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DISTRIBUTED EVENT–TRIGGERED TRACKING CONTROL OF LEADER–FOLLOWER MULTI–AGENT SYSTEMS WITH COMMUNICATION DELAYS

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As embedded microprocessors are applied widerly to multi-agent systems, control scheduling and time-delay problems arose in the case of limited energy and computational ability. It has been shown that the event-triggered actuation strategy is an effective methodology for designing distributed control of multi-agent systems with limited computational resources. In this paper, a tracking control problem of leader-follower multi-agent systems with/without communication delays is formulated and a distributed dynamic tracking control is designed by employing event-triggered technique. Then, the input-to-state stability of the closed-loop multi-agent system with directed interconnections is analyzed. Finally, a numerical example is given to validate the proposed control.

Keywords: leader-follower multi-agent system, event-triggered control, time-varying delay, directed topology

Classification: 93A14, 93C10

1. INTRODUCTION

Recently some great advances have been achieved in cooperative control of multi-agent systems. The research focus is mainly on communication environments which consequently require distributed control design. To this day, some control techniques have been proposed according to different communication conditions, such as time-varying networks [5, 16], subject to measurement noise [10, 13], time delays [9, 17], or disturbances [15, 21].

A future control design may equip agents with embedded micro-processors to collect information from neighboring agents so as to update the controller according to some pre-designed rules. Motivated by this observation, some protocols were proposed to deal with distributed algorithms of communication and controller actuation scheduling [3, 20, 22]. Since micro-processors are generally resource- and energy-limited, an event-triggered control was designed based on measurement errors for execution in [20]. A timing issue was investigated through the use of a distributed event-triggered feedback scheme in networked control systems in [22]. Very recently, some distributed event-triggered control strategies were proposed for multi-agent systems [2, 3, 14]. All these control design methods possess a common characteristics that the controller is updated only when the measurement error magnitude exceeds a certain threshold. In [3] and [2], centralized and decentralized event-triggered multi-agent control protocols were developed for a first-order agreement problem, which were proven to be input-to-state stable (ISS) [12]. The centralized cooperative controller was actuated according to a global event-trigger rule while the decentralized one was updated at a sequence of separate event-times encoded by a local trigger function for each agent. Furthermore, a centralized event-triggered cooperative control was constructed for higher-dimensional multi-agent consensus with a weighted topology in [14], an event-triggered cooperative control was proposed for first-order discrete-time multi-agent systems in [4], and a neighbor-based tracking control together with a distributed estimation was proposed for leader-follower multi-agent systems in [8].

In this paper, we consider a distributed event-triggered tracking control problem for leader-follower multi-agent systems in a fixed directed network topology with partial measurements and possible communication delays. In collective coordination of a group of autonomous agents, the leader-follower problem has been considered for tracking a single or multiple leaders in [1, 8, 10, 11, 18]. In reality, some state information of the leader cannot be measured, therefore a decentralized "observer" design plays a key role in cooperative control of leader-follower multi-agent systems. Within this context, an "observer"-based dynamic tracking control was proposed to estimate the unmeasurable state (i. e., velocity) of an active leader in [8] by collecting real-time measurements from neighbors. In this paper, inspired by the event-triggered scheduling strategy in multi-agent systems, we consider a dynamic tracking problem with event-triggered strategy involved in the control update. During the event-triggered tracking control process, we assume that every follower agent broadcasts its state information only if "needed", which requires the follower agent to update its state only if some measure of its state error is above a specified threshold. In the literature about event-triggered control of multi-agent systems, event-triggered cooperative controllers often keep constant between two consecutive broadcasts. However, in this paper we concern with the scenario of an independent active leader, who does not need the event-triggered control updates. Thus, a more sophisticated event-triggered strategy needs to be developed to continuously update every agent's partial control input, subject to its local computational resources availability. We adopt a decentralized event-triggered strategy to update the local controllers, and finally take into account the communication delays in the tracking control design.

The paper is organized as follows: Section 2 presents some preliminaries and formulates the tracking problem under investigation. In Section 3, a decentralized event-triggered tracking control is designed for two cases: time-delay free and time-varying delays. Then, the convergence of the tracking error evolution is analyzed in Section 4. Section 5 gives a numerical example to illustrate the tracking algorithm. Section 6 summarizes the results of the paper and suggests a couple of some future research topics.

Throughout this paper, I denotes an identity matrix; $\mathbf{1}$ denotes a column vector with all ones; the norm of a vector $x \in \mathbb{R}^n$ is defined as $\|x\| = \sqrt{x^T x}$; the spectral norm of matrix $A \in \mathbb{R}^{m\times n}$ is defined as $\|A\| = \max_{1\le i\le n} \sqrt{\lambda_i}$, where λ_i $(i = 1,\dots,n)$ are eigenvalues of A^TA; col$(\cdot)$ denotes the concatenation.

2. PRELIMINARIES AND PROBLEM FORMULATION

The multi-agent system under study is a group of n follower-agents (called followers for simplicity and labelled $1,\dots,n$) and one active leader-agent (called leader and labelled 0). The followers are moving based on the information exchange in their individual neighborhood while the leader is self-active hence moving independently. Thus, the information flow in the leader-follower multi-agent system can be conveniently described by a directed graph $\bar{\mathcal{G}}$. In graph theory [6], a directed graph $\bar{\mathcal{G}}$ consists of a vertex set $\bar{\mathcal{V}}$ and an arc set $\bar{\mathcal{E}}$. Here, $\bar{\mathcal{V}} = \{0, 1, 2, \dots, n\}$ and $\bar{\mathcal{E}} = \{(i, j)|i, j \in \bar{\mathcal{V}}\}$, where vertex $i \in \bar{\mathcal{V}}$ represents agent i, and (i, j) is in $\bar{\mathcal{E}}$ if and only if agent i receives information from agent j and a weight $a_{ij} > 0$ is defined simultaneously. A neighbor set of follower i is defined by $\mathcal{N}_i = \{j|(i, j) \in \bar{\mathcal{E}}\}$.

To define the connectivity of $\bar{\mathcal{G}}$, some concepts in graph theory are needed. A path in $\bar{\mathcal{G}}$ is a sequence $i_0, i_1, \dots, i_q$ of distinct vertices such that (i_{j-1}, i_j) is an arc, $j = 1,\dots,q$. If there exists a path from vertex i to vertex j, vertex j is said to be reachable from vertex i. Furthermore, if there exists a path from every vertex in $\bar{\mathcal{G}}$ to vertex j, then vertex j is a globally reachable vertex of $\bar{\mathcal{G}}$. A directed graph $\bar{\mathcal{G}}$ is strongly connected if there exists a path between any two distinct vertices. A directed graph $\mathcal{G}$ is a subgraph of $\bar{\mathcal{G}}$ if its vertex set $\mathcal{V}(\mathcal{G}) \subseteq \bar{\mathcal{V}}$, arc set $\mathcal{E}(\mathcal{G}) \subseteq \bar{\mathcal{E}}$ and every arc in $\mathcal{E}(\mathcal{G})$ has both end-vertices in $\bar{\mathcal{V}}$. A subgraph $\mathcal{G}$ is an induced subgraph provided that two vertices of $\mathcal{G}$ are adjacent in $\mathcal{G}$ if and only if they are adjacent in $\bar{\mathcal{G}}$. An induced subgraph $\mathcal{G}$ that is strongly connected and maximal (i. e., no more vertices can be added while preserving its connectedness) is called a strong component of $\bar{\mathcal{G}}$. It is noted that even if $\bar{\mathcal{G}}$ has a global reachable vertex, its subgraph $\mathcal{G}$ with vertex set $\mathcal{V} = \bar{\mathcal{V}}/\{0\}$ may not be strongly connected. In this paper, such a subgraph $\mathcal{G}$ will be employed to model the interconnection topology of the n followers.

The dynamics of the ith follower is assumed to be a first-order linear system:

$$\dot{x}_i(t) = u_i(t),\ i = 1,\dots,n, \tag{1}$$

where $x_i(t) \in \mathbb{R}^l$ and $u_i(t) \in \mathbb{R}^l$ are, respectively, the state and the control input. The active leader is described by a second-order linear system with a partially unknown acceleration:

$$\begin{cases} \dot{x}_0(t) = v_0(t), \\ \dot{v}_0(t) = u_0(t) + \delta(t), \\ y_0(t) = x_0(t), \end{cases} \tag{2}$$

where $x_0(t) \in \mathbb{R}^l, v_0(t) \in \mathbb{R}^l$ and $u_0(t) \in \mathbb{R}^l$ are, respectively, the position, velocity and acceleration, the disturbance $\delta(t) \in \mathbb{R}^l$ is bounded with an upper bound $\bar{\delta}$, and $y_0(t)$ is the only measured output. To simplify the notation, we assume $l = 1$ in the sequel.

Since only the position of the leader can be measured, each follower has to collect information from its neighbors and estimate the leader's velocity during the motion process. In [8], a distributed observer-based dynamic tracking control was proposed for each follower i:

$$\begin{aligned} u_i &= v_i - k\Big[\sum_{j\in\mathcal{N}_i} a_{ij}(x_i - x_j) + a_{i0}(x_i - x_0)\Big], \\ \dot{v}_i &= u_0 - \gamma k\Big[\sum_{j\in\mathcal{N}_i} a_{ij}(x_i - x_j) + a_{i0}(x_i - x_0)\Big], \end{aligned} \tag{3}$$

where $v_i(t)$ is the estimate of the leader's velocity $v_0(t)$ and a_{i0} is the leader's adjacency coefficient. The dynamic tracking control (3) assumes that the relative-position measurements $x_i - x_j$ are transmitted in continuous time. In practice, however, communication (especially wireless communication) takes place over digital networks therefore information is transmitted at discrete time instants. When the follower finds that a local "error" signal exceeds a given threshold, it broadcasts its state information to all neighboring agents. Under this scenario, the event-triggered dynamic tracking control is more preferable than that proposed in (3).

In the leader-follower problem under investigation, the active leader is independent and needs not broadcast its information in any event-triggered fashion. However, follower i's control, $u_i(t)$, has to be designed based on the latest states received from its neighboring followers and also the sate $x_0(t)$ if it is linked to the leader. Therefore, a new control protocol needs to be designed to solve the leader-following problem with an event-triggered scheduling strategy. The event-triggered tracking problem is said to be solved if one can find a distributed event-triggered control strategy such that

$$\|x_i(t) - x_0(t)\| \leq \zeta, \tag{4}$$

for some constant ζ depending on $\bar{\delta}$, $i = 1, \ldots, n$, as $t \to \infty$.

3. CONTROL DESIGN

In this section, we design event-triggered tracking controls for systems with or without communication delays.

In consensus control, typical information available for a follower is its relative positions with the neighbors. It is usually assumed that the relative-position measurement

$$y_{ij}(t) = x_i(t) - x_j(t) \tag{5}$$

is performed in continuous time, which implicitly implies that the multi-agent communication network bandwidth is unlimited or every agent has abundant energy. However, when followers transmit their state information in discrete time, distributed tracking control needs to be redefined to take into account event-triggered strategies. In order to model the event-triggers for followers, assume that there are n monotone increasing sequences of event times $\tau_i(s)$ $(s = 0, 1, \ldots; i = 1, \ldots, n)$. Let $\hat{x}_i(t) = x_i(\tau_i(s)), t \in [\tau_i(s), \tau_i(s+1))$, be the measured state of follower i. The measured relative-position measurements $\hat{y}_{ij}(t)$ depend on the measured states $\hat{x}_i(t)$ and $\hat{x}_j(t)$, $j \in \mathcal{N}_i$, that is,

$$\hat{y}_{ij}(t) = \hat{x}_i(t) - \hat{x}_j(t),\ i, j = 1, \ldots, n. \tag{6}$$

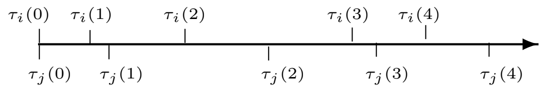


Fig. 1. The event times for follower i and follower j.

It should be noted that the event times $\tau_i(s)$ are mutually independent among followers and may take different values, as illustrated by Figure 1.

Furthermore, if the communication between agent i and agent j (or the leader) has a time-varying delay $r(t)$, then the measured relative-position measurement is described by

$$\hat{y}_{ij}(t-r(t)) = \hat{x}_i(t-r(t)) - \hat{x}_j(t-r(t)), \tag{7}$$

where $r(t)$ is a continuously differentiable function satisfying $0 \le r(t) \le \bar{r} < \infty$.

Due to unavailable measurement of the leader's velocity $v_0(t)$, each follower can have an estimate $v_i(t)$ by fusing the information obtained from its neighbors. When communication delay is not considered, the velocity estimate $v_i(t)$ is given with the measurements $\hat{y}_{ij}(t)$ and $y_{i0}(t)$, as follows:

$$\dot{v}_i(t) = u_0(t) - \gamma k \left[\sum_{j\in\mathcal{N}_i} a_{ij}\hat{y}_{ij}(t) + a_{i0}y_{i0}(t) \right], \tag{8}$$

where a_{ij} denotes the adjacency coefficient between follower i and follower j; constant $0 < \gamma < 1$; the gain k is to be designed. Moreover, an event-triggered tracking control is designed as follows:

$$u_i(t) = v_i(t) - k \left[\sum_{j\in\mathcal{N}_i} a_{ij}\hat{y}_{ij}(t) + a_{i0}y_{i0}(t) \right], \tag{9}$$

where the gain k is the same as above. It is noted that both the velocity estimate $v_i(t)$ and the control input $u_i(t)$ use the broadcasted measurements $\hat{y}_{ij}(t)$ from neighboring followers and the continuous-time measurement $y_{i0}(t)$ from the leader.

When communication delay is involved in the multi-agent coordination, a distributed event-triggered tracking control with time delays can be similarly formulated, as follows:

$$\begin{aligned} u_i(t) &= v_i(t) - k \left[\sum_{j\in\mathcal{N}_i} a_{ij}\hat{y}_{ij}(t-r) + a_{i0}y_{i0}(t-r) \right], \\ \dot{v}_i(t) &= u_0(t) - \gamma k \left[\sum_{j\in\mathcal{N}_i} a_{ij}\hat{y}_{ij}(t-r) + a_{i0}y_{i0}(t-r) \right]. \end{aligned} \tag{10}$$

4. CONVERGENCE ANALYSIS

In this section, we analyze the convergence of the tracking errors for all followers under distributed event-triggered control with/without communication delays.

4.1. No communication delays

Let $e_i(t) = \hat{x}_i(t) - x_i(t) = x_i(\tau_i(s)) - x_i(t),\ t \in [\tau_i(s), \tau_i(s+1))$. The event-time $\tau_i(s)$ is implicitly defined by an event-trigger, $f_i(e_i(t), \{e_j(t) | j \in \mathcal{N}_i\}) = 0$, which will be given below. Thus, $\hat{x}_i(t) = e_i(t) + x_i(t)$. With this variable change, the control (9) together with the velocity estimation (8) is applied to system (1), which yields the following closed-loop system:

$$\begin{cases} \dot{x} = v - k(L+B)x + kB\mathbf{1}x_0 - kLe, \\ \dot{v} = u_0\mathbf{1} - \gamma k(L+B)x + \gamma kB\mathbf{1}x_0 - \gamma kLe, \end{cases} \tag{11}$$

where $x = \mathrm{col}(x_1, \ldots, x_n) \in \mathbb{R}^n, v = \mathrm{col}(v_1, \ldots, v_n) \in \mathbb{R}^n$, and $e = \mathrm{col}(e_1, \ldots, e_n) \in \mathbb{R}^n$, respectively, denote the position, velocity estimation, measurement error of the leader-follower multi-agent system, $L = D - A \in \mathbb{R}^{n\times n}, A = [a_{ij}] \in \mathbb{R}^{n\times n}$ and $D \in \mathbb{R}^{n\times n}$ are, respectively, the Laplacian matrix, adjacency matrix and degree matrix of the directed subgraph $\mathcal{G}$, $B = \mathrm{diag}\{a_{10}, \ldots, a_{n0}\}$ is a diagonal matrix representing the leader-follower adjacency relationship, and $\mathbf{1} = \mathrm{col}(1, \ldots, 1) \in \mathbb{R}^n$. From algebraic graph theory [6], it is well known that L always has a zero eigenvalue associated with the right eigenvector $\mathbf{1}$. Moreover, if the subgraph $\mathcal{G}$ is balanced, L has a zero eigenvalue associated with the left eigenvector $\mathbf{1}$. Thus, we have

$$-(L+B)x + B\mathbf{1}x_0 = -(L+B)(x - x_0\mathbf{1}).$$

Define a new matrix $H = L + B$.

Lemma 4.1. (Hu and Hong [11]) The following statements are equivalent:

(I) vertex 0 is a globally reachable vertex of the directed graph $\bar{\mathcal{G}}$;

(II) $-H$ is a stable matrix whose eigenvalues have negative real-parts;

(III) $\mathcal{G}$ is balanced, and $H + H^T$ is a symmetric positive definite matrix.

Lemma 4.1 implies that if vertex 0 is a globally reachable vertex of the directed graph $\bar{\mathcal{G}}$ and if its subgraph $\mathcal{G}$ is balanced, then

$$\lambda_* = \min\{\lambda : \text{ eigenvalues of } H + H^T\} > 0.$$

For system (11), define two variable changes:

$$\begin{aligned} \bar{x} &= x - x_0\mathbf{1}, \\ \bar{v} &= v - v_0\mathbf{1}. \end{aligned} \tag{12}$$

Thus, system (11) can be further simplified as follows:

$$\begin{cases} \dot{\bar{x}} = \bar{v} - kH\bar{x} - kLe, \\ \dot{\bar{v}} = -\gamma kH\bar{x} - \gamma kLe - \mathbf{1}\otimes\delta, \end{cases} \tag{13}$$

which can be rewritten in a compact form:

$$\dot{\varepsilon} = F\varepsilon + Je + g, \tag{14}$$

where $\varepsilon = \mathrm{col}(\bar{x}, \bar{v})$, $F = \begin{pmatrix} -kH & I \\ -\gamma kH & 0 \end{pmatrix}$, $J = \begin{pmatrix} -kL \\ -\gamma kL \end{pmatrix}$ and $g = \begin{pmatrix} 0 \\ -\mathbf{1} \otimes \delta \end{pmatrix}$.

Now, a main result is established as follows.

Theorem 4.2. Assume that vertex 0 is globally reachable in the directed graph $\bar{\mathcal{G}}$, its subgraph $\mathcal{G}$ is balanced, and k satisfies

$$k > \frac{1}{2\gamma(1-\gamma^2)\lambda_*}. \tag{15}$$

Then, with control (9) and estimation (8), the event-triggered tracking problem is solved. Moreover, if the disturbance bound $\bar{\delta} = 0$, then $\lim_{t\to\infty} \|\varepsilon(t)\| = 0$.

Proof. Take a candidate ISS Lyapunov function $V(\varepsilon) = \varepsilon^T(t)P\varepsilon(t)$ with a symmetric positive definite matrix

$$P = \begin{pmatrix} I_n & -\gamma I_n \\ -\gamma I_n & I_n \end{pmatrix}, \ 0 < \gamma < 1.$$

Consider the derivative of $V(\varepsilon)$:

$$\begin{aligned} \dot{V}(\varepsilon)|_{(14)} &= \varepsilon^T(F^TP + PF)\varepsilon + 2\varepsilon^TPJe + 2\varepsilon^TPg \\ &= -\varepsilon^TQ\varepsilon + 2\varepsilon^TPJe + 2\varepsilon^TPg, \end{aligned}$$

where

$$Q = \begin{pmatrix} k(1-\gamma^2)(H+H^T) & -I_n \\ -I_n & 2\gamma I_n \end{pmatrix}.$$

If k satisfies (15), then the matrix Q will be positive definite according to the Schur complement formula. Simultaneously, we can get the minimum eigenvalue of Q:

$$\mu_* = \frac{1}{2}\Big[(1-\gamma^2)k\lambda_* + 2\gamma - \sqrt{[(1-\gamma^2)k\lambda_* - 2\gamma]^2 + 4}\Big], \tag{16}$$

which is a positive number when k satisfies (15). In addition, the eigenvalues of P are either $1-\gamma$ or $1+\gamma$, so

$$(1-\gamma)\|\varepsilon\|^2 \le V(\varepsilon) \le (1+\gamma)\|\varepsilon\|^2. \tag{17}$$

Therefore, from (16) and (17), we have

$$\begin{aligned} &\dot{V}(\varepsilon)|_{(14)} \le -\mu_*\|\varepsilon\|^2 + 2\varepsilon^TPJe + 2\varepsilon^TPg \\ &\le -\mu_*\|\bar{v}\|^2 - \mu_*\|\bar{x}\|^2 - 2(1-\gamma^2)k\sum_i \sum_{j\in\mathcal{N}_i} \bar{x}_i(e_i - e_j) + 2(1+\gamma)\|\varepsilon\|\bar{\delta} \\ &\le -\mu_*\|\bar{v}\|^2 - \mu_*\sum_i \left[\|\bar{x}_i\|^2 - \frac{2(1-\gamma^2)k\|\bar{x}_i\|}{\mu_*}\sum_{j\in\mathcal{N}_i}(\|e_i\| + \|e_j\|)\right] + 2(1+\gamma)\|\varepsilon\|\bar{\delta}. \end{aligned}$$

Enforcing the condition

$$\sum_{j\in\mathcal{N}_i}(\|e_i\| + \|e_j\|) \le \epsilon\frac{\mu_*\|\bar{x}_i\|}{2(1-\gamma^2)k} \tag{18}$$

with $0 < \epsilon < 1$, we have

$$\begin{aligned}\dot{V}(\varepsilon)|_{(14)} &\leq -(1-\epsilon)\mu_*\|\varepsilon\|^2 + 2(1+\gamma)\|\varepsilon\|\bar{\delta} \\ &\leq -\frac{1}{2}(1-\epsilon)\mu_*\|\varepsilon\|^2 + \frac{2(1+\gamma)^2\bar{\delta}^2}{(1-\epsilon)\mu_*}.\end{aligned} \tag{19}$$

Thus, for follower i, an event-trigger can be defined by

$$f_i(e_i(t), \{e_j(t)|j \in \mathcal{N}_i\}) = \sum_{j\in\mathcal{N}_i}(\|e_i\| + \|e_j\|) - \epsilon\frac{\mu_*\|\bar{x}_i\|}{(1-\gamma^2)k}. \tag{20}$$

When the event-trigger $f_i(e_i(t), \{e_j(t)|j \in \mathcal{N}_i\}) = 0$, the condition (18) is enforced.

Given the event-trigger (20), from (17) and (19), we have

$$\dot{V}(\varepsilon)|_{(14)} \leq -\frac{(1-\epsilon)\mu_*}{2(1+\gamma)}V(\varepsilon) + \frac{2(1+\gamma)^2\bar{\delta}^2}{(1-\epsilon)\mu_*}. \tag{21}$$

Thus, with $t_0 = 0$,

$$V(\varepsilon(t)) \leq e^{-\frac{(1-\epsilon)\mu_*}{2(1+\gamma)}t}V(\varepsilon(0)) + \frac{4(1+\gamma)^3\bar{\delta}^2}{(1-\epsilon)^2\mu_*^2}(1 - e^{-\frac{(1-\epsilon)\mu_*}{2(1+\gamma)}t}),$$

which implies

$$\lim_{t\to\infty}\|\varepsilon(t)\| \leq \zeta$$

with $\zeta = \frac{2(1+\gamma)\sqrt{1+\gamma}}{(1-\epsilon)\mu_*\sqrt{1-\gamma}}\bar{\delta}$. Furthermore, if $\bar{\delta} = 0$, then $\lim_{t\to\infty}\|\varepsilon(t)\| = 0$. The proof is thus completed. □

Remark 4.3. To simplify the simulations in the next section, the event-trigger condition (18) can be replaced by a conservative centralized one, given as follows:

$$\|e\| \leq \epsilon\frac{\mu_*\|\varepsilon\|}{2(1-\gamma^2)\|L\|}. \tag{22}$$

If the event-trigger condition (22) holds and if $\bar{\delta} = 0$, then there exists at least one agent for which the next inter-event interval is bounded from below by a time τ_D, implicitly determined by

$$\begin{cases}\tau_D = \frac{1}{\|F\|-\|J\|}\ln\left[\frac{1+\phi}{1+\frac{\|J\|}{\|F\|}\phi}\right], \\ \phi(\tau_D, 0) = \frac{\epsilon\mu_*}{2(1-\gamma^2)\|L\|},\end{cases} \tag{23}$$

where $y(t) = \frac{\|e\|}{\|\varepsilon\|} \leq \phi(t, \phi_0)$, $\phi(0, \phi_0) = \phi_0$, and $\phi(t, \phi_0)$ is the solution of

$$\dot{\phi} = \|F\|(1+\phi)\Big(1 + \frac{\|J\|}{\|F\|}\phi\Big). \tag{24}$$

Here, (24) is derived by differentiating $y(t)$ and employing (14).

4.2. With communication delays

Still, take the variable change $\hat{x}_i(t) = e_i(t) + x_i(t)$. With control (10), we have

$$\begin{cases} \dot{x} = v - k(L+B)x(t-r) + kB\mathbf{1}x_0(t-r) - kLe(t-r), \\ \dot{v} = u_0\mathbf{1} - \gamma k(L+B)x(t-r) + \gamma kB\mathbf{1}x_0(t-r) - \gamma kLe(t-r). \end{cases} \tag{25}$$

After making the variable changes (12), a further simplified closed-loop system is obtained in the form of time-delayed differential equations as follows:

$$\begin{cases} \dot{\bar{x}} = \bar{v} - kH\bar{x}(t-r) - kLe(t-r), \\ \dot{\bar{v}} = -\gamma kH\bar{x}(t-r) - \gamma kLe(t-r) - \mathbf{1}\otimes\delta, \end{cases} \tag{26}$$

or,

$$\dot{\varepsilon} = F_1\varepsilon + F_2\varepsilon(t-r) + Je(t-r) + g, \tag{27}$$

where

$$\varepsilon = \mathrm{col}(\bar{x},\bar{v}), F_1 = \begin{pmatrix} 0 & I \\ 0 & 0 \end{pmatrix}, \; F_2 = \begin{pmatrix} -kH & 0 \\ -\gamma kH & 0 \end{pmatrix}, \; J = \begin{pmatrix} -kL \\ -\gamma kL \end{pmatrix}, \text{ and } g = \begin{pmatrix} 0 \\ -\mathbf{1}\otimes\delta \end{pmatrix}.$$

Before establishing another main result, we state an important lemma on the stability of time-delayed systems.

Lemma 4.4. (Hale and Lunel [7]) Let ϕ_1, ϕ_2 and ϕ_3 be continuous, nonnegative, and nondecreasing functions, with $\phi_1(\ell) > 0, \phi_2(\ell) > 0, \phi_3(\ell) > 0$ for $\ell > 0$ and $\phi_1(0) = \phi_2(0) = 0$. Consider the following system:

$$\begin{aligned} \dot{x} &= f(x_t), \; t > 0, \\ x(\theta) &= \varphi(\theta), \; \theta \in [-r, 0], \end{aligned} \tag{28}$$

where $x_t(\theta) = x(t+\theta), \; \theta \in [-r, 0]$ and $f(0) = 0$. If there is a continuous function $V(t,x)$ satisfying:

- Condition (I):
$$\phi_1(\|x\|) \le V(t,x) \le \phi_2(\|x\|), t \in \mathbb{R}, \; x \in \mathbb{R}^n;$$
- Condition (II): for $\Delta > 0$, there exists a continuous nondecreasing function $\phi(\ell)$ with $\phi(\ell) > \ell$, for $\ell > 0$, such that
$$\dot{V}(t,x) \le -\phi_3(\|x\|) + \Delta,$$
whenever $V(t+\theta, x(t+\theta)) < \phi(V(t, x(t))), \; \theta \in [-r, 0]$;

then the solution $x = 0$ of (28) is uniformly ultimately bounded.

Theorem 4.5. Under the same conditions as in Theorem 4.2, the event-triggered tracking problem with communication delays is solved.

P r o o f. Take the same Lyapunov function $V(\varepsilon) = \varepsilon^T(t)P\varepsilon(t)$, and consider the derivative of $V(\varepsilon)$. Since

$$\begin{aligned}\varepsilon(t-r) &= \varepsilon(t) - \int_{t-r}^{t} \dot{\varepsilon}(\ell)\,\mathrm{d}\ell,\\ &= \varepsilon(t) - \int_{t-r}^{t} \Big[F_1\varepsilon(\ell) + F_2\varepsilon(\ell - r) + Je(\ell - r) + g(\ell)\Big]\,\mathrm{d}\ell,\end{aligned}$$

system (27) can be transformed to

$$\begin{aligned}\dot{\varepsilon} =& F\varepsilon - F_2F_1\int_{-r}^{0} \varepsilon(t+\ell)\,\mathrm{d}\ell - F_2^2\int_{-r}^{0}\varepsilon(t+\ell-r)\,\mathrm{d}\ell\\ &- F_2J\int_{-r}^{0} e(t+\ell-r)d\ell - F_2\int_{t-r}^{t} g(\ell)\,\mathrm{d}\ell,\end{aligned}$$

where $F = F_1 + F_2$. Then,

$$\begin{aligned}\dot{V}(\varepsilon)|_{(27)} =& \varepsilon^T(F^TP + PF)\varepsilon - 2\varepsilon^TPF_2F_1\int_{-r}^{0}\varepsilon(t+\ell)\,\mathrm{d}\ell - 2\varepsilon^TPF_2\int_{t-r}^{t} g(\ell)\,\mathrm{d}\ell\\ &- 2\varepsilon^TPF_2^2\int_{-r}^{0}\varepsilon(t+\ell-r)\,\mathrm{d}\ell - 2\varepsilon^TPF_2J\int_{-r}^{0} e(t+\ell-r)\,\mathrm{d}\ell.\end{aligned}$$

Observe that $2a^Tb \le a^T\Psi a + b^T\Psi^{-1}b$ holds for any appropriate positive definite matrix Ψ. Thus, for the second term in (4.2), with $a^T = -\varepsilon^TPF_2F_1, b = \varepsilon(t+\ell)$ and $\Psi = P^{-1}$, we have

$$-2\varepsilon^TPF_2F_1\int_{-r}^{0}\varepsilon(t+\ell)\,\mathrm{d}\ell \le r\varepsilon^TPF_2F_1P^{-1}F_1^TF_2^TP\varepsilon + \int_{-r}^{0}\varepsilon^T(t+\ell)P\varepsilon(t+\ell)\,\mathrm{d}\ell. \tag{29}$$

Similarly, for the third and fourth terms in (4.2), we have

$$\begin{aligned}&-2\varepsilon^TPF_2^2\int_{-r}^{0}\varepsilon(t+\ell-r)\,\mathrm{d}\ell \le r\varepsilon^TPF_2^2P^{-1}(F_2^2)^TP\varepsilon + \int_{-2r}^{-r}\varepsilon^T(t+\ell)P\varepsilon(t+\ell)\,\mathrm{d}\ell,\\ &-2\varepsilon^TPF_2J\int_{-r}^{0} e(t+\ell-r)\,\mathrm{d}\ell \le r\varepsilon^TPF_2JJ^TF_2^TP\varepsilon + \int_{-2r}^{-r} e^T(t+\ell)e(t+\ell)\,\mathrm{d}\ell,\\ &-2\varepsilon^TPF_2\int_{t-r}^{t} g(\ell)\,\mathrm{d}\ell \le r\varepsilon^TPF_2F_2^TP\varepsilon + n\bar{\delta}^2\bar{r}.\end{aligned} \tag{30}$$

It then follows from the above inequalities (29), (30), and Lemma 4.4 with

$$\phi(\ell) = q\ell \tag{31}$$

for some constant $q > 1$, that

$$\begin{aligned}\dot V(\varepsilon)|_{(27)} \le & -\varepsilon^T Q\varepsilon + r\varepsilon^T\Big[PF_2F_1P^{-1}F_1^TF_2^TP + PF_2^2P^{-1}(F_2^2)^TP \\ & + PF_2JJ^TF_2^TP + PF_2F_2^TP\Big]\varepsilon + \int_{-r}^{0}\varepsilon^T(t+\ell)P\varepsilon(t+\ell)\,\mathrm{d}\ell \\ & + \int_{-2r}^{-r}\varepsilon^T(t+\ell)P\varepsilon(t+\ell)\,\mathrm{d}\ell + \int_{-2r}^{-r}e^T(t+\ell)e(t+\ell)\,\mathrm{d}\ell + n\bar\delta^2\bar r \\ \le & -\varepsilon^T Q\varepsilon + r\varepsilon^T\Big[PF_2F_1P^{-1}F_1^TF_2^TP + PF_2^2P^{-1}(F_2^2)^TP \\ & + PF_2JJ^TF_2^TP + PF_2F_2^TP\Big]\varepsilon + \int_{-r}^{0}\varepsilon^T(t+\ell)P\varepsilon(t+\ell)\,\mathrm{d}\ell \\ & + \int_{-2r}^{-r}\varepsilon^T(t+\ell)P\varepsilon(t+\ell)\,\mathrm{d}\ell + \int_{-2r}^{-r}e^T(t+\ell)e(t+\ell)\,\mathrm{d}\ell + n\bar\delta^2\bar r \\ & + \frac{1}{1-\gamma}\int_{-2r}^{-r}\varepsilon^T(t+\ell)P\varepsilon(t+\ell)\,\mathrm{d}\ell - \int_{-2r}^{-r}\varepsilon^T(t+\ell)\varepsilon(t+\ell)\,\mathrm{d}\ell \\ \le & -\varepsilon^T\Big[Q - rPF_2F_1P^{-1}F_1^TF_2^TP - rPF_2^2P^{-1}(F_2^2)^TP - rPF_2JJ^TF_2^TP \\ & - rPF_2F_2^TP - \frac{3-2\gamma}{1-\gamma}qrP\Big]\varepsilon + n\bar\delta^2\bar r \\ & - \int_{-2r}^{-r}\Big[\varepsilon^T(t+\ell)\varepsilon(t+\ell) - e^T(t+\ell)e(t+\ell)\Big]\,\mathrm{d}\ell.\end{aligned}$$

Denote

$$\alpha = \|PF_2F_1P^{-1}F_1^TF_2^TP + PF_2^2P^{-1}(F_2^2)^TP + PF_2JJ^TF_2^TP + PF_2F_2^TP + \frac{3-2\gamma}{1-\gamma}qP\|.$$

If we choose the upper bound of the time delay to satisfy

$$\bar r < \frac{(1-\gamma)\mu_*}{(1+\gamma)\alpha - (1-\epsilon)(1-\gamma)q}, \tag{32}$$

where μ_* is given by (16), and the event-trigger function as

$$f_i(e_i, \{e_j | j \in \mathcal{N}_i\}) = \|e\|^2 - \epsilon\|\varepsilon\|^2, \tag{33}$$

for $0 < \epsilon < 1$, where $\bar x$ is defined in (12), then we have

$$\begin{aligned}\dot V(\varepsilon)|_{(27)} \le & -\mu_*\varepsilon^T\varepsilon + \alpha r\varepsilon^T\varepsilon - (1-\epsilon)\int_{-2r}^{-r}\varepsilon^T(t+\ell)\varepsilon(t+\ell)\,\mathrm{d}\ell + n\bar r\bar\delta^2 \\ \le & -\Big(\frac{\mu_*}{1+\gamma} - \frac{\alpha r}{1-\gamma} + \frac{(1-\epsilon)qr}{1+\gamma}\Big)V(\varepsilon) + n\bar r\bar\delta^2 \\ = & -\beta V(\varepsilon) + n\bar r\bar\delta^2,\end{aligned}$$

with $\beta = \frac{\mu_*}{1+\gamma} - \frac{\alpha r}{1-\gamma} + \frac{(1-\epsilon)qr}{1+\gamma}$. According to Lemma 4.4, $\varepsilon(t) = 0$ is uniformly ultimately bounded. In fact, $\|\varepsilon(t)\| \le \zeta = \sqrt{\frac{n\bar r}{(1-\gamma)\beta}}\bar\delta$, as $t \to \infty$. The proof is completed. □

Remark 4.6. The upper bound (32) of the time-varying communication delay is very conservative, as shown in the numerical example.

5. NUMERICAL EXAMPLE

In this section, we give a numerical example to illustrate the distributed event-triggered tracking controls (9) and (10) of the leader-follower multi-agent system (2) and system (1).

The considered directed graph $\bar{\mathcal{G}}$ associated with the leader-follower multi-agent system is depicted in Figure 2, where the corresponding subgraph $\mathcal{G}$ is balanced.

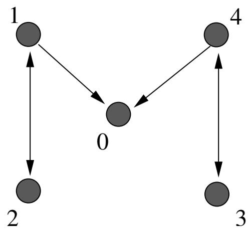
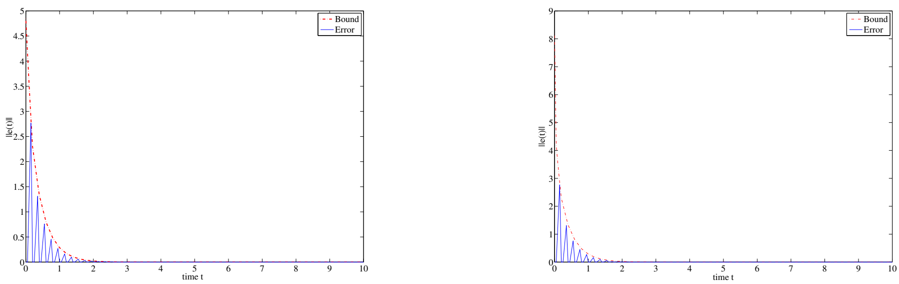


Fig. 2. Topology $\bar{\mathcal{G}}$ and its subgraph $\mathcal{G}$.



(a) Tracking without communication delays

(b) Tracking with a communication delay

Fig. 3. Evolution of $\|e(t)\|$ and its upper bound.

The Laplacian matrix L of $\mathcal{G}$ is

$$L = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix},$$

and the leader adjacency matrix is $B = \operatorname{diag}\{1, 0, 0, 1\}$. By Lemma 4.1, $H + H^T$ is positive definite. The minimum eigenvalue $\lambda_* = 0.7639$. Thus, from (15), we can take $k = 7.5$ when $\gamma = 0.5$. The acceleration of the active leader is assumed to be

$u_0(t) = t\sin(t)$. For event-triggered control (9) without communication delays, by taking $\epsilon = 0.8$ in (18), an alternative event-triggered condition $\|e(t)\| = \frac{\epsilon\|\varepsilon(t)\|}{2k(1-\gamma^2)\|L\|}$ is adopted to show the evolution of the measurement error, as illustrated in Figure 3 (a).

For event-triggered control (10) with communication delays, take $q = 2.5$ in (31), so that the upper bound $\bar{r} < 1.27 \times 10^{-6}$ and the time-varying delay is assumed to be $r(t) = 10^{-6} \cdot |\cos(t)| < \bar{r}$. An event-triggered condition $\|e(t)\| = \sqrt{\epsilon}\|\varepsilon(t)\|$ with $\epsilon = 0.7$ in (33) is used in this example. Then, the evolution of the measurement error is obtained as shown in Figure 3 (b).

6. CONCLUSION

A new distributed event-triggered tracking control was proposed for leader-follower multi-agent systems on a directed interconnection graph with/without communication delays. The input-to-state stability of the closed multi-agent system was analyzed by employing an ISS Lyapunov function. Simulations showed that the proposed control is effective. Some possible future research issues may be to provide rules that guarantee better bounds on the event-triggered intervals and to relax the requirements on updating control laws at the agents' own event times.

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