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ROBUST COORDINATION CONTROL OF SWITCHING MULTI-AGENT SYSTEMS VIA OUTPUT REGULATION APPROACH

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In this paper, the distributed output regulation problem of uncertain multi-agent systems with switching interconnection topologies is considered. All the agents will track or reject the signals generated by an exosystem (or an active leader). A systematic distributed design approach is proposed to handle output regulation via dynamic output feedback with the help of canonical internal model. With common solutions of regulator equations and Lyapunov functions, the distributed robust output regulation with switching interconnection topology is solved.

Keywords: distributed output regulation, multi-agent systems, switching topology, canonical internal model

Classification: 93A14, 93C10

1. INTRODUCTION

Recently, control and analysis of multi-agent dynamics have become a very active area in current science and technology in order to reveal tremendous and striking features/dynamics. Coordination of multi-agent networks has been widely studied, including consensus, formation, flocking, and coverage, and leader-follower coordination is an important problem [6, 10, 14, 16, 17].

On the other hand, the robust output regulation problem has been extensively studied for its strong theoretical and practical background. It is mainly concerned with designing a control law for an uncertain plant such that the closed-loop system satisfying the stability and asymptotically tracking a class of reference inputs in the presence of a kind of disturbances. Note that both the reference inputs and disturbances are generated by an autonomous differential equation called exosystem. A significant result on linear systems is the internal model principle which enables the conversion of the output regulation problem into an eigenvalue placement problem for an augmented linear system (referring to [2, 21]). Then many significant results were obtained for nonlinear systems ([1, 11]). For example, the so-called canonical internal model based on the notion of the steady-state generator was introduced for converting the robust output regulation problem of the considered nonlinear system into a robust stabilization problem of an augmented system [11, 15].

Distributed output regulation (DOR) of multi-agent systems extends the leader-follower problem with various practical applications including active leader following model and multi-agent synchronization with complex dynamics or environmental inputs ([4, 12, 17, 19]). In fact DOR has been studied from different viewpoints. Simple approaches based on distributed estimation for special class of multi-agent system (see [6, 8, 10]). A systematical approach is based on internal model to deal with general models. In fact, conventional internal model has been used in the distributed output regulation problem for multi-agent systems with fixed topologies [12, 19]. Unfortunately, this method based on conventional internal model failed to deal with linear multi-agent systems under switching topology.

The distributed robust output regulation can be viewed as a general framework of the conventional leader-following problem of multi-agent systems: (i) the interconnection topology is switching but keeps connected; (ii) the system contains uncertainties; (iii) the dynamic of the leader (or the exosystem) is different from the dynamics of the follower agents; (iv) there are unmeasurable variables for the exosystem and each agent and distributed dynamic output feedback is designed. Our contribution is threefold compared with [12, 19]. First, we consider uncertain linear multi-agent systems with general interaction topologies. Second, switching topology is considered here. In practice, switching topology is more realistic. To deal with switching interaction topologies, the constructed feedback law in [19] based on conventional internal model failed. Here, using the canonical internal model [11, 15], we can design a distributed dynamic feedback law to solve the output regulation problem.

This paper is organized as follows. In Section 2, problem formulation of distributed output regulation of multi-agent systems is introduced, along with preliminary knowledge. The existence of canonical internal model is shown for multi-agent systems in Section 3. Then in Section 4, a general design procedure is proposed based on canonical internal model. Finally, the concluding remarks are given in Section 5.

2. PROBLEM FORMULATION

In this section, the problem formulation along with preliminary knowledge is introduced.

First of all, we introduce some basic concepts and notations in graph theory (referring to [5] for details). A digraph is denoted as $\mathcal{G} = (\mathcal{O}, \mathcal{E})$, where $\mathcal{O} = \{1, 2, \cdots, \kappa\}$ is the set of nodes and $\mathcal{E}$ is the set of edges. $(i, j) \in \mathcal{E}$ denotes an edge leaving from node i and entering into node j if node i can get information from node j. In this case node j is said to be a neighbor of node i. The special case of digraph is undirected graph if $(i, j) \in \mathcal{E}$ once $(j, i) \in \mathcal{E}$. A path in digraph $\mathcal{G}$ is an alternating sequence $i_1 e_1 i_2 e_2 \cdots e_{k-1} i_k$ of nodes i_j and edges $e_j = (i_j, i_{j+1}) \in \mathcal{E}$ for $j = 1, 2, \cdots, k-1$. If there exists a path from node i to node j, then node j is said to be reachable from node i. A node which is reachable from every other node of $\mathcal{G}$ is called a globally reachable node of $\mathcal{G}$.

Here we consider a system consisting of κ agents and a leader (denoted as node 0). The corresponding digraph is denoted as $\bar{\mathcal{G}}$. Regarding the κ agents as the nodes, the relationships between κ agents can be conveniently described by an undirected graph $\mathcal{G}_0$ which is a subgraph of $\bar{\mathcal{G}}$. $\mathcal{N}_i$ $(i = 1, \ldots, \kappa)$ is called the neighbor set of agent i. The weighted adjacency matrix of $\mathcal{G}_0$ is denoted as $A^0 = (a_{ij})_{\kappa\times\kappa} \in \mathbb{R}^{\kappa\times\kappa}$, where $a_{ii} = 0$ and $a_{ij} \geq 0$ ($a_{ij} > 0$ if there is an edge from agent i to agent j). Its degree matrix $D^0 = diag\{\bar{a}_1^0, \ldots, \bar{a}_\kappa^0\} \in \mathbb{R}^{\kappa\times\kappa}$ is a diagonal matrix, where diagonal elements $\bar{a}_i^0 = \sum_{j=1}^{\kappa} a_{ij}$ for $i = 1, \ldots, \kappa$. Then the Laplacian of the weighted graph is defined as $L = D^0 - A^0$.

Furthermore, let us consider the digraph $\bar{\mathcal{G}}$ contains κ agents and the leader with directed edges from some agents to the leader by the connection weights $a_{i0} > 0$ if agent i can get information from the leader, otherwise $a_{i0} = 0$ (note that $\bar{\mathcal{G}}$ is directed though $\mathcal{G}_0$ is undirected). Set an $\kappa \times \kappa$ diagonal matrix $A_0 = diag\{a_{10}, \ldots, a_{\kappa 0}\}$. Define a matrix $H = L + A_0$ to describe the connectivity of the whole graph $\bar{\mathcal{G}}$. Obviously, we have $H\mathbf{1} = A_0\mathbf{1}$.

The following lemma is about the matrix H ([6]).

Lemma 2.1. *H is positive definite if and only if node 0 is globally reachable in $\bar{\mathcal{G}}$.*

In this paper, the multi-agent output regulation is considered with switching interaction topologies. To be strict, suppose that there is an infinite sequence of bounded, non-overlapping, contiguous time-intervals $[t_i, t_{i+1})$, $i = 0, 1, \cdots$, starting at $t_0 = 0$. To avoid infinite-switching within finite time interval and related non-smooth description, as usual, we assume that there is a constant $\tau_0 > 0$, often called dwell time, with $t_{i+1} - t_i \geq \tau_0$, $\forall i$. Denote $\mathcal{S} = \{\bar{\mathcal{G}}_1, \bar{\mathcal{G}}_2, \cdots, \bar{\mathcal{G}}_\mu\}$ as a set of the graphs with all possible interconnection topologies satisfying node 0 (the leader) is globally reachable in $\bar{\mathcal{G}}$. Take $\mathcal{P} = \{1, 2, \cdots, \mu\}$ as its index set. To describe the variable interconnection topology with a given dwell time, we define a switching signal $\sigma : [0, \infty) \to \mathcal{P}$, which is piecewise-constant. Therefore, Laplacian L_σ associated with the switching interconnection graph $\mathcal{G}_\sigma$ and $A_{0,\sigma}$ associated with the connections between agents and the leader are time-varying (switched at t_i, $i = 0, 1, \cdots$). Obviously, $H_\sigma = L_\sigma + A_{0,\sigma}$ is also time-varying. However, $L_p, A_{0,p}$ and H_p are time-invariant matrices noting that $\bar{\mathcal{G}}_p$ $(p \in \mathcal{P})$ is the graph during the time interval $[t_i, t_{i+1})$.

Here the exosystem (or the leader) in the output regulation problem is expressed as

$$\dot{v} = \Gamma v, \quad y_0 = Fv \in \mathbb{R}^m \tag{1}$$

where y_0 is the output and $v \in \mathbb{R}^q$ is the exogenous signal representing the disturbance input and/or the driving reference signal, while the dynamics of agents are described by:

$$\begin{cases} \dot{x}_i = A(w)x_i + B(w)u_i + E_i(w)v \\ y_i = C(w)x_i + D(w)u_i \end{cases} \quad i = 1, \ldots, \kappa, \tag{2}$$

where $x_i \in \mathbb{R}^n$, $y_i, u_i \in \mathbb{R}^m$ are the states, outputs, and control inputs of the agent i, and $w \in \mathbb{R}^l$ is the uncertain parameter vector. $E_i(w)$ is an input channel of the agent i, where the driving force or disturbance v can influence the agent dynamics.

The regulated output for agent i is denoted as

$$e_i = y_i - y_0 = C(w)x_i + D(w)u_i - Fv \in \mathbb{R}^m, \quad i = 1, \dots, \kappa. \tag{3}$$

The distributed control aim is to make $e_i(t) \to 0,\ i = 1, \dots, \kappa$ as $t \to \infty$.

Since not all the agents are connected to the exosystem, and some variables of the exosystem is unmeasurable, y_0 is not available to each agent, and therefore, $e_i = y_i - y_0$ cannot be used directly in its design unless agent i is connected to the exosystem. To achieve the aim, distributed algorithms are made, considering that each agent receives the external state measurements relative to its neighbors or the exosystem as follows:

$$e_{iv} = \sum_{j \in \mathcal{N}_i} a_{ij}(y_i - y_j) + a_{i0}e_i, \quad i = 1, \dots, \kappa, \tag{4}$$

which is available to agent i.

Denote

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_\kappa \end{pmatrix}, \quad z = \begin{pmatrix} z_1 \\ \vdots \\ z_\kappa \end{pmatrix}, \quad u = \begin{pmatrix} u_1 \\ \vdots \\ u_\kappa \end{pmatrix}, \quad e = \begin{pmatrix} e_1 \\ \vdots \\ e_\kappa \end{pmatrix}, \quad \mathbf{E}(w) = \begin{pmatrix} E_1(w) \\ \vdots \\ E_\kappa(w) \end{pmatrix}.$$

Then the multi-agent systems (1) – (3) become

$$\begin{cases} \begin{pmatrix} \dot{x} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} I_\kappa \otimes A(w) & 0 \\ 0 & \Gamma \end{pmatrix} \begin{pmatrix} x \\ v \end{pmatrix} + \begin{pmatrix} I_\kappa \otimes B(w) \\ 0 \end{pmatrix} u + \begin{pmatrix} \mathbf{E}(w) \\ 0 \end{pmatrix} v \\ e = (I_\kappa \otimes C(w))x + (I_\kappa \otimes D(w))u - (\mathbf{1} \otimes F)v \end{cases} \tag{5}$$

with $\mathbf{1} = (1 \cdots 1)^T \in \mathbb{R}^\kappa$.

The distributed control law for agent i mainly uses the information of e_{iv}.

In what follows, the distributed control laws are taken in the following dynamic form:

$$\begin{cases} u_i = K_z z_i \\ \dot{z}_i = E_z z_i + E_e e_{iv} \end{cases} \quad i = 1, \dots, \kappa, \tag{6}$$

where $z_i \in \mathbb{R}^{n_z}$ with the dimension n_z to be specified later.

Then the closed-loop system can be rewritten as

$$\begin{cases} \dot{x}_c = A_c^\sigma(w)x_c + B_c^\sigma(w)v \\ e = C_c(w)x_c - D_c v \end{cases} \quad x_c = \begin{pmatrix} x \\ z \end{pmatrix} \tag{7}$$

where

$$A_c^\sigma(w) = \begin{pmatrix} I_\kappa \otimes A(w) & I_\kappa \otimes (B(w)K_z) \\ H_\sigma \otimes (E_e C(w)) & I_\kappa \otimes E_z + H_\sigma \otimes (E_e D(w) K_z) \end{pmatrix}, \tag{8}$$

$$B_c^\sigma(w) = \begin{pmatrix} \mathbf{E}(w) \\ (H_\sigma \mathbf{1}) \otimes (E_e F) \end{pmatrix},$$

$$C_c(w) = \begin{pmatrix} I_\kappa \otimes C(w) & I_\kappa \otimes (D(w)K_z) \end{pmatrix}, \quad D_c = \mathbf{1} \otimes F.$$

The following definition mainly follows the conventional linear system in [11].

Definition 2.2. The distributed robust output regulation (DROR) problem is achieved for the system consisting of (2) and (1) under (6) if, for any initial condition $col(x_1(0), \ldots, x_\kappa(0))$, and all sufficiently small parameter perturbation w,

1) the closed-loop system (7) is asymptotically stable with $v = 0$;

2) for any initial condition $v(0)$,

$$\lim_{t \to +\infty} e_i(t) = 0, \quad i = 1, \ldots, \kappa. \tag{9}$$

In the following, we will list the assumptions for system (7) in the study of its distributed robust output regulation. For simplicity, denote

$$(A, B, C, D, E_i) = (A(0), B(0), C(0), D(0), E_i(0)).$$

Assumption 2.3. Node 0 (the exosystem) is always globally reachable in $\bar{\mathcal{G}}_{\sigma(t)}$.

Assumption 2.3 is given to describe the connectivity of the switching interconnection topology in order to ensure that the information of the leader (node 0) can be spread to all the agents somehow. The next assumption, which is widely used for simplicity in output regulation design, is given for the exosystem (1).

Assumption 2.4. The real parts of the eigenvalues of matrix Γ defined in (1) are nonnegative.

The following are standard assumptions for output regulation via output feedback.

Assumption 2.5. The pair (A, B) is stabilizable.

Assumption 2.6. The pair (C, A) is detectable.

Assumption 2.7. The rank condition

$$rank \begin{pmatrix} A - \lambda I & B \\ C & D \end{pmatrix} = n + m, \lambda \in \Lambda(\Gamma)$$

holds, where $\Lambda(\Gamma)$ denotes the spectrum of Γ.

3. INTERNAL MODEL AND CONTROL DESIGN

Conventional internal model was applied to linear multi-agent systems with fixed topologies [12, 19]. However, it does not work for many nonlinear systems and multi-agent systems with switching topologies. To solve the problem, we employ a generalized internal model, called canonical internal model, which has been widely used for nonlinear systems ([11, 13, 15]).

Recalling [11], Assumption 2.7 can guarantee the solvability of regulator equation:

$$\begin{cases} X_i(w)\Gamma = A(w)X_i(w) + B(w)U_i(w) + E_i(w) \\ C(w)X_i(w) + D(w)U_i(w) = F \end{cases} \tag{10}$$

$i = 1, \ldots, \kappa$, for any matrices $E_i(w)$, F. Denote the solution of (10) is $X_i(w), U_i(w)$, $i = 1, \ldots, \kappa$.

Let us first introduce the steady-state generator ([15]).

Definition 3.1. The multi-agent systems consisting of (2), (3) and (1) is said to have a steady-state generator with output $u_i = U_i(w)v$, $i = 1, \ldots, \kappa$, if there is a triple matrix $\{\Theta_i, \Upsilon_i, \Xi_i\}$, $i = 1, \ldots, \kappa$, where $\Theta_i \in R^{s\times q}$, $\Upsilon_i \in R^{s\times s}$ and $\Xi_i \in R^{m\times s}$ for some integer s, such that, for all $w \in \mathcal{W}$, some neighborhood of the origin of $\mathbb{R}^l$,

$$\Theta_i(w)\Gamma = \Upsilon_i\Theta_i(w), \quad U_i(w) = \Xi_i\Theta_i(w), \quad i = 1, \ldots, \kappa. \tag{11}$$

If, in addition, the pair $(\Xi_i\Theta_i, \Upsilon_i\Theta_i)$ is observable for $i = 1, \ldots, \kappa$, then the system is said to have an observable steady-state generater with output $u_i = U_i(w)v$, $i = 1, \ldots, \kappa$.

The definition of canonical internal model is introduced as follows.

Definition 3.2. Under Assumptions 2.4 and 2.7, suppose the multi-agent systems (2), (3) and (1) has a steady-state generator $\{\Theta_i, \Upsilon_i, \Xi_i\}$ with output $u_i = U_i(w)v$, $i = 1, \ldots, \kappa$. Then we call the following system:

$$\dot{\eta}_i = M\eta_i + M_g u_i, \ i = 1, \ldots, \kappa \tag{12}$$

a canonical internal model (candidate) of (2), (3) and (1) with output u_i if

$$M\Theta_i(w) + M_g U_i(w) = \Upsilon_i\Theta_i(w). \tag{13}$$

Note that the distributed internal model takes the same form for all the agents (that is, M and M_g are independent of i). The internal model candidate (12) is a system which asymptotically approaches the steady-state generator (11).

Remark 3.3. In fact, the conventional internal model given in [2] only contains the information of the exosystem without information of the controlled systems, while the canonical internal model [11, 15] contains the information of both the exosystem and the controlled systems. Therefore, the canonical internal model is more powerful than conventional internal model in control design.

The following lemma shows the existence of an observable steady-state generator and a canonical internal model candidate for the multi-agent systems consisting of (2), (3) and (1).

Lemma 3.4. Under Assumptions 2.4 and 2.7, it is always possible to find an observable steady-state generator and an internal model candidate with output $u_i = U_i(w)v$, $i = 1, \ldots, \kappa$ for the multi-agent systems (2), (3) and (1), independent of any switching $\sigma(t)$.

Proof. Denote the minimal polynomial of Γ as

$$poly(\Gamma) = \lambda^r + \gamma_1\lambda^{(r-1)} + \cdots + \gamma_{(r-1)}\lambda + \gamma_r,$$

and then

$$\tilde{\Theta}_i(w) = T \begin{pmatrix} U_i(w) \\ U_i(w)\Gamma \\ \vdots \\ U_i(w)\Gamma^{r-1} \end{pmatrix}, \ i = 1, \ldots, \kappa, \tag{14}$$

where $U_i(w),\ i = 1, \ldots, \kappa$ are the solution of (10) and T is a nonsingular matrix to be defined. Take

$$\tilde{\Phi} = \begin{pmatrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ -\gamma_r & -\gamma_{(r-1)} & \cdots & -\gamma_2 & -\gamma_1 \end{pmatrix}, \tilde{\Psi} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}^T,$$

$$\Phi = blockdiag(\underbrace{\tilde{\Phi}, \ldots, \tilde{\Phi}}_{m}), \quad blockdiag(\underbrace{\tilde{\Psi}, \ldots, \tilde{\Psi}}_{m}). \tag{15}$$

Therefore, the systems (2), (3) and (1) has a linearly observable steady-state generator $\{\Theta_i, \Upsilon, \Xi\}$ with output u_i as follows:

$$\begin{cases} \Theta_i(w) = (\underbrace{\tilde{\Theta}_i^T(w), \cdots, \tilde{\Theta}_i^T(w)}_{m})^T \\ \Upsilon = T\Phi T^{-1} \\ \Xi = \Psi T^{-1} \end{cases} \tag{16}$$

Then we propose a special class of internal model candidate based on the constructed steady-state generator (16).

Pick any controllable pairs $\tilde{M} \in R^{r\times r}$ and $\tilde{M}_g \in R^{r\times 1}$ with $\tilde{M}$ Hurwitz and has disjoint spectra with $\tilde{\Phi}$. Set

$$M = blockdiag(\underbrace{\tilde{M}, \ldots, \tilde{M}}_{m}), \quad blockdiag(\underbrace{\tilde{M}_g, \ldots, \tilde{M}_g}_{m}),$$

and then we claim that

$$\dot{\eta}_i = M\eta_i + M_g u_i, \quad \eta_i \in \mathbb{R}^{mr} \tag{17}$$

is an internal model candidate of (2), (3) and (1) with output $u_i,\ i = 1, \ldots, \kappa$.

Since the spectra of the matrices $\tilde{\Phi}$ and $\tilde{M}$ are disjoint, and $(\tilde{\Psi}, \tilde{\Phi})$ is observable, according to Proposition A.2 in [11], there exists a unique and nonsingular matrix $\tilde{T}$ satisfying the following Sylvester equation:

$$\tilde{T}\tilde{\Phi} - \tilde{M}\tilde{T} = \tilde{M}_g\tilde{\Psi}. \tag{18}$$

Set $T = blockdiag(\underbrace{\tilde{T}, \dots, \tilde{T}}_{m})$, and then for $i = 1, \dots, \kappa$

$$\begin{aligned} & M\Theta_i(w) + M_g U_i(w) \\ = \; & M\Theta_i(w) + M_g \Psi T^{-1}\Theta_i(w) \\ = \; & T\Phi T^{-1}\Theta_i(w) \\ = \; & \Upsilon\Theta_i(w). \end{aligned}$$

Therefore, (17) is an internal model candidate of (2), (3) and (1) with output u_i for $i = 1, \dots, \kappa$. □

From Lemma 3.4, systems (2), (3) and (1) has an observable steady-state generator (16) and an internal model candidate (17) with output u_i, $i = 1, \dots, \kappa$. Then we obtain an augmented system:

$$\begin{cases} \dot{x}_i = A(w)x_i + B(w)u_i + E_i(w)v \\ \dot{\eta}_i = M\eta_i + M_g u_i & i = 1, \dots, \kappa. \\ e_i = C(w)x_i + D(w)u_i - Fv \end{cases} \tag{19}$$

To solve the DROR of the switching multi-agent systems (2), (3) and (1), we construct the following observer-based feedback:

$$\begin{cases} u_i = \Psi T^{-1}\eta_i + K_\xi \xi_i \\ \dot{\eta}_i = M\eta_i + M_g(\Psi T^{-1}\eta_i + K_\xi \xi_i) \\ \dot{\xi}_i = (A_\xi + B_\xi K_\xi)\xi_i + L_\xi(e_{iv} - \hat{e}_{iv}) \end{cases} \tag{20}$$

where

$$A_\xi(w) = \begin{pmatrix} A(w) & B(w)\Psi T^{-1} \\ 0 & M + M_g \Psi T^{-1} \end{pmatrix}, \; A_\xi = A_\xi(0), \tag{21}$$

$$B_\xi(w) = \begin{pmatrix} B(w) \\ M_g \end{pmatrix}, \; B_\xi = B_\xi(0), \tag{22}$$

and

$$\begin{aligned} \hat{e}_{iv} \;=\; & \sum_{j \in \mathcal{N}_i} a_{ij}(C(w)\xi_{i1} + D(w)(\Psi T^{-1}\xi_{i2} + K_\xi \xi_i)) \\ - \; & \sum_{j \in \mathcal{N}_i} a_{ij}(C(w)\xi_{j1} + D(w)(\Psi T^{-1}\xi_{j2} + K_\xi \xi_j)) \\ + \; & a_{i0}(C(w)\xi_{i1} + D(w)(\Psi T^{-1}\xi_{i2} + K_\xi \xi_i)), \end{aligned} \tag{23}$$

with $col(\xi_{i1}, \xi_{i2}) = \xi_i$, $\quad i = 1, \dots, \kappa$ as the estimation of $col(x_i, \eta_i)$, $\quad i = 1, \dots, \kappa$ and the feedback gain matrices K_ξ, L_ξ to be determined later (in the next section).

It is not hard to obtain the following formula:

$$\begin{aligned} e_{iv} - \hat{e}_{iv} \;=\; & \sum_{j \in \mathcal{N}_i} a_{ij}(C(w)(x_i - \xi_{i1} - x_j + \xi_{j1}) + D(w)\Psi T^{-1}(\eta_i - \xi_{i2} - \eta_j + \xi_{j2})) \\ + \; & a_{i0}(C(w)(x_i - \xi_{i1}) + D(w)\Psi T^{-1}(\eta_i - \xi_{i2}) - Fv). \end{aligned} \tag{24}$$

Then the augmented multi-agent system (19) under observer-based feedback (20) becomes

$$\begin{cases} \dot{x}_i = A(w)x_i + B(w)(\Psi T^{-1}\eta_i + K_\xi \xi_i) + E_i(w)v \\ \dot{\eta}_i = M\eta_i + M_g(\Psi T^{-1}\eta_i + K_\xi \xi_i) \\ \dot{\xi}_i = (A_\xi + B_\xi K_\xi)\xi_i + L_\xi(e_{iv} - \hat{e}_{iv}) \\ e_i = C(w)x_i + D(w)(\Psi T^{-1}\eta_i + K_\xi \xi_i) - Fv \end{cases} \quad i = 1, \dots, \kappa \tag{25}$$

4. DISTRIBUTED DESIGN FOR SWITCHING CASES

Both state and output feedback laws based on conventional internal model were proposed to solve the distributed output regulation problem of heterogeneous agents for a specific fixed topology [19], but the method cannot be extended to deal with distributed output regulation problem in the case of switching interaction topologies. Here we proposed a distributed dynamic output feedback (20) to solve the robust regulation problem of linear multi-agent systems with switching connected topologies, with the help of a canonical internal model.

For the following analysis, we introduce a lemma, whose different versions can be found in many books (for example [21]).

Lemma 4.1. Consider system $\dot{x} = \tilde{A}x + \tilde{B}u \in \mathbb{R}^n,\ y = \tilde{C}x$, if $(\tilde{C}, \tilde{A})$ is detectable and matrices $\hat{M},\ \tilde{M}$ are positive definite, then there is a unique positive definite matrix P to satisfy the Riccati equation:

$$P\tilde{A}^T + \tilde{A}P - P\tilde{C}^T\hat{M}^{-1}\tilde{C}P + \tilde{M} = 0.$$

Furthermore, $\tilde{A}^T - \tilde{C}^T\hat{M}^{-1}\tilde{C}P$ is stable.

The next lemma was given in [9], to check the positive definiteness of a matrix.

Lemma 4.2. Suppose that a symmetric matrix is partitioned as

$$R_0 = \begin{pmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{pmatrix}$$

where R_1 and R_3 are square. R_0 is positive definite if and only if both R_1 and $R_3 - R_2^T R_1^{-1} R_2$ are positive definite.

Recalling (25), let $x_c = col(x_1, \eta_1, x_2, \eta_2, \dots, x_\kappa, \eta_\kappa, \xi_1, \dots, \xi_\kappa)$, and then we have the closed-loop system consisting of (2) and (1) under the feedback (20) as follows:

$$\begin{cases} \dot{x}_c = A_c^\sigma(w)x_c + B_c^\sigma(w)v \\ e = C_c(w)x_c - D_c v \end{cases} \tag{26}$$

with

$$A_c^\sigma(w) = \begin{pmatrix} I_\kappa \otimes A_\xi(w) & I_\kappa \otimes (B_\xi(w)K_\xi) \\ H_\sigma \otimes (L_\xi C_\xi(w)) & I_\kappa \otimes (A_\xi + B_\xi K_\xi) - H_\sigma \otimes (L_\xi C_\xi(w)) \end{pmatrix}, \tag{27}$$

$A_\xi(w),\ B_\xi(w)$ are defined in (21), (22), respectively,

$$C_\xi(w) = \begin{pmatrix} C(w) & D(w)\Psi T^{-1} \end{pmatrix}, \tag{28}$$

and

$$B_c^\sigma(w) = \begin{pmatrix} E_1(w) \\ 0 \\ \vdots \\ E_\kappa(w) \\ 0 \\ -H_\sigma \otimes (L_\xi F) \end{pmatrix},$$

$$C_c(w) = \begin{pmatrix} I_\kappa \otimes C_\xi(w) & I_\kappa \otimes (D(w)K_\xi) \end{pmatrix}, \quad D_c = \mathbf{1} \otimes F.$$

At first, we prove the existence of the common regulation matrix $X_c(w)$ as the solution of regulator equation:

$$\begin{cases} X_c(w)\Gamma = A_c^\sigma(w)X_c(w) + B_c^\sigma(w) \\ C_c(w)X_c(w) = D_c \end{cases} \tag{29}$$

for system (26).

Theorem 4.3. There is a common regulation matrix $X_c(w)$ for the closed-loop system (26), independent of the topology switching.

P r o o f. Set

$$X_c(w) = \begin{pmatrix} X_1(w) \\ \Theta_1(w) \\ \vdots \\ X_\kappa(w) \\ \Theta_\kappa(w) \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \tag{30}$$

where $X_i(w),\ i = 1, \dots, \kappa$ are the solution of the regulator equation (10), which is guaranteed by Assumption 2.7 and $\Theta_i(w),\ i = 1, \dots, \kappa$ are defined in (16). From (10), $X_i(w),\ U_i(w),\ i = 1, \dots, \kappa$, only based on $A(w), B(w), C(w), D(w), E_i(w), F(w)$, are independent of $\sigma(t)$. According to (14) and (16), $\Theta_i(w),\ i = 1, \dots, \kappa$ are also independent of $\sigma(t)$. Therefore, $X_c(w)$ is independent of $\sigma(t)$. In the following we will show $X_c(w)$ is the solution of the regulator equation (29) with $A_c^\sigma(w),\ B_c^\sigma(w),\ C_c(w),\ D_c$ defined in (26). Since $X_i(w),\ i = 1, \dots, \kappa$ are the solution of the regulator equation (10),

$$X_i(w)\Gamma = A(w)X_i(w) + B(w)U_i(w) + E_i(w),\ i = 1, \dots, \kappa.$$

From the proof of Lemma 3.4, we have

$$U_i(w) = \Psi T^{-1}\Theta_i(w),\ i = 1, \dots, \kappa.$$

Then

$$X_i(w)\Gamma = A(w)X_i(w) + B(w)\Psi T^{-1}\Theta_i(w) + E_i(w),\ i = 1, \dots, \kappa. \tag{31}$$

According to the proof of Lemma 3.4, we have

$$(M + M_g\Psi T^{-1})\Theta_i(w) = T\Phi T^{-1}\Theta_i(w),\ i = 1, \dots, \kappa, \tag{32}$$

bringing

$$\Theta_i(w) = (\underbrace{\tilde{\Theta}_i^T(w), \cdots, \tilde{\Theta}_i^T(w)}_{m})^T,\ i = 1, \dots, \kappa,$$

$$\tilde{\Theta}_i(w) = T\begin{pmatrix} U_i(w) \\ U_i(w)\Gamma \\ \vdots \\ U_i(w)\Gamma^{r-1} \end{pmatrix},\ i = 1, \dots, \kappa$$

into the right side of (32).

Note that

$$\Phi\begin{pmatrix} U_i(w) \\ U_i(w)\Gamma \\ \vdots \\ U_i(w)\Gamma^{r-1} \end{pmatrix} = \begin{pmatrix} U_i(w)\Gamma \\ U_i(w)\Gamma^2 \\ \vdots \\ U_i(w)\Gamma^{r} \end{pmatrix},\ i = 1, \dots, \kappa.$$

Thus

$$\Theta_i(w)\Gamma = (M + M_g\Psi T^{-1})\Theta_i(w),\ i = 1, \dots, \kappa. \tag{33}$$

Recalling $X_i(w),\ i = 1, \dots, \kappa$ are the solution of the regulator equation (10), and $U_i(w) = \Psi T^{-1}\Theta_i(w),\ i = 1, \dots, \kappa$, thus

$$C_\xi(w)\begin{pmatrix} X_i(w) \\ \Theta_i(w) \end{pmatrix} - F = 0,\ i = 1, \dots, \kappa,\ C_\xi(w) = \begin{pmatrix} C(w) & D(w)\Psi T^{-1} \end{pmatrix}.$$

Then

$$H_\sigma \otimes (L_\xi C_\xi(w))\begin{pmatrix} X_1(w) \\ \Theta_1(w) \\ \vdots \\ X_\kappa(w) \\ \Theta_\kappa(w) \end{pmatrix} - H_\sigma \otimes (L_\xi F) = 0. \tag{34}$$

According to (31), (33) and (34), $X_c(w)$ is the solution of the regulator equation (29) with $A_c^\sigma(w),\ B_c^\sigma(w),\ C_c(w),\ D_c$ defined in (26). □

Then we obtain the distributed output regulation for the switched multi-agent system (26):

Theorem 4.4. With Assumptions 2.3-2.7, the distributed robust output regulation problem of the system consisting of (2), (3) and (1) can be solved by distributed output feedback (20).

P r o o f. Based on Theorem 4.3, there is a common regulation matrix $X_c(w)$. Therefore, we only need to find a common Lyapunov function to prove the convergence for the switched system.

Since

$$\begin{pmatrix} A-\lambda I & B\Psi T^{-1} & B \\ 0 & M+M_g\Psi T^{-1}-\lambda I & M_g \end{pmatrix} = \begin{pmatrix} A-\lambda I & 0 & B \\ 0 & M-\lambda I & M_g \end{pmatrix} \begin{pmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & \Psi T^{-1} & I \end{pmatrix}$$

and M is Hurwitz together with Assumption 2.5, we can get that (A_ξ, B_ξ) (defined in (21) and(22)) is stabilizable. Then there exists K_ξ such that

$$A_\xi + B_\xi K_\xi \tag{35}$$

is stable.

Considering

$$\begin{pmatrix} A-\lambda I & B\Psi T^{-1} \\ 0 & M+M_g\Psi T^{-1}-\lambda I \\ C & D\Psi T^{-1} \end{pmatrix}, \tag{36}$$

it has full rank for all $\lambda \bar{\in} \sigma(\Phi)$ due to Assumption 2.6 and the fact that $M + M_g\Psi T^{-1} = T\Phi T^{-1}$.

Using the decomposition

$$\begin{pmatrix} A-\lambda I & B\Psi T^{-1} \\ 0 & M+M_g\Psi T^{-1}-\lambda I \\ C & D\Psi T^{-1} \end{pmatrix} = \begin{pmatrix} A-\lambda I & 0 & B \\ 0 & M-\lambda I & M_g \\ C & 0 & D \end{pmatrix} \begin{pmatrix} I & 0 \\ 0 & I \\ 0 & \Psi T^{-1} \end{pmatrix}$$

and Assumption 2.7, it easily follows that (36) has full rank for all $\lambda \in \sigma(\Phi)$. Thus,

$$(A_\xi, \ \ C_\xi)\,, \tag{37}$$

is detectable, where $C_\xi = C_\xi(0)$, $C_\xi(w)$ is defined in (28). Since (37) is detectable, according to Lemma 4.1, $A_\xi^T - C_\xi^T C_\xi P_\xi$ is stable, where P_ξ is the unique solution of the following Riccati equation

$$A_\xi P_\xi + P_\xi A_\xi^T - P_\xi C_\xi^T C_\xi P_\xi + I = 0. \tag{38}$$

Set

$$L_\xi^T = \max\{1, \frac{1}{\bar{\lambda}}\} C_\xi P_\xi, \tag{39}$$

where

$$\bar{\lambda} = \min\{\text{eigenvalues of } H_p \in R^{\kappa\times\kappa},\ p \in \mathcal{P},\ \text{Assumption 2.3 holds}\}. \tag{40}$$

Under Assumption 2.3, according to Lemma 2.1, all eigenvalues of the matrices H_p ($p \in \mathcal{P}$) are positive. Moreover, since the set $\mathcal{P}$ is finite, $\bar{\lambda} > 0$ is fixed.

Obviously,

$$\dot{\bar{x}}_c = A_c^\sigma(w)\bar{x}_c, \quad \bar{x}_c = x_c - X_c(w)v, \tag{41}$$

where $A_c^\sigma(w)$ is defined in (27).

In what follows, we will show $\lim_{t\to\infty} \bar{x}_c(t) = 0$ in some open neighborhood W around $w = 0$. To this end, we will show there is a common Lyapunov function for system (41) with $w \in W$.

Let T_σ be an orthogonal transformation such that $U_\sigma = T_\sigma H_\sigma T_\sigma^{-1}$ is a diagonal matrix with the eigenvalues of H_σ along the diagonal. Clearly, $T_\sigma \otimes I_\kappa$ transforms $H_\sigma \otimes I_\kappa$ into $U_\sigma \otimes I_\kappa$. Setting

$$\tilde{x}_c = \begin{pmatrix} I & 0 \\ 0 & T_\sigma \otimes I_\kappa \end{pmatrix} \begin{pmatrix} I & 0 \\ -I & I \end{pmatrix} \bar{x}_c,$$

we obtain

$$\dot{\tilde{x}}_c = \begin{pmatrix} I_\kappa \otimes (A_\xi(w) + B_\xi(w)K_\xi) & I_\kappa \otimes (B_\xi(w)K_\xi) \\ I_\kappa \otimes [A_\xi + B_\xi K_\xi - A_\xi(w) - B_\xi(w)K_\xi] & I_\kappa \otimes [A_\xi + (B_\xi - B_\xi(w))K_\xi] - U_\sigma \otimes (L_\xi C_\xi(w)) \end{pmatrix} \tilde{x}_c. \tag{42}$$

Set $\tilde{x}_c = (\tilde{x}_{c1}, \ldots, \tilde{x}_{c\kappa})^T$, and then (42) becomes

$$\dot{\tilde{x}}_{ci} = \begin{pmatrix} A_\xi(w) + B_\xi(w)K_\xi & B_\xi(w)K_\xi \\ A_\xi + B_\xi K_\xi - A_\xi(w) - B_\xi(w)K_\xi & A_\xi + (B_\xi - B_\xi(w))K_\xi - \lambda_{i\sigma} L_\xi C_\xi(w) \end{pmatrix} \tilde{x}_{ci},$$

for $i = 1, \ldots, \kappa,$, where $\lambda_{i\sigma}$ is the ith eigenvalue of H_σ.

Since (35) is stable, there exists an open neighborhood W_1 of $w = 0$ such that for each $w \in W_1$, $A_\xi(w) + B_\xi(w)K_\xi$ is stable. Therefore, there exist positive definite matrices $P_*(w)$ and $Q_*(w)$ such that for each $w \in W_1$,

$$P_*(w)(A_\xi(w) + B_\xi(w)K_\xi) + (A_\xi(w) + B_\xi(w)K_\xi)^T P_*(w) = -Q_*(w).$$

From (37) and Lemma 4.1, all real parts of the eigenvalues of $A_\xi - \lambda(H_\sigma)L_\xi C_\xi$ (L_ξ defined in (39)) are negative, since $A_\xi^T - \alpha C_\xi^T C_\xi P_\xi$, ($P_\xi$ defined in (38)) are so for any $\alpha \geq 1$, where $\lambda(H_\sigma)$ denotes any eigenvalue of matrix H_σ.

Set $\bar{A}_\xi(w) = A_\xi + (B_\xi - B_\xi(w))K_\xi - \lambda(H_\sigma)L_\xi C_\xi(w)$. Then

$$\begin{aligned} & P_\xi \bar{A}_\xi^T(w) + \bar{A}_\xi(w) P_\xi \\ = \;& -I + P_\xi C_\xi^T C_\xi P_\xi \\ - \;& 2\lambda(H_\sigma) \max\{1, \frac{1}{\bar{\lambda}}\} P_\xi C_\xi^T C_\xi P_\xi \\ + \;& \lambda(H_\sigma) \max\{1, \frac{1}{\bar{\lambda}}\} P_\xi (C_\xi - C_\xi(w))^T C_{\xi i} P_\xi \\ + \;& \lambda(H_\sigma) \max\{1, \frac{1}{\bar{\lambda}}\} P_\xi C_\xi^T (C_\xi - C_\xi(w)) P_\xi \\ + \;& P_\xi [B_\xi - B_\xi(w))K_\xi]^T + [B_\xi - B_\xi(w))K_\xi] P_\xi. \end{aligned}$$

Due to $C_\xi = C_\xi(0), B_\xi = B_\xi(0)$, there exists an open neighborhood W_2 of $w = 0$ such that, for each $w \in W_2$

$$\begin{aligned}
&\lambda(H_\sigma)\max\{1, \frac{1}{\bar{\lambda}}\}P_\xi(C_\xi - C_\xi(w))^T C_\xi P_\xi \\
&+\lambda(H_\sigma)\max\{1, \frac{1}{\bar{\lambda}}\}P_\xi C_\xi^T(C_\xi - C_\xi(w))P_\xi \\
&+P_\xi[B_\xi - B_\xi(w))K_\xi]^T + [B_\xi - B_\xi(w))K_\xi]P_\xi \\
&\leq I/2.
\end{aligned}$$

Therefore,

$$P_\xi \bar{A}_\xi^T(w) + \bar{A}_\xi(w)P_\xi \leq -I/2 - P_\xi C_\xi^T C_\xi P_\xi, \quad w \in W_2,$$

which implies that

$$P_\xi^{-1}\bar{A}_\xi(w) + \bar{A}_\xi^T(w)P_\xi^{-1} \leq -Q_\xi, \quad Q_\xi := -(P_\xi^{-1})^2/2 - C_\xi^T C_\xi, \quad w \in W_2, \tag{43}$$

where Q_ξ is obviously positive definite.

Take a Lyapunov function for system (41):

$$V(\bar{x}_c) = \bar{x}_c^T J_c^T(I_\kappa \otimes P(w))J_c\bar{x}_c = \hat{x}_c^T(I_\kappa \otimes P(w))\hat{x}_c, \tag{44}$$

where

$$P(w) = \begin{pmatrix} P_*(w)/\varpi & 0 \\ 0 & P_\xi \end{pmatrix}$$

with $\varpi > 0$ to be determined. Clearly, V keeps unchanged with switching signal σ, which is a candidate of a common Lyapunov function independent of switching. Moreover,

$$V = \hat{x}_c^T(I_\kappa \otimes P(w))\hat{x}_c = \tilde{x}_c^T(I_\kappa \otimes P(w))\tilde{x}_c = \sum_{i=1}^{\kappa}\tilde{x}_{ci}^T P(w)\tilde{x}_{ci}$$

because $T_\sigma^T = T_\sigma^{-1}$.

The interconnection graph associated with $H_p,\ p \in \mathcal{P}$ is unchanged and connected on an interval $[t_i, t_{i+1})$. Therefore, $A_c^p(w)$ is constant in the interval. Consider the derivative of V with $t \in [t_i, t_{i+1})$:

$$\dot{V}|_{(42)} = -\sum_{i=1}^{\kappa}\tilde{x}_{ci}^T\tilde{Q}(w)\tilde{x}_{ci}, \tag{45}$$

where

$$\begin{aligned}
\tilde{Q}(w) &= \begin{pmatrix} Q_*(w)/\varpi & \Pi(w) \\ \Pi^T(w) & Q_\xi \end{pmatrix}, \\
\Pi(w) &= P_*/\varpi(B_\xi K_\xi) + (A_\xi + B_\xi K_\xi - A_\xi(w) - B_\xi(w)K_\xi)^T P_\xi.
\end{aligned}$$

From Lemma 4.2, the positive definite of $\tilde{Q}(w)$ can be guaranteed by the positive definiteness of the matrices

$$Q_*(w)/\varpi, \quad Q_\xi - \varpi\Pi^T(w)Q_*^{-1}(w)\Pi(w).$$

Note that the positive definiteness of

$$\tilde{Q}_\xi = Q_\xi - \frac{1}{\varpi}[P_*(B_\xi K_\xi)]^T Q_*^{-1}(w)[P_*(B_\xi K_\xi)]$$

can be obtained when ϖ is sufficiently large. Then, due to $A_\xi + B_\xi K_\xi = A_\xi(0) + B_\xi(0)K_\xi$, the local positive definiteness of $Q_\xi - \varpi\Pi^T(w)Q_*^{-1}(w)\Pi(w)$ results from

$$Q_\xi - \varpi\Pi^T(w)Q_*^{-1}(w)\Pi(w) = \tilde{Q}_\xi - \varpi[(A_\xi+B_\xi K_\xi-A_\xi(w)-B_\xi(w)K_\xi)^T P_\xi]^T Q_*^{-1}(w)[(A_\xi+B_\xi K_\xi-A_\xi(w)-B_\xi(w)K_\xi)^T P_\xi]$$

with the selected ϖ and a small open neighborhood $W_3(\subset W_1 \cap W_2)$ of $w = 0$ such that, for any $w \in W_3$,

$$[(A_\xi+B_\xi K_\xi-A_\xi(w)-B_\xi(w)K_\xi)^T P_\xi]^T Q_*^{-1}(w)[(A_\xi+B_\xi K_\xi-A_\xi(w) - B_\xi(w)K_\xi)^T P_\xi] \le \frac{\tilde{Q}_\xi}{2\varpi}.$$

Recalling the dwell-time assumption and (45) gives

$$\dot{V} \le -\hat{\lambda}V/\sqrt{||P(w)||_2}, \quad \forall t \ge 0,$$

with $\hat{\lambda} = \min\{eigenvalues\ of\ \tilde{Q}(w)\}$, which implies system (42) is asymptotically stable for each $w \in W_3$. Consequently, system (41) is asymptotically stable for each $w \in W_3$. Moreover, $e(t) = C_c(w)\bar{x}_c(t) + C_c(w)X_c(w)v - D_c v$, which implies $\lim_{t\to\infty} e(t) = \lim_{t\to\infty} C_c(w)\bar{x}_c(t) = 0$. Thus, the conclusion follows. □

Here is an example for illustration.

Example 1. Consider the exosystem

$$\dot{v} = \Gamma v, \quad \Gamma = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & -2 & 0 \end{pmatrix}, \; v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix},$$

with output $y_0 = v_3$ (that is, $F = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}$). and κ agents in the form of

$$\begin{cases} \dot{x}_{i1} = x_{i1} + x_{i2} + v_1 \\ \dot{x}_{i2} = x_{i3} \\ \dot{x}_{i3} = -(1+w)x_{i1} - x_{i2} + u_i \\ y_i = x_{i2}, \; i = 1, \ldots, \kappa, \end{cases}$$

namely

$$A(w)=\begin{pmatrix}1 & 1 & 0\\ 0 & 0 & 1\\ -(1+w) & -1 & 0\end{pmatrix},\ B(w)=\begin{pmatrix}0\\0\\1\end{pmatrix},\ E(w)=\begin{pmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix},$$

and $C(w)=\begin{pmatrix}0 & 1 & 0\end{pmatrix},\ D(w)=0$.

Obviously, the regulated output $e_i=y_i-y_0=x_{i2}-v_3$.

The solution of the regulator equation is

$$X_i(w)=\begin{pmatrix}-\frac{1}{2} & -\frac{1}{2} & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 2\end{pmatrix},\quad U_i(w)=\begin{pmatrix}-\frac{1+w}{2} & -\frac{1+w}{2} & -3 & 0\end{pmatrix},\ i=1,\dots,\kappa.$$

With the calculation based on Lemma 3.4, we obtain a steady-state generator $\{\Theta_i,\Upsilon,\Xi\}$

$$\Theta_i(w)=\begin{pmatrix}-\frac{1+w}{2} & -\frac{1+w}{2} & -3 & 0\\ \frac{1+w}{2} & -\frac{1+w}{2} & 0 & -6\\ \frac{1+w}{2} & \frac{1+w}{2} & 12 & 0\\ -\frac{1+w}{2} & \frac{1+w}{2} & 0 & 24\end{pmatrix},\ i=1,\dots,\kappa, \tag{46}$$

$$\Phi=\begin{pmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\-4&0&-5&0\end{pmatrix},\quad \Psi=\begin{pmatrix}1&0&0&0\end{pmatrix}.$$

Note that the eigenvalues of Φ are $\pm i,\ \pm 2i$ $(i^2=-1)$. Thus the rank condition holds.

Then we can take

$$M=\begin{pmatrix}-1&0&0&0\\1&-1&0&0\\0&1&-1&0\\0&0&1&-1\end{pmatrix},\ M_g=\begin{pmatrix}1\\1\\1\\1\end{pmatrix}.$$

Then the dynamic output feedback in the form (20) solves the robust distributed output regulation problem of the considered systems, where K_ξ and L_ξ satisfying (35) and (39), respectively are given by

$$K_\xi=\begin{pmatrix}-7 & -6 & -4 & -15 & 25 & -24 & 10\end{pmatrix},$$

$$L_\xi=\max\left\{1,\frac{1}{\min\limits_{i=1,\dots,\kappa} Re(\bar{\lambda}_i)}\right\}\begin{pmatrix}8.16\ 6.7461\ 3.1\ 13.0285\ -22.4252\ 22.44\ -9.2397\end{pmatrix}.$$

The topology of the multi-agent system with 5 agents switches between the two given topologies periodically carried out in an alternative order: $\{\mathcal{G}_1,\ \mathcal{G}_2,\ \mathcal{G}_1,\dots\}$ with switching period $t=10$. $\mathcal{G}_1$ is a graph with weights $a_{23}=a_{32}=a_{14}=a_{41}=a_{34}=a_{43}=a_{45}=a_{54}=a_{10}=a_{20}=1$, while other weights as 0. $\mathcal{G}_2$ is described by $a_{23}=a_{32}=a_{14}=a_{41}=a_{31}=a_{13}=a_{52}=a_{25}=a_{10}=1$, while other weights are 0. Figure 1 demonstrates the regulated errors of the five agents.

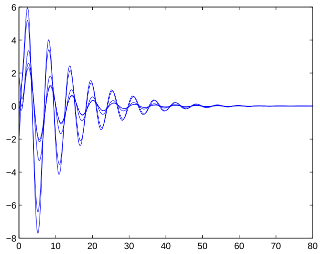

Fig. 1. Regulated errors of the agents.

5. CONCLUSIONS

In this paper, we analyzed the distributed robust output regulation problem for a group of mobile agents with uncertainty and switching topologies. We provided that sufficient conditions for the convergence of all the agents to the leader by constructing dynamic feedback with canonical internal model.

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