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CHANCE CONSTRAINED BOTTLENECK TRANSPORTATION PROBLEM WITH PREFERENCE OF ROUTES

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This paper considers a variant of the bottleneck transportation problem. For each supply-demand point pair, the transportation time is an independent random variable. Preference of each route is attached. Our model has two criteria, namely: minimize the transportation time target subject to a chance constraint and maximize the minimal preference among the used routes. Since usually a transportation pattern optimizing two objectives simultaneously does not exist, we define non-domination in this setting and propose an efficient algorithm to find some non-dominated transportation patterns. We then show the time complexity of the proposed algorithm. Finally, a numerical example is presented to illustrate how our algorithm works.

Keywords: bottleneck transportation, random transportation time, chance constraint, preference of routes, non-domination

Classification: 90C35, 90C15, 90C70, 68Q25

1. INTRODUCTION

The classical transportation problem is defined by minimization of variable transportation costs while meeting a set of demands from a set of available supplies. It is also known as the cost minimizing transportation problem, which has been extensively studied in the literature and several algorithms [4, 6, 7, 12, 14, 17] are available to solve it. One well studied variant of the classical transportation problem is known as the bottleneck transportation problem, which determines a single bottleneck for the transportation system by minimizing the maximum time for transport of all goods. It is also known as the time minimizing transportation problem. Similarly, many efficient algorithms have been proposed by Garfinkel and Rao [8], Hammer [11], Russell et al. [16], Srinivasan and Thompson [18] and Szwarc [19] for solving it. As for stochastic programming, the basic properties and algorithms [15], the stability and robustness [3] and the chance constraint condition [13] are well studied. Recently, Adeyefa and Luhandjulawe [1] presented the main principle of multiobjective stochastic linear programming. Stochastic versions of bottleneck transportation problem are considered [9, 10]. Ge and Ishii [9] proposed a stochastic bottleneck transportation problem with fuzzy supply and demand. Geetha and Nair [10] presented a single criterion bottleneck transportation problem with random transportation cost. Besides, Chen et al. [5] studied a fuzzy transportation problem with preference of routes.

The model considered in this paper is an extension of these previous models. We extend the bottleneck transportation problem by considering randomness of transportation time and preference of route. Randomness means that the transportation time may change according to many factors. The preference of route reflects the degree of satisfaction with respect to the chosen route. So two criteria are taken into account. One is to minimize the transportation time target subject to a chance constraint. The other is to maximize the minimal preference among the used routes. But usually a transportation pattern optimizing two objectives simultaneously does not exist. So we seek some non-dominated transportation patterns.

The rest of this paper is organized as follows. Our problem is formulated in Section 2, and then in Section 3 we present an efficient algorithm to find some non-dominated transportation patterns. Section 4 shows how our algorithm works using a numerical example. Finally, Section 5 concludes this paper and discusses further research problems.

2. PROBLEM FORMULATION

In this paper, we consider the following bi-criteria bottleneck transportation problem with randomness of transportation time and preference of route.

(C1) There exist m supply points $\{S_1, \dots, S_m\}$ and n demand points $\{T_1, \dots, T_n\}$. The total upper limit provided from each supply point S_i is a_i and the total lower limit to each demand point T_j is b_j. Further, we assume that these a_i, b_j are positive integers and $\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j$.

(C2) Let (i, j) denote the route from supply point S_i to demand point T_j, $i = 1, \dots, m$, $j = 1, \dots, n$. Preference of route is attached and it is assumed to be measured by a real number μ_{ij} between 0 and 1. This number reflects the degree of satisfaction with respect to the chosen route. The value $\mu_{ij} = 1$ corresponds to complete satisfaction, while $\mu_{ij} = 0$ corresponds to complete dissatisfaction. For intermediate numbers, a higher value corresponds to a higher degree of satisfaction.

(C3) For each route (i, j), the transportation time t_{ij} is an independent random variable according to a normal distribution $N(m_{ij}, \sigma_{ij}^2)$ with mean m_{ij} and variance σ_{ij}^2. We denote the transportation quantity using the route (i, j) by x_{ij} and assume that these x_{ij} are nonnegative decision variables. The following chance constraint is attached:

$$\Pr\{t_{ij} \leq f\} \geq \alpha, \ (i, j) | x_{ij} > 0 \tag{1}$$

where $\alpha > 0.5$ and f is also a decision variable denoting the target of bottleneck transportation time to be minimized.

(C4) We consider two criteria: one is to maximize the minimal preference among the used routes and the other is to minimize f.

Under the above setting, our chance constrained bottleneck transportation problem with preference of routes can be formulated as follows:

$$
\begin{aligned}
\text{TP}: \quad & \text{minimize} && f \\
& \text{maximize} && \min_{i,j}\{\mu_{ij} | x_{ij} > 0\} \\
& \text{subject to} && \Pr\{t_{ij} \le f\} \ge \alpha, \ (i,j)|x_{ij} > 0, \ i = 1, \ldots, m, \ j = 1, \ldots, n \\
& && \sum_{j=1}^{n} x_{ij} \le a_i, \ i = 1, \ldots, m \\
& && \sum_{i=1}^{m} x_{ij} \ge b_j, \ j = 1, \ldots, n \\
& && x_{ij}: \ \text{nonnegative}, \ i = 1, \ldots, m, \ j = 1, \ldots, n.
\end{aligned}
$$

In order to solve problem TP, first we introduce the following equivalent parametric programming formulations.

The chance constraint (1) reduces to:

$$
F\left(\frac{f - m_{ij}}{\sigma_{ij}}\right) \ge \alpha, \ (i,j)|x_{ij} > 0
$$

where $F(\cdot)$ is the cumulative distribution function of the standard normal distribution $N(0,1)$. That is,

$$
\begin{aligned}
(1) \iff & \frac{f - m_{ij}}{\sigma_{ij}} \ge K_\alpha, \ (i,j)|x_{ij} > 0 \\
\iff & f \ge m_{ij} + K_\alpha \sigma_{ij}, \ (i,j)|x_{ij} > 0
\end{aligned}
$$

where $K_\alpha = F^{-1}(\alpha)$.

Since f should be minimized, then problem TP reduces to:

$$
\begin{aligned}
\text{P}: \quad & \text{minimize} && \max_{i,j}\{m_{ij} + K_\alpha \sigma_{ij} | x_{ij} > 0\} \\
& \text{maximize} && \min_{i,j}\{\mu_{ij} | x_{ij} > 0\} \\
& \text{subject to} && \sum_{j=1}^{n} x_{ij} \le a_i, \ i = 1, \ldots, m \\
& && \sum_{i=1}^{m} x_{ij} \ge b_j, \ j = 1, \ldots, n \\
& && x_{ij}: \ \text{nonnegative}, \ i = 1, \ldots, m, \ j = 1, \ldots, n.
\end{aligned}
$$

Next, we define the bi-objective vector $\mathrm{v}(\mathbf{x})$ of a transportation pattern $\mathbf{x} = (x_{ij})$ feasible for P as

$$
\mathrm{v}(\mathbf{x}) = (v(\mathbf{x})_1, v(\mathbf{x})_2) = \left(\max_{i,j}\{m_{ij} + K_\alpha \sigma_{ij} | x_{ij} > 0\}, \min_{i,j}\{\mu_{ij} | x_{ij} > 0\}\right).
$$

Generally, a transportation pattern optimizing two objectives simultaneously does not exist. Therefore, we seek some non-dominated transportation patterns, the definition of which is given as follows.

Definition 2.1. Let $\mathbf{x}^a$, $\mathbf{x}^b$ be two transportation patterns that are feasible for P. Then, we say that $\mathbf{x}^a$ dominates $\mathbf{x}^b$, if $v(\mathbf{x}^a)_1 \leq v(\mathbf{x}^b)_1$, $v(\mathbf{x}^a)_2 \geq v(\mathbf{x}^b)_2$ and $(v(\mathbf{x}^a)_1, v(\mathbf{x}^a)_2) \neq (v(\mathbf{x}^b)_1, v(\mathbf{x}^b)_2)$. If there exists no transportation pattern dominating $\mathbf{x}$, $\mathbf{x}$ is called a non-dominated transportation pattern.

3. SOLUTION PROCEDURE

Sorting μ_{ij}, $i = 1, \ldots, m$, $j = 1, \ldots, n$, and let the result be

$$0 < \mu^1 < \ldots < \mu^g \leq 1$$

where g is the number of different values of them.

Compute $m_{ij} + K_\alpha \sigma_{ij}$, $i = 1, \ldots, m$, $j = 1, \ldots, n$, and arrange these values in ascending order. Let the result be

$$c^1 < \ldots < c^l$$

where l is the number of different values of them. Let $\mathbf{C} \triangleq (m_{ij} + K_\alpha \sigma_{ij})_{m \times n}$.

For $u = 1, \ldots, g$, $k = 1, \ldots, l$, set

$$c_{ij}^{u,k} = \begin{cases} 0 & \text{if } \mu_{ij} \geq \mu^u,\ m_{ij} + K_\alpha \sigma_{ij} \leq c^k \\ M & \text{otherwise} \end{cases}, \quad i = 1, \ldots, m,\ j = 1, \ldots, n,$$

where M is a sufficiently large value.

For $u = 1, \ldots, g$, $k = 1, \ldots, l$, denote the cost minimizing transportation problem with the above defined cost values as P_u^k:

$$\begin{aligned} \mathrm{P}_u^k: \quad & \text{minimize} && \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{u,k} x_{ij} \\ & \text{subject to} && \sum_{j=1}^{n} x_{ij} \leq a_i,\ i = 1, \ldots, m \\ & && \sum_{i=1}^{m} x_{ij} \geq b_j,\ j = 1, \ldots, n \\ & && x_{ij}: \text{ nonnegative},\ i = 1, \ldots, m,\ j = 1, \ldots, n. \end{aligned}$$

For fixed $u \in \{1, \ldots, g\}$ and $k \in \{1, \ldots, l\}$, note that P_u^k is a restricted transportation problem, therefore it is not always have a feasible solution with optimal value 0. If it exists a feasible solution, then it is feasible only using the route (i, j) with $\mu_{ij} \geq \mu^u$, $m_{ij} + K_\alpha \sigma_{ij} \leq c^k$.

Denote

$$S(u) = \{(i,j) | \mu_{ij} = \mu^u, i = 1, \ldots, m,\ j = 1, \ldots, n\},\ u = 1, \ldots, g$$

$$T(k) = \{(i,j) | m_{ij} + K_\alpha \sigma_{ij} = c^k, i = 1, \ldots, m,\ j = 1, \ldots, n\},\ k = 1, \ldots, l$$

$$p = \max \left\{ t \middle| \sum_{r=t}^{g} |S(r)| \geq n \right\}$$

$$q = \min\left\{ t \Big| \sum_{r=1}^{t} |T(r)| \geq n \right\}.$$

It is obvious that P_u^k is infeasible when $u \in \{p+1, \dots, g\}$ or $k \in \{1, \dots, q-1\}$.

For each $u \in \{1, \dots, p\}$, we then give the algorithm to find the smallest $k \in \{q, \dots, l\}$ such that P_u^k is feasible. If such k exists, denote it by k_u. Otherwise, see the following Remark 3.1.

Remark 3.1. If exists $u_0 \in \{1, \dots, p\}$, such that $\mathrm{P}_{u_0}^k$ is infeasible for any $k \in \{q, \dots, l\}$, then P_u^k, $u = u_0 + 1, \dots, p$ are also infeasible for any $k \in \{q, \dots, l\}$.

For each $u \in \{1, \dots, p\}$, we need to find the smallest k_u such that $\mathrm{P}_u^{k_u}$ is feasible. The smallest k_u corresponds to the biggest c^{k_u}, which is the first component of the bi-objective vector (c^{k_u}, μ^u). The main idea to find the smallest k_u such that $\mathrm{P}_u^{k_u}$ is feasible is based on a binary method, which is given as follows in detail.

Algorithm (To find the smallest k such that P_1^k is feasible)

Step 1 Set $L = q$ and check whether P_1^L is feasible or not. If feasible, terminate after setting $k_1 = L$. Otherwise, set $U = l$ and check whether P_1^U is feasible or not. If feasible, go to Step 2. Otherwise, terminate due to infeasibility.

Step 2 When $U - L > 1$, set $K = \lfloor (L+U)/2 \rfloor$ and check whether P_1^K is feasible or not, where $\lfloor \cdot \rfloor$ denotes the greatest integer not greater than $\cdot$. If feasible, set $U = K$ and repeat Step 2. Otherwise, set $L = K$ and repeat Step 2. When $U - L = 1$, go to Step 3.

Step 3 If P_1^L is feasible, set $k_1 = L$. Otherwise, set $k_1 = U$.

For P_u^k, $u = 2, \dots, p$, the algorithm is very similar to that of P_1^k; the only difference is we first set $L = k_{u-1}$.

Denote $A = \{(u, k_u) | k_u \text{ exists}, u = 1, \dots, p\}$. If there exists (u_1, k_{u_1}) and $(u_2, k_{u_2}) \in A$ such that $u_1 \neq u_2$ but $k_{u_1} = k_{u_2}$, then delete $(\min\{u_1, u_2\}, k_{\min\{u_1, u_2\}})$ from A. Let obtained set after deletion be B. Note that all elements in B have different first components and also different second components.

For all $(u, k_u) \in B$, solve problems $\mathrm{P}_u^{k_u}$'s, and let denote the optimal transportation patterns by $\mathbf{x}_u^{k_u}$'s. Then we find a set of some non-dominated transportation patterns and that of the corresponding bi-objective vectors of problem P, denoted by NDT and NDV respectively.

The validity of our solution procedure is shown in the following proposition.

Proposition 3.2. The solution procedure for P is valid.

Proof. For each u, the algorithm to find the smallest k such that P_u^k is feasible is a binary feasibility checking method. For each $(u, k_u) \in A$, $\mathrm{P}_u^{k_u}$ is feasible, conversely, for each feasible P_u^k, we have $(u, k) \in A$. For each $(u, k_u) \in B$, an optimal transportation

$i\backslash j$	1	2	3	a_i
1	$N(3, 0.5^2)$	$N(7, 0.4^2)$	$N(4, 1.2^2)$	50
2	$N(6, 0.8^2)$	$N(5, 0.3^2)$	$N(1, 0.7^2)$	85
3	$N(7, 0.3^2)$	$N(4, 0.6^2)$	$N(8, 1.0^2)$	30
b_j	60	35	55	–

Tab. 1 The values of a_i, b_j and the distribution of t_{ij}.

pattern $\mathbf{x}_u^{k_u}$ of problem $\mathrm{P}_u^{k_u}$ is a non-dominated transportation pattern of problem P and (c^{k_u}, μ^u) is the corresponding bi-objective vector, that is, $NDT = \{\mathbf{x}_u^{k_u} | (u, k_u) \in B\}$, $NDV = \{(c^{k_u}, \mu^u) | (u, k_u) \in B\}$. Therefore, our solution procedure is valid. □

Next we show the time complexity of our solution procedure for P.

Theorem 3.3. The time complexity of our solution procedure for P is

$$\mathrm{O}(mn(m+n)^3 \log(m+n)).$$

Proof. Note that $g = l = \mathrm{O}(mn)$, so sorting μ_{ij} and $m_{ij} + K_\alpha \sigma_{ij}$ both takes at most $\mathrm{O}(mn \log(mn))$ operations. For each u, the time complexity of the algorithm to find the smallest $k \in \{q, \dots, l\}$ such that P_u^k is feasible follows from the fact that the binary search over l values has time complexity $\mathrm{O}(\log l)$, and each feasibility checking takes $\mathrm{O}(mn)$ because at most $\mathrm{O}(mn)$ elements should be checked. So for each u, checking totally needs $\mathrm{O}(mn \log(mn))$ computational times. The algorithm is executed at most $\mathrm{O}(g)$ times to find the smallest $k \in \{q, \dots, l\}$ such that P_u^k is feasible. So checking totally needs $\mathrm{O}((mn)^2 \log(mn))$. Solving each feasible classical transportation problem takes at most $\mathrm{O}((m+n)^3 \log(m+n))$ (see [2]) and totally at most $\mathrm{O}(mn)$ classical transportation problems should be solved, therefore this part takes at most $\mathrm{O}(mn(m+n)^3 \log(m+n))$ computational times. Consequently, the time complexity is $\mathrm{O}(\max\{(mn)^2 \log(mn), mn(m+n)^3 \log(m+n)\}) = \mathrm{O}(mn(m+n)^3 \log(m+n))$. □

4. NUMERICAL EXAMPLE

Consider problem TP with $\alpha = 0.9987$, $t_{ij} \sim N(m_{ij}, \sigma_{ij}^2)$ and the values of a_i, b_j are given in Table 1. The preference of routes are given in the following matrix:

$$\mathbf{U} = \begin{pmatrix} 0.5 & 0.8 & 0.4 \\ 0.75 & 0.6 & 0.7 \\ 0.85 & 1 & 0.6 \end{pmatrix}.$$

Our problem TP reduces to problem P:

$$\begin{array}{lll} \mathrm{P}: & \text{minimize} & \max\limits_{i,j}\{m_{ij}+3.0\sigma_{ij}|x_{ij}>0\} \\ & \text{maximize} & \min\limits_{i,j}\{\mu_{ij}|x_{ij}>0\} \\ & \text{subject to} & \sum\limits_{j=1}^{3} x_{ij} \le a_i,\ i=1,2,3 \\ & & \sum\limits_{i=1}^{3} x_{ij} \ge b_j,\ j=1,2,3 \\ & & x_{ij}:\ \text{nonnegative},\ i,j=1,2,3. \end{array}$$

Sorting μ_{ij}, $i,j=1,2,3$, we obtain

$$0<\mu^1=0.4<\mu^2=0.5<\mu^3=0.6<\mu^4=0.7<\mu^5=0.75$$
$$<\mu^6=0.8<\mu^7=0.85<\mu^8=1.$$

Compute $m_{ij}+3.0\sigma_{ij}$, $i,j=1,2,3$, we obtain

$$\mathbf{C}=\begin{pmatrix} 4.5 & 8.2 & 7.6 \\ 8.4 & 5.9 & 3.1 \\ 7.0 & 5.8 & 11.0 \end{pmatrix}.$$

Arrange these values in ascending order, that is,

$$c^1=3.1<c^2=4.5<c^3=5.8<c^4=5.9<c^5=7.6<c^6=7.9$$
$$<c^7=8.2<c^8=8.4<c^9=11.0.$$

For $u=1,\dots,8$, $k=1,\dots,9$, set

$$c_{ij}^{u,k}=\begin{cases} 0 & \text{if } \mu_{ij}\ge\mu^u,\ m_{ij}+3.0\sigma_{ij}\le c^k \\ M & \text{otherwise} \end{cases},\quad i,j=1,2,3,$$

where M is a sufficiently large value.

It is obvious that $p=6$, $q=3$.

For $u=1,\dots,6$, $k=3,\dots,9$, problem P_u^k has the following form:

$$\begin{array}{lll} \mathrm{P}_u^k: & \text{minimize} & \sum\limits_{i=1}^{3}\sum\limits_{j=1}^{3} c_{ij}^{u,k}x_{ij} \\ & \text{subject to} & \sum\limits_{j=1}^{3} x_{ij} \le a_i,\ i=1,2,3 \\ & & \sum\limits_{i=1}^{3} x_{ij} \ge b_j,\ j=1,2,3 \\ & & x_{ij}:\ \text{nonnegative},\ i,j=1,2,3. \end{array}$$

Next we give the solution procedure for problem P.

Find the smallest $k \in \{3, \dots, 9\}$ such that $\mathbf{P}_1^k$ is feasible:

Step 1. Set $L = 3$ and P_1^3 is infeasible. Set $U = 9$ and P_1^9 is feasible. Go to Step 2.

Step 2. $U - L = 6 \neq 1$. Set $K = 6$ and P_1^6 is feasible. Set $U = 6$, repeat Step 2.

Step 2. $U - L = 3 \neq 1$. Set $K = 4$ and P_1^4 is infeasible. Set $L = 4$, repeat Step 2.

Step 2. $U - L = 2 \neq 1$. Set $K = 5$ and P_1^5 is infeasible. Set $L = 5$, repeat Step 2.

Step 2. $U - L = 1$, so go to Step 3.

Step 3. P_1^5 is infeasible, so set $k_1 = 6$.

Find the smallest $k \in \{6, \dots, 9\}$ such that $\mathbf{P}_2^k$ is feasible:

Step 1. Set $L = 6$ and P_2^6 is feasible. Set $k_2 = 6$.

Find the smallest $k \in \{6, \dots, 9\}$ such that $\mathbf{P}_3^k$ is feasible:

Step 1. Set $L = 6$ and P_3^6 is infeasible. Set $U = 9$ and P_3^9 is feasible. Go to Step 2.

Step 2. $U - L = 3 \neq 1$. Set $K = 7$ and P_3^7 is infeasible. Set $L = 7$, repeat Step 2.

Step 2. $U - L = 2 \neq 1$. Set $K = 8$ and P_3^8 is feasible. Set $U = 8$, repeat Step 2.

Step 2. $U - L = 1$, so go to Step 3.

Step 3. P_3^7 is infeasible, so set $k_3 = 8$.

Find the smallest $k \in \{8, 9\}$ such that $\mathbf{P}_4^k$ is feasible:

Step 1. Set $L = 8$ and P_4^8 is feasible. Set $k_4 = 8$.

Find the smallest $k \in \{8, 9\}$ such that $\mathbf{P}_5^k$ is feasible:

Step 1. Set $L = 8$ and P_5^8 is infeasible. Set $U = 9$ and P_5^9 is infeasible. Therefore, there exists no $k \in \{8, 9\}$ such that P_5^k is feasible. From Remark 3.1, such a case also holds for P_6^k.

Therefore $B = \{(2, 6), (4, 8)\}$. Solve P_2^6 and P_4^8, we obtain the optimal transportation patterns $\mathbf{x}_2^6$ and $\mathbf{x}_4^8$, respectively:

$$\mathbf{x}_2^6: \quad x_{11} = 50,\ x_{22} = 15,\ x_{23} = 55,\ x_{31} = 10,\ x_{32} = 20,\ \text{other } x_{ij} = 0,$$

$$\mathbf{x}_4^8: \quad x_{12} = 35,\ x_{21} = 30,\ x_{23} = 55,\ x_{31} = 30,\ \text{other } x_{ij} = 0,$$

which are the non-dominated transportation patterns of problem P, and the corresponding bi-objective vectors are $(7.9, 0.5)$ and $(8.4, 0.7)$, respectively. That is,

$$NDT = \{\mathbf{x}_2^6, \mathbf{x}_4^8\},$$

$$NDV = \{(7.9, 0.5), (8.4, 0.7)\}.$$

5. CONCLUSION

In this paper, we have considered a bi-criteria chance constrained bottleneck transportation problem with preference of routes and developed an algorithm to find some non-dominated transportation patterns. Further, we have shown the validity and time complexity of the algorithm. Besides, our algorithm is illustrated using a numerical example. As a further research problem, we should consider the flexibility of supply and demand quantity, which is the case that the total quantity from supplies is less than that to demand customers. This case makes the problem three criteria one and we are now attacking this case. Additionally, there remain many other variants of bottleneck transportation problem to be considered and solved.

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