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# DETECTION OF TRANSIENT CHANGE IN MEAN – A LINEAR BEHAVIOR INSIDE EPIDEMIC INTERVAL

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A procedure for testing occurrance of a transient change in mean of a sequence is suggested where inside an epidemic interval the mean is a linear function of time points. Asymptotic behavior of considered trimmed maximum-type test statistics is presented. Approximate critical values are obtained using an approximation of exceedance probabilities over a high level by Gaussian fields with a locally stationary structure.

*Keywords:* detection of transient change, trimmed maximum-type test statistic, extremes of Gaussian fields

*Classification:* 62F05, 60G60, 60G70

## 1. INTRODUCTION

A sequence of independent identically distributed random variables $X_1, \dots, X_n$ that correspond to some observations taken at time points $i = 1, \dots, n$ represents a basic stochastic model. However, it can happen that in an interval $[k_1+1, k_2] \subset [1, n]$ with an unknown beginning $k_1 + 1$ and an unknown end $k_2$ a subsequence $X_{k_1+1}, \dots, X_{k_2}$ behaves differently from the rest of the sequence, e. g., the behavior of the mean of the observations inside this interval differs from the mean of the observations outside it. The simplest situation occurs when the observations taken in the interval $[k_1+1, k_2]$ vary around some constant mean value while the observations taken outside the interval vary around another constant mean. Such model was called by Levin and Kline [10] an "epidemic model" because they applied it for modeling an epidemic outbreak in medical applications. The epidemic model was studied in details by Antoch and Hušková [1] and it was also treated by Csörgő and Horváth [5]. The epidemic models as well as other models with a "transient change" may be applied not only in medicine but also in signal detection or pattern recognition. Motivated by these applications Loader [11], Siegmund [13], Siegmund and Venkatranan [14] and Siegmund and Yakir [15] obtained very interesting results that are closely related to the results of this paper.

It is clear that the mean in an "epidemic interval" $[k_1+1, k_2]$ need not be constant but it can behave for instance as a function of time points $i = k_1 + 1, \dots, k_2$ that is linear in some unknown parameters. In this paper we study the situation when the mean in $[k_1 + 1, k_2]$ is a linear function of time while outside the interval it is constant.

The epidemic models or models with a transient change belong to the change point models. In change point analysis statistical inference has usually two steps. In the first step we decide whether there is a change and if the answer is positive then in the second step we estimate change points together with parameters of the model. Our paper is devoted to the decision problems. The problem of estimation with a constant mean in an epidemic interval was solved by Hušková [7]. General asymptotic theory for change points estimators in linear models can be found in Feder [6] and Bai and Perron [2].

The decision problems in change point analysis are solved by hypotheses testing. For decision, whether the null hypothesis that claims that all observations have the same mean is to be rejected, we suggest to apply maximum-type test statistics. The maximum-type test statistics arise in a natural way. First, we choose a test statistic for testing the null hypothesis against the corresponding alternative with fixed known change point(s). Then, we calculate values of such statistics for all considered positions of the change point(s). The null hypothesis is rejected if at least one of these values is larger than a chosen critical value, i. e., if the maximal value is larger than a chosen critical value. In linear models with known change points the test statistic is often based on the least squares estimates of the parameters of the models that describe stochastic behavior of the observations taken in intervals between the change points. To get reasonable estimates of these parameters we will assume that at least a certain given proportion of all time points (say $100\,\alpha\,\%$, where $\alpha$ is a chosen small number) is situated inside the epidemic interval and in case the mean outside the epidemic interval is also unknown then we assume that a certain given proportion of time points is situated outside the interval. In the other words we propose to use so called trimmed maximum-type test statistics (the maximum is taken over all "reasonable" pairs of time points) instead of over-all maximum-type test statistics. The disadvantage of the trimmed maximum-type test statistics is that the value of $\alpha$ has to be chosen subjectively and the choice affects critical values significantly. On the other side, using the "functional central limit theorem", see Bickel and Wichura [3], we can easily obtain asymptotic distributions of test statistics under the null hypothesis. The limit variables have a form of maxima of Gaussian random fields and with the help of the theory of extremes of Gaussian fields with a locally stationary structure, see Piterbarg [12], we can obtain approximations of distribution functions for large argument values. These approximations provide us with approximate critical values.

## 2. DETECTION OF TRANSIENT CHANGE WHEN THE MEAN IN EPIDEMIC INTERVAL IS CONSTANT

We start with the alternative claiming that the mean inside an epidemic interval is constant. The problem was solved by Jarušková and Piterbarg [8] and that is why we treat it only briefly.

Suppose that we observe a random sequence $X_1, \ldots, X_n$. We would like to test the null hypothesis $H_0$ against the alternative $A_1$:

$$\begin{aligned} H_0 : X_i &= \mu + e_i, & i &= 1,\dots,n, \\ A_1 : \exists \quad 1 &\le k_1 < k_2 < n & &\text{such that} \\ X_i &= \mu + e_i, & i &= 1,\dots,k_1, \\ X_i &= \mu + \Delta_\mu + e_i, & i &= k_1+1,\dots,k_2, \\ X_i &= \mu + e_i, & i &= k_2+1,\dots,n. \end{aligned} \tag{1}$$

We suppose that $\Delta_\mu \neq 0$ is an unknown real number. The error terms $\{e_i\}$ are i.i.d. and $E\,e_i = 0$, $E\,e_i^2 = \sigma^2$ (known) and $E\,|e_i|^{2+\Delta} < \infty$ for some $\Delta > 0$.

First, we start with a known baseline value $\mu$. Without a loss of generality we can set $\mu = 0$. (This holds not only here but in all our testing problems.) For a given $\alpha$ the trimmed maximum-type test statistics look as follows:

$$\max_{\substack{1\le k_1<k_2<n \\ [\alpha\, n]\le k_2-k_1}} \frac{\sum_{i=k_1+1}^{k_2} X_i}{\sigma\sqrt{k_2-k_1}} \quad \text{(for one-sided alternative)}, \tag{2}$$

$$\max_{\substack{1\le k_1<k_2<n \\ [\alpha\, n]\le k_2-k_1}} \frac{\left|\sum_{i=k_1+1}^{k_2} X_i\right|}{\sigma\sqrt{k_2-k_1}} \quad \text{(for two-sided alternative)}. \tag{3}$$

For an unknown baseline $\mu$ the test statistics have the form:

$$\max_{\substack{1\le k_1<k_2<n \\ [\alpha\, n]\le k_2-k_1\le[(1-\alpha)\, n]}} \frac{\sum_{i=k_1+1}^{k_2} (X_i-\bar X)}{\sigma\sqrt{(k_2-k_1)\big(1-(k_2-k_1)/n\big)}} \quad \text{(for one-sided alternative)}, \tag{4}$$

$$\max_{\substack{1\le k_1<k_2<n \\ [\alpha\, n]\le k_2-k_1\le[(1-\alpha)\, n]}} \frac{\left|\sum_{i=k_1+1}^{k_2} (X_i-\bar X)\right|}{\sigma\sqrt{(k_2-k_1)\big(1-(k_2-k_1)/n\big)}} \quad \text{(for two-sided alternative)}. \tag{5}$$

We introduce random fields $\{U_W(t_1,t_2) = \big(W(t_2)-W(t_1)\big)/\sqrt{t_2-t_1},\ 0\le t_1<t_2\le 1\}$ and $\{U_B(t_1,t_2) = \big(B(t_2)-B(t_1)\big)/\sqrt{(t_2-t_1)\big(1-(t_2-t_1)\big)},\ 0\le t_1<t_2\le 1\}$, where $\{W(t), 0\le t\le 1\}$ is a Wiener process and $\{B(t), 0\le t\le 1\}$ is a Brownian bridge. Using the "functional central limit theorem", see Bickel and Wichura [3], we get (as

$n \to \infty$) that under $H_0$ it holds:

$$\max_{\substack{1\le k_1<k_2<n\\ [\alpha n]\le k_2-k_1}} \frac{\sum_{i=k_1+1}^{k_2} X_i}{\sigma\sqrt{k_2-k_1}} \xrightarrow{D} \max_{\substack{0\le t_1<t_2<1\\ \alpha\le t_2-t_1}} U_W(t_1,t_2),$$

$$\max_{\substack{1\le k_1<k_2<n\\ [\alpha n]\le k_2-k_1}} \frac{\left|\sum_{i=k_1+1}^{k_2} X_i\right|}{\sigma\sqrt{k_2-k_1}} \xrightarrow{D} \max_{\substack{0\le t_1<t_2<1\\ \alpha\le t_2-t_1}} |U_W(t_1,t_2)|,$$

$$\max_{\substack{1\le k_1<k_2<n\\ [\alpha n]\le k_2-k_1\le[(1-\alpha)n]}} \frac{\frac{1}{\sqrt{n}}\sum_{i=k_1+1}^{k_2}(X_i-\bar{X})}{\sigma\sqrt{\frac{k_2-k_1}{n}\left(1-\frac{k_2-k_1}{n}\right)}} \xrightarrow{D} \max_{\substack{0\le t_1<t_2<1\\ \alpha\le t_2-t_1\le 1-\alpha}} U_B(t_1,t_2),$$

$$\max_{\substack{1\le k_1<k_2<n\\ [\alpha n]\le k_2-k_1\le[(1-\alpha)n]}} \frac{\frac{1}{\sqrt{n}}\left|\sum_{i=k_1+1}^{k_2}(X_i-\bar{X})\right|}{\sigma\sqrt{\frac{k_2-k_1}{n}\left(1-\frac{k_2-k_1}{n}\right)}} \xrightarrow{D} \max_{\substack{0\le t_1<t_2<1\\ \alpha\le t_2-t_1\le 1-\alpha}} |U_B(t_1,t_2)|.$$

**Theorem 2.1.** As $u \to \infty$, it holds

$$P\Big(\max_{\substack{0\le t_1<t_2<1\\ \alpha\le t_2-t_1}} U_W(t_1,t_2)>u\Big) \sim \frac{1}{4}\Big(\frac{1}{\alpha}+\log\alpha-1\Big)\,u^4\,\big(1-\Phi(u)\big),$$

$$P\Big(\max_{\substack{0\le t_1<t_2<1\\ \alpha\le t_2-t_1}} |U_W(t_1,t_2)|>u\Big) \sim \frac{1}{2}\Big(\frac{1}{\alpha}+\log\alpha-1\Big)\,u^4\,\big(1-\Phi(u)\big),$$

$$P\Big(\max_{\substack{0\le t_1<t_2<1\\ \alpha\le t_2-t_1\le 1-\alpha}} U_B(t_1,t_2)>u\Big) \sim \frac{1}{4}\Big(\frac{1}{\alpha}+2\log\big(\frac{1-\alpha}{\alpha}\big)-\frac{1}{1-\alpha}\Big)u^4\big(1-\Phi(u)\big),$$

$$P\Big(\max_{\substack{0\le t_1<t_2<1\\ \alpha\le t_2-t_1\le 1-\alpha}} |U_B(t_1,t_2)|>u\Big) \sim \frac{1}{2}\Big(\frac{1}{\alpha}+2\log\big(\frac{1-\alpha}{\alpha}\big)-\frac{1}{1-\alpha}\Big)u^4\big(1-\Phi(u)\big).$$

P r o o f.
The zero mean unit variance Gaussian field $\{U_W(t_1,t_2),\, 0\le t_1<t_2\le 1\}$ has the covariance function $r_W(t_1,t_2;s_1,s_2)$ satisfying

$$r_W(t_1,t_2;t_1+h,t_2+k) = 1-\frac{1}{2(t_2-t_1)}|h|-\frac{1}{2(t_2-t_1)}|k|+o(|h|+|k|)\ \text{as}\ h\to 0, k\to 0.$$

The zero mean unit variance Gaussian field $\{U_B(t_1,t_2),\, 0\le t_1<t_2\le 1\}$ has the covariance function $r_B(t_1,t_2;s_1,s_2)$ satisfying

$$r_B(t_1,t_2;t_1+h,t_2+k) = 1-\frac{1}{2(t_2-t_1)\big(1-(t_2-t_1)\big)}|h| - \frac{1}{2(t_2-t_1)\big(1-(t_2-t_1)\big)}|k|+o(|h|+|k|) \quad \text{as} \quad h\to 0, k\to 0.$$

Both fields are fields with a locally stationary structure and Theorem A.1 may be applied with

$$I_A = \iint_{\substack{0\le t_1<t_2\le 1\\ \alpha\le t_2-t_1}} \frac{1}{4\,(t_2-t_1)^2}\,\mathrm{d}t_1\mathrm{d}t_2 = \frac{1}{4}\Big(\frac{1}{\alpha}+\log\alpha-1\Big),$$

respectively

$$I_A = \iint_{\substack{0\le t_1<t_2\le 1\\ \alpha\le t_2-t_1\le(1-\alpha)}} \frac{1}{4\,(t_2-t_1)^2(1-(t_2-t_1))^2}\,\mathrm{d}t_1\mathrm{d}t_2$$
$$= \frac{1}{4}\Big(\frac{1}{\alpha}+2\log\big(\frac{1-\alpha}{\alpha}\big)-\frac{1}{1-\alpha}\Big).$$

□

Tables 1 − 4 present approximate critical values of (2), (3), (4), (5) calculated according to Theorem 2.1.

| | 5% crit.v. | 1% crit.v. |
|---|---|---|
| $\alpha = 0.05$ | 3.862 | 4.343 |
| $\alpha = 0.10$ | 3.559 | 4.093 |

**Table 1.** Approximate critical values of (2) – $\mu$ is known.

| | 5% crit.v. | 1% crit.v. |
|---|---|---|
| $\alpha = 0.05$ | 4.002 | 4.462 |
| $\alpha = 0.10$ | 3.801 | 4.291 |

**Table 2.** Approximate critical values of (4) – $\mu$ is unknown.

| | 5% crit.v. | 1% crit.v. |
|---|---|---|
| $\alpha = 0.05$ | 4.080 | 4.528 |
| $\alpha = 0.10$ | 3.803 | 4.294 |

**Table 3.** Approximate critical values of (3) – $\mu$ is known.

| | 5% crit.v. | 1% crit.v. |
|---|---|---|
| $\alpha = 0.05$ | 4.209 | 4.641 |
| $\alpha = 0.10$ | 4.023 | 4.480 |

**Table 4.** Approximate critical values of (5) – $\mu$ is unknown.

## 3. DETECTION OF TRANSIENT CHANGE WHEN THE MEAN IN EPIDEMIC INTERVAL IS A LINEAR FUNCTION WITH DISCONTINUITIES AT CHANGE POINTS

We consider the following testing problem:

$$\begin{aligned}
&H_0 : X_i = \mu + e_i, && i = 1,\dots,n, \qquad (6)\\
&A_2 : \exists \quad 1 \le k_1 < k_2 < n && \text{such that}\\
&\quad X_i = \mu + e_i, && i = 1,\dots,k_1,\\
&\quad X_i = \mu + \Delta_\mu + b\Big(\frac{i}{n} - \frac{k_1+k_2+1}{2n}\Big) + e_i, && i = k_1+1,\dots,k_2,\\
&\quad X_i = \mu + e_i, && i = k_2+1,\dots,n,
\end{aligned}$$

where $\Delta_\mu \neq 0$ and/or $b \neq 0$. The random errors $\{e_i\}$ have the same properties as before. Notice that the general linear function inside the interval $[k_1+1,\dots,k_2]$ is parametrized as $y_i = \Delta_\mu + b\Big(i/n - (k_1+k_2+1)/(2n)\Big)$, $i = k_1+1,\ldots,k_2$, with $(k_1+k_2+1)/2n$ being the average of $(k_1+1)/n,\ldots,k_2/n$.

First again, we consider the situation when $\mu$ is known so that we can assume $\mu = 0$. Under the assumption that $k_1$ and $k_2$ are known and fixed, the least squares estimates of $\Delta_\mu$ and $b$ have the form:

$$\widehat{\Delta}_\mu = \frac{\sum_{i=k_1+1}^{k_2} X_i}{k_2-k_1}, \quad \widehat{b} = \frac{\sum_{i=k_1+1}^{k_2}\left(\frac{i}{n} - \frac{k_1+k_2+1}{2n}\right)X_i}{\sum_{i=k_1+1}^{k_2}\left(\frac{i}{n} - \frac{k_1+k_2+1}{2n}\right)^2}.$$

Under $H_0$ the estimates $\widehat{\Delta}_\mu$ and $\widehat{b}$ are uncorrelated (due to the parametrization of the linear function inside the epidemic interval) and for $n$ large the statistic

$$\chi_1^2(k_1,k_2) = \frac{\left(\sum_{i=k_1+1}^{k_2} X_i\right)^2}{\sigma^2(k_2-k_1)} + \frac{\left(\sum_{i=k_1+1}^{k_2}\left(\frac{i}{n} - \frac{k_1+k_2+1}{2n}\right)X_i\right)^2}{\sigma^2\sum_{i=k_1+1}^{k_2}\left(\frac{i}{n} - \frac{k_1+k_2+1}{2n}\right)^2}$$

has approximately a $\chi^2$ distribution with two degrees of freedom. The trimmed maximum-type test statistic has the form:

$$\max_{\substack{1\le k_1<k_2<n\\ [\alpha\, n]\le k_2-k_1}} \chi_1^2(k_1,k_2) \tag{7}$$

and using the "functional central limit theorem", see Bickel and Wichura [3], it follows that under $H_0$ it converges in distribution (as $n\to\infty$) to a maximum of a $\chi^2$ random field:

$$\max_{\substack{1\le k_1<k_2<n\\ [\alpha\, n]\le k_2-k_1}} \chi_1^2(k_1,k_2) \xrightarrow{D} \max_{\substack{0\le t_1<t_2<1\\ \alpha\le t_2-t_1}} \big(U_W(t_1,t_2)\big)^2 + \big(V(t_1,t_2)\big)^2.$$

Similarly as $\{U_W(t_1,t_2)\}$, the field

$$\left\{V(t_1,t_2) = \frac{\int_{t_1}^{t_2}\left(s - \frac{t_2+t_1}{2}\right)\mathrm{d}W(s)}{\sqrt{\frac{(t_2-t_1)^2}{12}}},\ 0\le t_1<t_2\le 1\right\}$$

is a zero mean unit variance Gaussian field. A traditional way how to deal with a maximum of a $\chi^2$ process or a field is to use the identity:

$$\begin{aligned}\max_{\substack{0\le t_1<t_2<1\\ \alpha\le t_2-t_1}} &\sqrt{\big(U_W(t_1,t_2)\big)^2 + \big(V(t_1,t_2)\big)^2}\\ &= \max_{\substack{0\le t_1<t_2<1\\ \alpha\le t_2-t_1}} \max_{-\pi\le\theta\le\pi} U_W(t_1,t_2)\cos\theta + V(t_1,t_2)\sin\theta.\end{aligned}$$

The random field $\{X_1(t_1,t_2,\theta) = U_W(t_1,t_2)\cos\theta + V(t_1,t_2)\sin\theta, 0\le t_1<t_2\le 1, -\pi\le\theta\le\pi\}$ is a zero mean unit variance Gaussian field with a covariance function

$$\begin{aligned}r_{X1}(t_1,t_2,\theta;s_1,s_2,\psi) &= r_{11}(t_1,t_2;s_1,s_2)\cos\theta\cos\psi + r_{22}(t_1,t_2;s_1,s_2)\sin\theta\sin\psi\\ &\quad + r_{12}(t_1,t_2;s_1,s_2)\cos\theta\sin\psi + r_{21}(t_1,t_2;s_1,s_2)\sin\theta\cos\psi,\end{aligned}$$

where $r_{11}(t_1,t_2;s_1,s_2)$ is the covariance function of $\{U_W(t_1,t_2)\}$, $r_{22}(t_1,t_2;s_1,s_2)$ is the covariance function of $\{V(t_1,t_2)\}$ and $r_{12}(t_1,t_2;s_1,s_2)$ and $r_{21}(t_1,t_2;s_1,s_2)$ are the mixed covariance functions. For $h\to 0$, $k\to 0$ and $\phi\to 0$ it holds

$$r_{X1}(t_1,t_2,\theta;t_1+h,t_2+k,\theta+\phi)=1-\frac{\phi^2}{2}-\frac{1}{2(t_2-t_1)}\big(\cos\theta-\sqrt{3}\sin\theta\big)^2|h| \\ +\frac{1}{2(t_2-t_1)}\big(\cos\theta+\sqrt{3}\sin\theta\big)^2|k|+o(|h|+|k|+\phi^2).$$

Now, we consider the case of an unknown $\mu$. For the known fixed values of change points $1\le k_1<k_2<n$ the least squares estimates of $b$ is the same as before, while the least squares estimates of $\mu$ and $\Delta_\mu$ are:

$$\widehat{\mu}=\frac{\sum_{i=1}^{k_1}X_i+\sum_{i=k_2+1}^{n}X_i}{n-(k_2-k_1)},\quad \widehat{\Delta}_\mu=\frac{n\sum_{i=k_1+1}^{k_2}(X_i-\bar{X})}{(k_2-k_1)\big(n-(k_2-k_1)\big)}.$$

For large $n$ the statistic

$$\chi_2^2(k_1,k_2)=\frac{\Big(\sum_{i=k_1+1}^{k_2}(X_i-\bar{X})\Big)^2}{\sigma^2(k_2-k_1)(1-\frac{k_2-k_1}{n})}+\frac{\Big(\sum_{i=k_1+1}^{k_2}\big(\frac{i}{n}-\frac{k_1+k_2+1}{2n}\big)X_i\Big)^2}{\sigma^2\sum_{i=k_1+1}^{k_2}\big(\frac{i}{n}-\frac{k_1+k_2+1}{2n}\big)^2}$$

has approximately $\chi^2$ distribution with two degrees of freedom. The trimmed maximum-type test statistic has a form:

$$\max_{\substack{1\le k_1<k_2<n\\ [\alpha\, n]\le k_2-k_1\le[(1-\alpha)\, n]}}\chi_2^2(k_1,k_2) \tag{8}$$

and under $H_0$ it converges in distribution (as $n\to\infty$) to a limit variable:

$$\max_{\substack{1\le k_1<k_2<n\\ [\alpha\, n]\le k_2-k_1\le[(1-\alpha)\, n]}}\chi_2^2(k_1,k_2)\ \xrightarrow{D}\ \max_{\substack{0\le t_1<t_2<1\\ \alpha\le t_2-t_1\le 1-\alpha}}\big(U_B(t_1,t_2)\big)^2+\big(V(t_1,t_2)\big)^2.$$

Similarly as before,

$$\max_{\substack{0\le t_1<t_2<1\\ \alpha\le t_2-t_1<(1-\alpha)}}\sqrt{\big(U_B(t_1,t_2)\big)^2+\big(V(t_1,t_2)\big)^2}=\max_{\substack{0\le t_1<t_2<1\\ \alpha\le t_2-t_1\le 1-\alpha}}\max_{-\pi\le\theta\le\pi}X_2(t_1,t_2,\theta),$$

where $\{X_2(t_1,t_2,\theta)=U_B(t_1,t_2)\cos\theta+V(t_1,t_2)\sin\theta, 0\le t_1<t_2\le 1, -\pi\le\theta\le\pi\}$ is a zero mean unit variance Gaussian field with a covariance function $r_{X2}(t_1,t_2,\theta;s_1,s_2,\psi)$. For $h\to 0$, $k\to 0$ and $\varphi\to 0$ it holds:

$$r_{X2}(t_1,t_2,\theta;t_1+h,t_2+k,\theta+\varphi)=1-\frac{\varphi^2}{2} \\ -\frac{1}{2(t_2-t_1)}\Big(\frac{\cos\theta}{\sqrt{1-(t_2-t_1)}}-\sqrt{3}\sin\theta\Big)^2|h| \\ +\frac{1}{2(t_2-t_1)}\Big(\frac{\cos\theta}{\sqrt{1-(t_2-t_1)}}+\sqrt{3}\sin\theta\Big)^2|k|+o(|h|+|k|+\varphi^2).$$

**Theorem 3.1.** For $u \to \infty$ it holds

$$P\Big(\max_{\substack{0\le t_1<t_2<1\\ \alpha\le t_2-t_1}} \big(U_W(t_1,t_2)\big)^2 + \big(V(t_1,t_2)\big)^2 > u^2\Big) \quad \sim \frac{1}{\sqrt{\pi}} C_W\, u^5\big(1-\Phi(u)\big),$$

$$P\Big(\max_{\substack{0\le t_1<t_2<1\\ \alpha\le t_2-t_1\le 1-\alpha}} \big(U_B(t_1,t_2)\big)^2 + \big(V(t_1,t_2)\big)^2 > u^2\Big) \sim \frac{1}{\sqrt{\pi}} C_B\, u^5\big(1-\Phi(u)\big),$$

where $C_W = \frac{3}{2\sqrt{2}}\Big(\frac{1}{\alpha} + \log\alpha - 1\Big)\pi$ and $C_B = \frac{\pi}{16\sqrt{2}}\Big(24\big(\frac{1}{\alpha} - \frac{1}{1-\alpha}\big) + 21\log\frac{\alpha}{1-\alpha}\Big)$.

Proof.

The Gaussian fields $\{X_1(t_1,t_2,\theta)\}$ and $\{X_2(t_1,t_2,\theta)\}$ are fields with a locally stationary structure and Theorem A.1 may be applied with

$$I_A = C_W = \frac{1}{\sqrt{2}}\frac{1}{4}\iint_{\substack{0\le t_1<t_2\le 1\\ \alpha\le t_2-t_1}} \frac{1}{(t_2-t_1)^2}\int_{-\pi}^{\pi}(\cos^2\theta - 3\sin^2\theta)^2\, \mathrm{d}\theta \mathrm{d}t_1 \mathrm{d}t_2,$$

$$I_A = C_B$$

$$= \frac{1}{\sqrt{2}}\frac{1}{4}\iint_{\substack{0\le t_1<t_2\le 1\\ \alpha\le t_2-t_1\le 1-\alpha}} \int_{-\pi}^{\pi}\Big(\frac{\cos^2\theta}{(1-t_2+t_1)(t_2-t_1)} - \frac{3\sin^2\theta}{(t_2-t_1)}\Big)^2 \mathrm{d}\theta \mathrm{d}t_1 \mathrm{d}t_2.$$

□

Tables 5 – 6 present approximate critical values of (7), (8) calculated according to Theorem 3.1.

| | 5% crit.v. | 1% crit. v. |
|---|---|---|
| $\alpha = 0.05$ | 4.849 | 5.230 |
| $\alpha = 0.10$ | 4.624 | 5.029 |

**Table 5.** Approximate critical values of (7) – $\mu$ is known.

| | 5% crit.v. | 1% crit. v. |
|---|---|---|
| $\alpha = 0.05$ | 4.855 | 5.235 |
| $\alpha = 0.10$ | 4.635 | 5.038 |

**Table 6.** Approximate critical values of (8) – $\mu$ is unknown.

## 4. DETECTION OF TRANSIENT CHANGE WHEN THE MEAN IN EPIDEMIC INTERVAL IS A LINEAR FUNCTION CONTINUOUS AT ONE CHANGE POINT

We consider the following testing problem:

$$\begin{aligned}
&H_0 : X_i = \mu + e_i, && i = 1,\dots,n, \\
&A_3 : \exists \quad 1\le k_1 < k_2 < n && \text{such that} \\
&\qquad X_i = \mu + e_i, && i = 1,\dots,k_1, \\
&\qquad X_i = \mu + c\Big(\frac{k_2}{n} - \frac{i}{n}\Big) + e_i, && i = k_1+1,\dots,k_2, \\
&\qquad X_i = \mu + e_i, && i = k_2+1,\dots,n,
\end{aligned} \tag{9}$$

where $c \neq 0$. The random errors $\{e_i\}$ have the same properties as before. Notice that the linear function $y_i = \mu + c\,(k_2/n - i/n)$, $i = k_1+1,\dots,k_2$, is continuous at $k_2$.

First again, we consider the situation when $\mu$ is known and therefore, we may assume $\mu = 0$. Under the assumption that $k_1$ and $k_2$ are known and fixed, the least squares estimates of $c$ has the form:

$$\widehat{c} = \frac{\sum_{i=k_1+1}^{k_2}\left(\frac{k_2}{n} - \frac{i}{n}\right)X_i}{\sum_{i=k_1+1}^{k_2}\left(\frac{k_2}{n} - \frac{i}{n}\right)^2}.$$

Under $H_0$ the trimmed maximum-type test statistics

$$\max_{\substack{1\le k_1<k_2<n\\ [\alpha\, n]\le k_2-k_1}} \frac{\sum_{i=k_1+1}^{k_2}\left(\frac{k_2}{n} - \frac{i}{n}\right)X_i}{\sigma\sqrt{\sum_{i=k_1+1}^{k_2}\left(\frac{k_2}{n} - \frac{i}{n}\right)^2}} \quad \text{(for one-sided alternative)} \tag{10}$$

$$\max_{\substack{1\le k_1<k_2<n\\ [\alpha\, n]\le k_2-k_1}} \frac{\left|\sum_{i=k_1+1}^{k_2}\left(\frac{k_2}{n} - \frac{i}{n}\right)X_i\right|}{\sigma\sqrt{\sum_{i=k_1+1}^{k_2}\left(\frac{k_2}{n} - \frac{i}{n}\right)^2}} \quad \text{(for two-sided alternative)} \tag{11}$$

converge in distribution (as $n \to \infty$):

$$\max_{\substack{1\le k_1<k_2<n\\ [\alpha\, n]\le k_2-k_1}} \frac{\sum_{i=k_1+1}^{k_2}\left(\frac{k_2}{n} - \frac{i}{n}\right)X_i}{\sigma\sqrt{\sum_{i=k_1+1}^{k_2}\left(\frac{k_2}{n} - \frac{i}{n}\right)^2}} \xrightarrow{D} \max_{\substack{0\le t_1<t_2<1\\ \alpha\le t_2-t_1}} \frac{\int_{t_1}^{t_2}(t_2-s)\,\mathrm{d}W(s)}{\sqrt{(t_2-t_1)^3/3}},$$

$$\max_{\substack{1\le k_1<k_2<n\\ [\alpha\, n]\le k_2-k_1}} \frac{\left|\sum_{i=k_1+1}^{k_2}\left(\frac{k_2}{n} - \frac{i}{n}\right)X_i\right|}{\sigma\sqrt{\sum_{i=k_1+1}^{k_2}\left(\frac{k_2}{n} - \frac{i}{n}\right)^2}} \xrightarrow{D} \max_{\substack{0\le t_1<t_2<1\\ \alpha\le t_2-t_1}} \frac{\left|\int_{t_1}^{t_2}(t_2-s)\,\mathrm{d}W(s)\right|}{\sqrt{(t_2-t_1)^3/3}}.$$

The covariance function $r_{X3}(t_1,t_2;s_1,s_2)$ of a zero mean unit variance Gaussian field $\{X_3(t_1,t_2) = \int_{t_1}^{t_2}(t_2-s)\,\mathrm{d}W(s)/\sqrt{(t_2-t_1)^3/3};\ 0\le t_1 < t_2 \le 1\}$ has an expansion:

$$r_{X3}(t_1,t_2;t_1+h,t_2+k) = 1 - \frac{3}{2(t_2-t_1)}|h| - \frac{3}{8(t_2-t_1)^2}k^2 + o(|h|+k^2)$$
$$\text{as} \quad h\to 0, k\to 0.$$

For $\mu$ unknown the trimmed maximum-type test statistics have the form for the one-sided alternative:

$$\max_{\substack{1\le k_1<k_2<n\\ [\alpha\, n]\le k_2-k_1\le[(1-\alpha)n]}} \frac{\sum_{i=k_1+1}^{k_2}\left(\frac{k_2}{n} - \frac{i}{n}\right)(X_i-\bar{X})}{\sigma\sqrt{\sum_{i=k_1+1}^{k_2}\left(\frac{k_2}{n} - \frac{i}{n}\right)^2 - \frac{1}{n}\left(\sum_{i=k_1+1}^{k_2}\left(\frac{k_2}{n} - \frac{i}{n}\right)\right)^2}} \tag{12}$$

and for the two-sided alternative:

$$\max_{\substack{1\le k_1<k_2<n\\ [\alpha\, n]\le k_2-k_1\le[(1-\alpha)n]}} \frac{\left|\sum_{i=k_1+1}^{k_2}\left(\frac{k_2}{n} - \frac{i}{n}\right)(X_i-\bar{X})\right|}{\sigma\sqrt{\sum_{i=k_1+1}^{k_2}\left(\frac{k_2}{n} - \frac{i}{n}\right)^2 - \frac{1}{n}\left(\sum_{i=k_1+1}^{k_2}\left(\frac{k_2}{n} - \frac{i}{n}\right)\right)^2}}. \tag{13}$$

Under $H_0$ they converge in distribution (as $n \to \infty$) to:

$$\max_{\substack{0 \le t_1 < t_2 < 1 \\ \alpha \le t_2 - t_1 < (1-\alpha)}} \frac{\int_{t_1}^{t_2} (t_2 - s)\,\mathrm{d}W(s) - W(1)\frac{(t_2-t_1)^2}{2}}{\sqrt{\frac{(t_2-t_1)^3}{3} - \frac{(t_2-t_1)^4}{4}}},$$

resp. to:

$$\max_{\substack{0 \le t_1 < t_2 < 1 \\ \alpha \le t_2 - t_1 < (1-\alpha)}} \frac{\left| \int_{t_1}^{t_2} (t_2 - s)\,\mathrm{d}W(s) - W(1)\frac{(t_2-t_1)^2}{2} \right|}{\sqrt{\frac{(t_2-t_1)^3}{3} - \frac{(t_2-t_1)^4}{4}}}.$$

The covariance function $r_{X4}(t_1, t_2; s_1, s_2)$ of a zero mean unit variance Gaussian field $\{X_4(t_1, t_2) = \big(\int_{t_1}^{t_2} (t_2 - s)\,\mathrm{d}W(s) - W(1)\frac{(t_2-t_1)^2}{2}\big)/\sqrt{\frac{(t_2-t_1)^3}{3} - \frac{(t_2-t_1)^4}{4}}; 0 \le t_1 < t_2 \le 1\}$ has an expansion:

$$\begin{aligned} r_{X4}(t_1, t_2; t_1 + h, t_2 + k) &= 1 - \frac{6}{(t_2 - t_1)\big(4 - 3(t_2 - t_1)\big)} |h| \\ &- \frac{6\big(1 - (t_2 - t_1)\big)\big(10 - 9(t_2 - t_1)\big)}{(t_2 - t_1)^2 \big(4 - 3(t_2 - t_1)\big)^2} k^2 + o(|h| + k^2) \quad \text{as} \quad h \to 0, k \to 0. \end{aligned}$$

**Theorem 4.1.** For $u \to \infty$ it holds

$$P\Big(\max_{\substack{0 \le t_1 < t_2 < 1 \\ \alpha \le t_2 - t_1}} X_3(t_1, t_2) > u\Big) \sim \frac{1}{\sqrt{\pi}} \frac{3\sqrt{3}}{4\sqrt{2}} \Big(\frac{1}{\alpha} + \log \alpha - 1\Big) u^3 \big(1 - \Phi(u)\big),$$

$$P\Big(\max_{\substack{0 \le t_1 < t_2 < 1 \\ \alpha \le t_2 - t_1}} |X_3(t_1, t_2)| > u\Big) \sim 2\,\frac{1}{\sqrt{\pi}} \frac{3\sqrt{3}}{4\sqrt{2}} \Big(\frac{1}{\alpha} + \log \alpha - 1\Big) u^3 \big(1 - \Phi(u)\big),$$

$$P\Big(\max_{\substack{0 \le t_1 < t_2 < 1 \\ \alpha \le t_2 - t_1 \le (1-\alpha)}} X_4(t_1, t_2) > u\Big) \sim \frac{1}{\sqrt{\pi}} C_{4\alpha}\, u^3 \big(1 - \Phi(u)\big),$$

$$P\Big(\max_{\substack{0 \le t_1 < t_2 < 1 \\ \alpha \le t_2 - t_1 \le (1-\alpha)}} |X_4(t_1, t_2)| > u\Big) \sim 2\,\frac{1}{\sqrt{\pi}} C_{4\alpha}\, u^3 \big(1 - \Phi(u)\big),$$

where $C_{4\alpha} = 6\sqrt{6} \int_{\alpha}^{1-\alpha} \frac{(1-x)^{3/2}(10-9x)^{1/2}}{x^2(4-3x)^2}\,\mathrm{d}x$.

P r o o f.
The Gaussian fields $\{X_3(t_1, t_2), 0 \le t_1 < t_2 \le 1\}$ and $\{X_4(t_1, t_2), 0 \le t_1 < t_2 \le 1\}$ are fields with a locally stationary structure and Theorem A.1 can be applied. □

Tables 7 – 10 present approximate critical values of (10), (11), (12), (13) calculated with the help of Theorem 4.1.

| | 5% crit.v. | 1% crit. v. |
|---|---|---|
| $\alpha = 0.05$ | 3.668 | 4.146 |
| $\alpha = 0.10$ | 3.370 | 3.897 |

**Table 7.** Approximate critical values of (10) – $\mu$ is known.

| | 5% crit.v. | 1% crit. v. |
|---|---|---|
| $\alpha = 0.05$ | 4.039 | 4.467 |
| $\alpha = 0.10$ | 3.795 | 4.254 |

**Table 8.** Approximate critical values of (12) – $\mu$ is unknown.

| | 5% crit.v. | 1% crit. v. |
|---|---|---|
| $\alpha = 0.05$ | 3.883 | 4.331 |
| $\alpha = 0.10$ | 3.610 | 4.097 |

**Table 9.** Approximate critical values of (11) – $\mu$ is known.

| | 5% crit.v. | 1% crit. v. |
|---|---|---|
| $\alpha = 0.05$ | 4.230 | 4.636 |
| $\alpha = 0.10$ | 4.001 | 4.434 |

**Table 10.** Approximate critical values of (13) – $\mu$ is unknown.

## 5. REMARKS

**Remark 5.1.** In the considered hypotheses testing problems we supposed that the variance $\sigma^2$ is known. If $\sigma^2$ is unknown it can be replaced by any consistent estimate, e. g. by $\widehat{\sigma^2} = \sum_{i=1}^n (X_i - \bar{X})^2/n$, but the procedures have a larger power if we estimate $\sigma^2$ by $\widetilde{\sigma^2} = RSS(\widehat{k_1}, \widehat{k_2})/n$, where $RSS(k_1, k_2)$ is a residual sum of squares of the corresponding problem calculated at time points $(\widehat{k_1}, \widehat{k_2})$ that maximize the trimmed maximum-type test statistic.

**Remark 5.2.** Let $\{X_i\}$ be i.i.d. standard normal variables. Kabluchko [9] has shown that for the over-all maximum-type test statistic

$$\max_{1 \le k_1 < k_2 \le n} \frac{\sum_{i=k_1+1}^{k_2} X_i}{\sqrt{k_2 - k_1}}$$

it holds

$$\lim_{n\to\infty} P\Big( \max_{1 \le k_1 < k_2 \le n} \frac{\sum_{i=k_1+1}^{k_2} X_i}{\sqrt{k_2 - k_1}} \le a_n + b_n\, x \Big) = \exp\big( - e^{-x} \big), \quad x \in R^1,$$

where $a_n$ and $b_n$ are given by

$$a_n = \sqrt{2\log n} + \frac{(1/2)\log\log n + \log H - \log 2\sqrt{\pi}}{\sqrt{2\log n}}, \quad b_n = \frac{1}{\sqrt{2\log n}},$$

where

$$H = \int_0^\infty \exp\Big\{ -4 \sum_{k=1}^\infty \frac{1}{k} \Phi\big( -\sqrt{k/(2y)} \big) \Big\}\, \mathrm{d}y \approx 0.21.$$

It seems plausible that similar results might be obtain for other over-all maximum-type test statistics as well. However, application of these asymptotic distributions for calculation of approximate critical values is limited.

**Remark 5.3.** Consider any of our three test procedures. Suppose that the corresponding alternative holds true with $k_1^* = [n\tau_1]$, $k_2^* = [n\tau_2]$, where $0 \le \tau_1 < \tau_2 \le 1$, $\alpha \le \tau_2 - \tau_1$ (for a known baseline $\mu$) or $\alpha \le \tau_2 - \tau_1 \le 1 - \alpha$ (for an unknown baseline $\mu$). Clearly, the trimmed maximum-type test statistic is stochastically larger than the corresponding test statistic for known fixed change points $k_1^* < k_2^*$ which is asymptotically consistent. It follows that the trimmed maximum-type test statistic is also asymptotically consistent.

## A. APPENDIX

Results of this paper are based on Theorem 7.1 of Piterbarg [12] where the approximation of exceedance probability over a high threshold for locally stationary processes is presented. For applications it seems more natural to study directly the expansion of the covariance function of the studied Gaussian field than to study the behavior of this covariance function in the transformed coordinates as in Piterbarg's definition of locally stationary fields. Moreover, we need to consider the situation when the functions in the expansion may be equal to zero on zero Lebesque measure sets. Therefore, we state Theorem 7.1 of Piterbarg [12] in a slight modification.

**Theorem A.1.** Let $\{X(\boldsymbol{x}),\, \boldsymbol{x} \in R^m\}$ be a zero mean unit variance Gaussian field defined on a compact set $A \subset R^m$ with a covariance function $r(\boldsymbol{x};\boldsymbol{y}) = E\,X(\boldsymbol{x})X(\boldsymbol{y})$. We suppose that for $\boldsymbol{x} \in A$, $\boldsymbol{y} \in A$ the covariance function $r(\boldsymbol{x};\boldsymbol{y})$ has the following expansion:

$$\begin{aligned}
&r(x_1,\dots,x_m;\, x_1+h_1,\dots,x_m+h_m)\\
&\qquad = 1 - c_1(x_1,\dots,x_m)|h_1| - \dots - c_p(x_1,\dots,x_m)|h_p|\\
&\qquad - c_{p+1}(x_1,\dots,x_m)h_{p+1}^2 - \dots - c_m(x_1,\dots,x_m)h_m^2\\
&\qquad + o(|h_1| + \dots + |h_p| + h_{p+1}^2 + \dots + h_m^2) \quad \text{as} \quad h_1 \to 0,\, \dots, h_m \to 0,
\end{aligned}$$

where $c_1(x_1,\dots,x_m)$, $\dots$, $c_m(x_1,\dots,x_m)$ are continuous functions on $A$. If we suppose that the Lebesque measure $mes\big\{\boldsymbol{x};\, c_1(\boldsymbol{x}) = 0 \cup \dots \cup c_m(\boldsymbol{x}) = 0\big\} = 0$ then

$$P\big(\max_{\boldsymbol{x}\in A} X(\boldsymbol{x}) > u\big) = \frac{1}{\pi^{(m-p)/2}}\, I_A\, u^{m+p}\big(1-\Phi(u)\big)\big(1+o(1)\big) \quad \text{as} \quad u \to \infty, \tag{14}$$

where $I_A = \int \underset{A}{\cdots} \int c_1(\boldsymbol{x}) \dots c_p(\boldsymbol{x})\big(c_{p+1}(\boldsymbol{x})\big)^{1/2} \dots\ \big(c_m(\boldsymbol{x})\big)^{1/2}\, \mathrm{d}x_1 \dots \mathrm{d}x_m$.

Proof.

First, suppose that all $c_1(\boldsymbol{x}),\dots,c_m(\boldsymbol{x})$ are strictly positive. Then there exist constants $K_1 > 0$, $K_2 > 0$ such that $K_1 \le c_1(\boldsymbol{x}) \le K_2, \dots, K_1 \le c_m(\boldsymbol{x}) \le K_2$ for all $\boldsymbol{x} \in A$. For $\varepsilon > 0$ we can find $\delta > 0$ such that for any $\boldsymbol{x} \in A$ and $\boldsymbol{y} \in A$ such that $||\boldsymbol{x} - \boldsymbol{y}|| < \delta$ it holds

$$\begin{aligned}
1 - c_1(\boldsymbol{x})|y_1 - x_1| - \dots - c_m(\boldsymbol{x})(y_m - x_m)^2 - \frac{\varepsilon}{2}\big(|||y - x|||\big)&\\
\le r(\boldsymbol{x};\boldsymbol{y})&\\
\le 1 - c_1(\boldsymbol{x})|y_1 - x_1| - \dots - c_m(\boldsymbol{x})(y_m - x_m)^2 + \frac{\varepsilon}{2}\big(|||y - x|||\big),&
\end{aligned}$$

($|||a||| = |a_1| + \dots + |a_p| + a_{p+1}^2 + \dots + a_m^2$) and $|c_1(\boldsymbol{x}) - c_1(\boldsymbol{y})| \le \varepsilon/2, \dots, |c_m(\boldsymbol{x}) - c_m(\boldsymbol{y})| \le \varepsilon/2$. Then for any $\boldsymbol{\eta} \in A$ and any $\boldsymbol{x} \in A$, $\boldsymbol{y} \in A$ such that $||\boldsymbol{x} - \boldsymbol{\eta}|| \le \delta$ and $||\boldsymbol{y} - \boldsymbol{\eta}|| \le \delta$ it holds

$$\begin{gathered} 1 - c_1(\boldsymbol{\eta})|y_1 - x_1| - \dots - c_m(\boldsymbol{\eta})(y_m - x_m)^2 - \varepsilon\big(|||y - x|||\big) \\ \le r(\boldsymbol{x}; \boldsymbol{y}) \le \\ 1 - c_1(\boldsymbol{\eta})|y_1 - x_1| - \dots - c_m(\boldsymbol{\eta})(y_m - x_m)^2 + \varepsilon\big(|||y - x|||\big). \end{gathered}$$

Therefore, $\{X(\boldsymbol{x})\}$ is a Gaussian field with a locally stationary structure and according to Theorem 7.1 of Piterbarg [12] the assertion (14) holds true.

Now suppose that $c_1(\boldsymbol{x}) \ge 0, \dots, c_m(\boldsymbol{x}) \ge 0$ but $mes\big\{\boldsymbol{x};\, c_1(\boldsymbol{x}) = 0 \cup \dots \cup c_m(\boldsymbol{x}) = 0\big\} = 0$. There exist constants $K > 0$ and $\delta > 0$ such that for $|||\boldsymbol{y} - \boldsymbol{x}||| \le \delta$

$$r(\boldsymbol{x}; \boldsymbol{y}) \ge 1 - K\, |||\boldsymbol{y} - \boldsymbol{x}|||.$$

Further there exists a number $n_0 \in N$ such that $A \subset \cup_{i=1}^{n_0} A_i$ and for any $i = 1, \dots, n_0$ and $\boldsymbol{x} \in A_i$ and $\boldsymbol{y} \in A_i$ it holds $|||\boldsymbol{x} - \boldsymbol{y}||| \le \delta$. Moreover for any $i$ there exists a homogeneous zero mean unit variance Gaussian field $\{\widetilde{X}_i(\boldsymbol{x}),\, \boldsymbol{x} \in A_i\}$ with a covariance function $r_i(\boldsymbol{x}; \boldsymbol{y})$ satisfying:

$$r_i(\boldsymbol{x}; \boldsymbol{y}) = 1 - K|||y - x||| + o(|||y - x|||) \quad \text{as} \quad ||x - y|| \to 0,$$

and

$$r_i(\boldsymbol{x};\, \boldsymbol{y}) \le r(\boldsymbol{x}; \boldsymbol{y}) \quad \text{for all} \quad \boldsymbol{x} \in A_i,\, \boldsymbol{y} \in A_i.$$

Clearly

$$P\big(\max_{\boldsymbol{x} \in A_i} \widetilde{X}_i(\boldsymbol{x}) > u\big) \ge P\big(\max_{\boldsymbol{x} \in A_i} X_i(\boldsymbol{x}) > u\big).$$

Further, for any $\varepsilon > 0$ we can find a compact set $A_\varepsilon$ such that $mes(A_\varepsilon) \le \varepsilon$ and for $\boldsymbol{x} \in \overline{A - A_\varepsilon}$ (a closure of $A - A_\varepsilon$) it holds $c_1(\boldsymbol{x}) > 0 \cap \dots \cap c_m(\boldsymbol{x}) > 0$. Therefore, for all $u > 0$

$$P\big(\max_{\boldsymbol{x} \in A_\varepsilon} X(\boldsymbol{x}) > u\big) \le \sum_{i=1}^{n_0} P\big(\max_{\boldsymbol{x} \in A_i \cap A_\varepsilon} \widetilde{X}_i(\boldsymbol{x}) > u\big)$$

and it holds

$$\limsup_{u \to \infty} \frac{P\big(\max_{\boldsymbol{x} \in A_\varepsilon} X(\boldsymbol{x}) > u\big)}{u^{m+p}(1 - \Phi(u))} \le n_0\, \frac{1}{\pi^{(m-p)/2}} K^{(m+p)/2}\, \varepsilon.$$

It follows that for any $\varepsilon' > 0$

$$\begin{aligned} \frac{1}{\pi^{(m-p)/2}}\big(I_A - \varepsilon'\big) &\le \liminf_{u \to \infty} \frac{P\big(\max_{\boldsymbol{x} \in A} X(\boldsymbol{x}) > u\big)}{u^{m+p}(1 - \Phi(u))} \\ &\le \limsup_{u \to \infty} \frac{P\big(\max_{\boldsymbol{x} \in A} X(\boldsymbol{x}) > u\big)}{u^{m+p}(1 - \Phi(u))} \le \frac{1}{\pi^{(m-p)/2}}\big(I_A + \varepsilon'\big). \end{aligned}$$

□

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