \begin{pmatrix} 0 \\ 0 \\ (1.625+\Delta_5)\sin(t+\Delta_6) \end{pmatrix}.$$

From function $\tilde{f}(y)$, the matrix $M(x,y)$ mentioned in Hypothesis 2.3 equals

$$M(x,y) = \begin{pmatrix} -0.002(x_1^2+x_1y_1+y_1^2) & (-1+\Delta_1)y_3 & (-1+\Delta_1)x_2 \\ y_3 & (-0.002+\Delta_3)(x_2^2+x_2y_2+y_2^2) & x_1 \\ 0 & 0 & -0.002(x_3^2+x_3y_3+y_3^2) \end{pmatrix}.$$

Hence

$$Q(t) = \tilde{A}(t) + M(x,y) + (\tilde{A}(t)+M(x,y))^T + I - \beta I =$$

$$\begin{pmatrix} 0.996-0.004(x_1^2+x_1y_1+y_1^2)-\beta & \Delta_2(1+6.5\cos t)+\Delta_1y_3 & 0.2+(-1+\Delta_1)x_2 \\ \Delta_2(1+6.5\cos t)+\Delta_1y_3 & 0.996+2\Delta_3+(-0.004+2\Delta_3)(x_2^2+x_2y_2+y_2^2)-\beta & -0.2+\Delta_4+x_1 \\ 0.2+(-1+\Delta_1)x_2 & -0.2+\Delta_4+x_1 & 0.596-0.004(x_3^2+x_3y_3+y_3^2)-\beta \end{pmatrix}.$$

In order to choose β to satisfy the condition (i) in Theorem 3.1, our object is to choose β such that $Q(t)$ is negative definite. To this end, we introduce the Gerschgorin disc theorem.

Lemma 4.1. (see Horn and Johnson [9]) Let $D = (d_{ij}) \in R^{n\times n}$ and $p_1, p_2, \dots, p_n$ be positive number, then all the eigenvalues of D lie in the region

$$\bigcup_{i=1}^{n}\Big\{\lambda \in C : \mid \lambda - d_{ii} \mid \leq \frac{1}{p_i}\sum_{j=1, j\neq i}^{n} p_j \mid d_{ij} \mid \Big\},$$

where C is the set of complex numbers.

According to the Gerschgorin disc theorem, the matrix $Q(t)$ is negative definite if β satisfies

$$\beta > \max\{H_1, H_2, H_3\}, \tag{27}$$

with

$$\begin{aligned} H_1 &= 0.996 - 0.004(x_1^2 + x_1y_1 + y_1^2) + \frac{p_2}{p_1}|\Delta_2(1 + 6.5\cos t) + \Delta_1 y_3| \\ &\quad + \frac{p_3}{p_1}|0.2 + (-1 + \Delta_1)x_2|, \qquad (28) \\ H_2 &= 0.996 + 2\Delta_3 + (-0.004 + 2\Delta_3)(x_2^2 + x_2y_2 + y_2^2) + \frac{p_1}{p_2}|\Delta_2(1 + 6.5\cos t) + \Delta_1 y_3| \\ &\quad + \frac{p_3}{p_2}| - 0.2 + \Delta_4 + x_1|, \qquad (29) \\ H_3 &= 0.596 - 0.004(x_3^2 + x_3y_3 + y_3^2) + \frac{p_1}{p_3}|0.2 + (-1 + \Delta_1)x_2| + \frac{p_2}{p_3}| - 0.2 + \Delta_4 + x_1|. \qquad (30) \end{aligned}$$

Based on the Hypothesis 2.2, the bounds of $H_1,\ H_2,\ H_3$ can be expressed as

$$\begin{aligned} H_1 &\leq 0.996 + 0.004M_1^2 + \frac{7.5p_2}{p_1}|\Delta_2| + \frac{p_2}{p_1}|\Delta_1|M_3 + \frac{0.2p_3}{p_1} \\ &\quad + \frac{p_3}{p_1}(1 + |\Delta_1|)M_2, \qquad (31) \\ H_2 &\leq 0.996 + 2|\Delta_3| + 0.004M_2^2 + 6|\Delta_3|M_2^2 + \frac{7.5p_1}{p_2}|\Delta_2| \\ &\quad + \frac{p_1}{p_2}|\Delta_1|M_3 + \frac{p_3}{p_2}(0.2 + |\Delta_4| + M_1), \qquad (32) \\ H_3 &\leq 0.596 + 0.004M_3^2 + 0.2\Big(\frac{p_1}{p_3} + \frac{p_2}{p_3}\Big) + \frac{p_1}{p_3}(1 + |\Delta_1|)M_2 + \frac{p_2}{p_3}(|\Delta_4| + M_1). \qquad (33) \end{aligned}$$

On the other hand, we have

$$\Delta A(t) = \begin{pmatrix} 0 & -\Delta_2(1 + 6.5\cos t) & 0 \\ 0 & -\Delta_3 & 0 \\ 0 & -\Delta_4 & 0 \end{pmatrix},\ \Delta f(x) = \begin{pmatrix} -\Delta_1 x_2 x_3 \\ -\Delta_3 x_2^3 \\ 0 \end{pmatrix},$$

and $\Delta m(t) = (0,\ 0,\ 1.625 \sin t - (1.625 + \Delta_5)\sin(t + \Delta_6))^T$. Hence, we can obtain

$$\begin{aligned}\|\Delta m(t)\| &= |1.625 \sin t - (1.625 + \Delta_5)\sin(t + \Delta_6)| \\ &\le |1.625 \sin t - (1.625 + \Delta_5)\sin t| + |(1.625 + \Delta_5)(\sin t - \sin(t + \Delta_6))| \\ &\le |\Delta_5| + (1.625 + |\Delta_5|)|\sin t - \sin(t + \Delta_6)|.\end{aligned}$$

By the differential mean-value theorem, yields

$$\|\Delta m(t)\| \le |\Delta_5| + (1.625 + |\Delta_5|)|\Delta_6|. \tag{34}$$

Thus, we can get the following inequalities

$$\begin{aligned} & ||\Delta A(t)x + \Delta f(x) + \Delta m(t)|| \\ \le\ & \|\Delta A(t)x\| + \|\Delta f(x)\| + \|\Delta m(t)\| \\ =\ & |x_2|\sqrt{\Delta_2^2(1 + 6.5\cos t)^2 + \Delta_3^2 + \Delta_4^2} + |x_2|\sqrt{\Delta_1^2 x_3^2 + \Delta_3^2 x_2^4} + \|\Delta m(t)\| \\ \le\ & M_2\sqrt{56.25\Delta_2^2 + \Delta_3^2 + \Delta_4^2} + M_2\sqrt{\Delta_1^2 M_3^2 + \Delta_3^2 M_2^4} + |\Delta_5| \\ & +(1.625 + |\Delta_5|)|\Delta_6|. \end{aligned} \tag{35}$$

From condition (ii) in Theorem 3.1, gives

$$\alpha \ge M_2\sqrt{56.25\Delta_2^2+\Delta_3^2+\Delta_4^2}+M_2\sqrt{\Delta_1^2 M_3^2+\Delta_3^2 M_2^4}+|\Delta_5|+(1.625+|\Delta_5|)|\Delta_6|. \tag{36}$$

From Figure 1, we know that the bounds of the chaotic attractor are $-3 < x_1 < 2.4$, $-3 < x_2 < 2$ and $-2 < x_3 < 1.4$. Namely, $M_1 = 3$, $M_2 = 3$ and $M_3 = 2$. Usually, the values of Δ_i can be uncertain, $i = 1, 2, \ldots, 6$. Taking the bounds of the parameter mismatch are $|\Delta_1| \le 0.2$, $|\Delta_2| \le 0.02$, $|\Delta_3| \le 0.0002$, $|\Delta_4| \le 0.02$, $|\Delta_5| \le 0.02$ and $|\Delta_6| \le 0.02$, then $\alpha \ge 1.707$ is obtained from inequality (36). From inequalities (31)–(33), it can be seen that the values of p_1, p_2 and p_3 can adjust the magnitudes of H_1, H_2 and H_3. In order to get the optimal value of β, taking $p_1 = 1.5p_2$ and $p_3 = 1.7p_2$, we can further obtain $H_1 \le 5.705$, $H_2 \le 5.518$ and $H_3 \le 5.859$ from inequalities (31)–(33). Therefore, $\beta > 5.859$. For given a desired synchronization error bound $\varepsilon = 0.01$, by taking $\gamma^2 = 0.2 < 1$, together with the above data, we can get $\delta < 0.0026$ from inequality (22).

Based on the above data, take $\alpha = 1.75$, $\beta = 5.86$, $\delta = 0.002$, $\Delta_1 = 0.15$, $\Delta_2 = 0.015$, $\Delta_3 = 0.0001$, $\Delta_4 = 0.015$, $\Delta_5 = -0.01$ and $\Delta_6 = -0.015$, which are chosen within their bounds, respectively. We plot the time-response curve of the norm of the synchronization error in Figure 2 with the initial values $(x_1(0), x_2(0), x_3(0)) = (1, 1, 1)$, and $(y_1(0), y_2(0), y_3(0)) = (-1, -1, -1)$, which are taken arbitrarily. From Figure 2, we can see that the real synchronization error bound is smaller than the desired one $\varepsilon = 0.01$, which means the robustness of the synchronization for the parameter mismatch. It further demonstrates the effectiveness of the presented control technique.

If we take $\gamma^2 = 0.1$ and $\delta = 0.005$ satisfying inequality (20), and the values of α and β are the same as the above data, then $\sigma = 0.017$ is obtained from analytical expression (23). In this case, the synchronization scheme (25)–(26) also can achieve practical

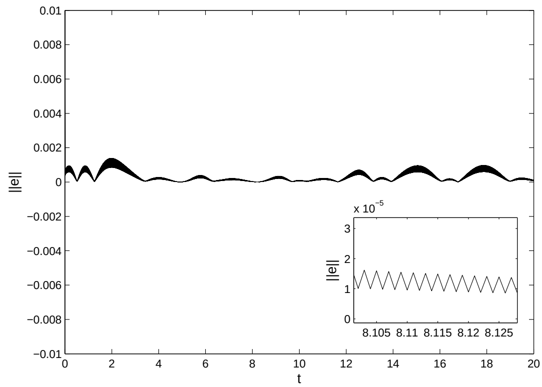

Fig. 2. Synchronization error curves with $\gamma^2 = 0.2$ and $\delta = 0.002$ for the master-slave system (25) – (26).

synchronization, as shown in Figure 3, and the initial values are the same as in Figure 2. From Figure 3, we also can see that the real synchronization error bound is smaller than the estimated one $\sigma = 0.017$, which also means the robustness of the synchronization for the parameter mismatch. Therefore, the simulation results have a good agreement with the theoretical analysis obtained in the paper.

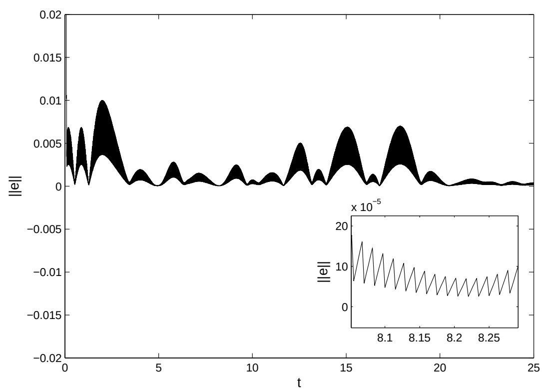

Fig. 3. Synchronization error curves with $\gamma^2 = 0.1$ and $\delta = 0.005$ for the master-slave system(25) – (26).

For given $\gamma^2 = 0.3$, α and β are the same as mentioned above, then $\delta < 0.205$ is obtained from inequality (20). In practice, we hope to get the synchronization error as small as possible. Thus, we plot the estimated synchronization error bound σ in expression (23) with different values of the impulse interval δ only from 0.001 to 0.02, as visualized by Figure 4. In addition, if for given $\delta = 0.001$, α and β are the same as mentioned above, then from inequality (20) or (10), $\gamma^2 < 0.994$ is obtained. Thus, one can get $k \in (-1.997,\ -1) \cup (-1,\ -0.003)$. Consider the practical significance, we also plot part of the curve about the estimated synchronization error bound σ in expression (24) with different values of control gain $k \in (-1.9,\ -1) \cup (-1,\ -0.1)$, as shown in Figure 5. Subsequently, the relationship between γ^2 and the impulse interval δ is considered. For given a desired synchronization error $\varepsilon = 0.01$, from inequality (21), the maximum value of impulse interval denoted by $\delta_{\max}$ versus γ^2 can be shown in Figure 6. According to Figs. 4−6, δ and k can be selected properly for purpose of practical control strategy, respectively.

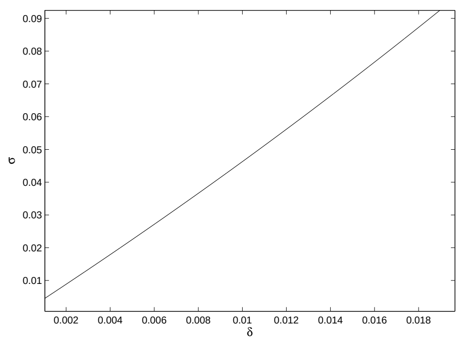

Fig. 4. Synchronization error bound estimated by Eq. (24) increases as the impulse interval gets bigger in the case of $\gamma^2 = 0.3$.

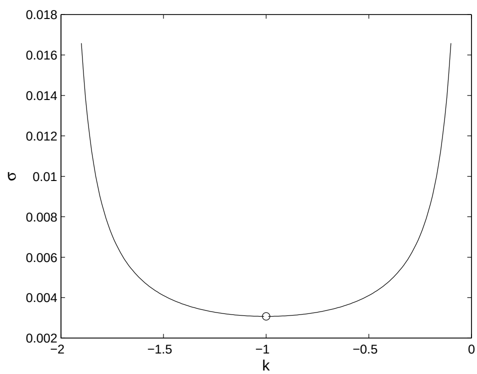

Fig. 5. Estimated synchronization error bound versus the control gain k in the case of $\delta = 0.001$.

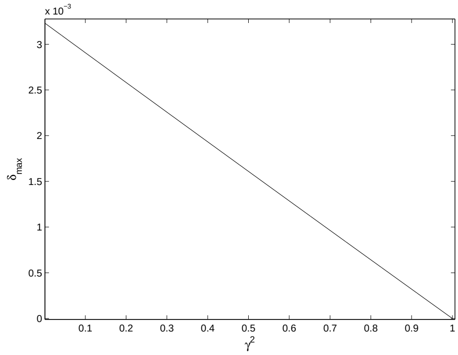

Fig. 6. The impulse interval $\delta_{\max}$ versus γ^2 for given desired synchronization error bound $\varepsilon = 0.01$.

5. CONCLUSION

By using impulsive control, this paper has derived some general criteria to make n-dimensional nonautonomous chaotic systems with parameter mismatch achieve robust practical synchronization. The values of the parameter mismatch can be uncertain. Furthermore, the synchronization error bound can be estimated by an analytical expression, which shows that the synchronization error can be controlled as small as possible by selecting control gain and impulse interval properly. As a direct application of the new theoretical results, a representative example is simulated and discussed in detail. Numerical experiments verify the effectiveness and robustness of the proposed control technique. It is believed that the presented technique may provide a valuable insight into the underlying use of chaos synchronization in practical designs and engineering applications.

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