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EDITORIAL: QUANTUM STRUCTURES, STATES AND RELATED TOPICS

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This special issue "Quantum structures, states and related topics" is devoted to problems presented at two conferences. Namely the conference "Fuzzy set theory and applications 2010" which was held in Liptovský Ján, February 1 – 5, 2010 and the conference Analytic and Algebraic Methods in Physics VI which was held in Prague, May 9 – 11, 2010. The motivation of this common issue was the just started cooperation of a group of participants of both conferences.

In the classical probability theory (called Kolmogorovian) the set S of (experimentally verifiable) events forms a Boolean σ-algebra in which every pair of elements is compatible and every element is sharp.

Another situation arises in quantum mechanics. For example, from Heisenberg uncertainty principle it follows that the position x and the momentum p of an elementary particle cannot be measured simultaneously with arbitrarily prescribed accuracy (events x and p are non-compatible elements in non-classical logic).

G. Birkhoff and J. von Neumann (1936) suggested the lattice of (orthogonal) projection operators on a Hilbert space H as a non-classical logic for calculus of propositions of a quantum-mechanical system. G. Birkhoff and J. von Neumann proposed that this logic should be an orthocomplemented modular lattice. But the lattice of projections on a Hilbert space is modular iff H is finitely dimensional. K. Husimi introduced new law which holds in arbitrary Hilbert spaces called (in present time) the orthomodular law. Thus orthomodular lattices are nondistributive generalizations of Boolean algebras which may include non-compatible pairs of elements.

MV-algebras introduced as model of multi valued logic by C. C. Chang (1958) are another generalization of Boolean algebras. MV-algebras are distributive lattices with further binary operation $\oplus$ in which the orthocomplementation is weakened to supplementation.

The notion of an effect algebra has been introduced by D. J. Foulis and M. K. Bennett (1994). It is worth to note that in fact it generalizes the algebraic structure of the set $\mathcal{E}(\mathcal{H})$ Hilbert space effects, that are self-adjoint operators between the zero and identity operators on a Hilbert space $\mathcal{H}$.

From related topics involved in this special issue, recall the probability theory, generalized measure and integral theory, as well as statistical and fuzzy approaches

to uncertainty modelling.

In the first paper, "MacNeille Completion of Centers and Centers of MacNeille Completions of Lattice Effect Algebras" by Martin Kalina (Department of Mathematics, Faculty of Civil Engineering, Slovak University of Technology, Bratislava, Slovak Republic) a natural question whether the MacNeille completion of the center of an atomic Archimedean lattice effect algebra coincides with the center of the MacNeille completion of this lattice effect algebra is solved in the negative.

In the second paper, "Every Uniformly Archimedean Atomic MV-effect Algebra is Sharply Dominating" by Vladimír Olejček (Department of Mathematics, Faculty of Electrical Engineering and Information Technology, Slovak University of Technology, Bratislava, Slovak Republic) it is proved that if an atomic MV-effect algebra is uniformly Archimedean then it is sharply dominating.

The third paper, "Almost Orthogonality and Hausdorff Interval Topologies of Atomic Lattice Effect Algebras" by Jan Paseka (Department of Mathematics and Statistics, Masaryk University, Brno, Czech Republic), Zdenka Riečanová (Department of Mathematics, Faculty of Electrical Engineering and Information Technology, Slovak University of Technology, Bratislava, Slovak Republic) and Wu Junde (Department of Mathematics, Zhejiang University, Hangzhou, China) shows that an atomic Archimedean lattice effect algebra is almost orthogonal iff it is (o)-continuous and its interval topology is Hausdorff.

The fourth paper "On The Structure of Numerical Event Spaces" by Gerhard Dorfer, Dietmar Dorninger and Helmut Länger (Institute of Discrete Mathematics and Geometry, Vienna University of Technology, Vienna, Austria) gives necessary and sufficient conditions for systems of numerical events to be a lattice and characterizes those systems which are Boolean.

New approach to characterization of orthomodular lattices by means of special types of bivariable functions G is suggested in the fifth paper "Quantum Logics and Bivariable Functions" by Eva Drobná (Department of Informatics, Armed Forces Academy of General Milan Rastislav Štefánik, Slovak Republic), Oľga Nánásiová and Ľubica Valášková (Department of Mathematics, Faculty of Civil Engineering, Slovak University of Technology, Bratislava, Slovak Republic). There is shown that it is possible to describe finite orthomodular lattices with states (quantum logics) using bivariable function G, which under special marginal conditions can operate as infimum measure, supremum measure or symmetric difference measure for two compatible elements of an orthomodular lattice.

The main contribution of the sixth paper "Compatibility and Central Elements in Pseudo-Effect Algebras" by Paolo Vitolo (Dipartimento di Matematica e Informatica, Universitá della Basilicata, Potenza, Italy) is a new characterization of compatible elements in pseudo-effect algebras and henceforth a characterization of pseudo-MV-algebras as those lattice pseudo-effect algebras in which all elements are pairwise compatible.

The seventh paper "Fuzzification of Crisp Domains" by Roman Frič (Mathematical Institute, Slovak Academy of Sciences, Košice, Catholic University in Ružomberok) and Martin Papčo (Mathematical Institute, Slovak Academy of Sciences, Bratislava, Slovak Republic) is devoted to the transition from crisp domains of probability to

fuzzy domains of probability. One of the aims is to analyze the process of fuzzification of classical crisp domains of probability within the category ID of D-posets of fuzzy sets and to put into perspective their earlier results concerning categorical aspects of fuzzification.

The theme of the eighth paper "On Fuzzification of the Notion of Quantaloid" by Sergey A. Solovyov (Department of Mathematics, University of Latvia, Riga, Latvia) is the study of a fuzzification of the notion of quantaloid resulting in the structure of quantale algebroid.

In the ninth paper "Some New Results about Brooks-Jewett and Dieudonné-type Theorems in (l)-groups" by Antonio Boccuto and Domenico Candeloro (Dipartimento di Matematica e Informatica, Perugia, Italy) some new versions of Brooks-Jewett and Dieudonné-type theorems for (l)-group-valued measures are presented.

In the tenth paper "Quantum Bochner Theorems and Incompatible Observables" by Robin Hudson (Mathematics Department, Loughborough University, Loughborough, Great Britain) the quantum Bochner theorem for spin components is recovered.

In the eleventh paper entitled "DUCS Copulas", Radko Mesiar and Monika Pekárová (Department of Mathematics, Faculty of Civil Engineering, Slovak University of Technology Bratislava, Slovak Republic) have introduced a new class of copulas based on a distortion of univariate conditioning stable copulas generated by a generator f, DUCS copulas in short. These copulas show a striking similarity in properties with the class of Archimax copulas (generalizing both Archimedean and Extreme Values copulas).

The paper number twelve "T-extensions as a Method of Construction of a Generalized Aggregation Operator" is due to Julija Lebedinska (Department of Mathematics, University of Latvia, Riga, Latvia) and it deals with aggregation functions acting on fuzzy numbers. Her approach is based on a (continuous) triangular norm and follows the ideas of Zadeh's extension principle. For different orderings of fuzzy numbers, properties of proposed aggregation functions are discussed.

The Choquet integral is a genuine extension of the Lebesgue integral not requiring the additivity of the underlying measure. In the thirteenth paper "The Choquet Integral as Lebesgue Integral and related Inequalities", Radko Mesiar (Department of Mathematics, Faculty of Civil Engineering, Slovak University of Technology, Bratislava, Slovak Republic), Jun Li (School of Science, Communication University of China, Beijing, China) and Endre Pap (Department of Mathematics and Informatics, University of Novi Sad, Serbia) discuss the circumstances when the Choquet integral (with respect to a monotone measure m) can be represented as the Lebesgue integral (with respect to a measure M). Subsequently, several integral inequalities, such as the Jensen or Minkowski inequalities, for the Choquet integral are examined.

The fourteenth paper "Formula for Unbiased Bases" by Maurice R. Kibler (Institut de Physique Nucléaire, Université de Lyon, Lyon, France) deals with mutually unbiased bases for systems of qubits in d dimensions.

The main goal of the fifteenth paper "Analytical Derivation of Time Spectral Rigidity for Thermodynamic Traffic Gas" by Milan Krbálek (Department of Mathematics, Faculty of Nuclear Sciences and Physical Engineering, Czech Technical

University in Prague, Prague, Czech Republic) is to obtain meaningful predictions for the microstructure of traffic sample and compare the results obtained to freeway measurements.

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