KYBERNETIKA — VOLUME 45 (2009), NUMBER 5, PAGES 768 – 784

ON ROBUST CONSENSUS OF MULTI–AGENT SYSTEMS WITH COMMUNICATION DELAYS

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In this paper, two robust consensus problems are considered for a multi-agent system with various disturbances. To achieve the robust consensus, two distributed control schemes for each agent, described by a second-order differential equation, are proposed. With the help of graph theory, the robust consensus stability of the multi-agent system with communication delays is obtained for both fixed and switching interconnection topologies. The results show the leaderless consensus can be achieved with some disturbances or time delays.

Keywords: multi-agent consensus, robust consensus, disturbances, neighbor-based rules, time delays

AMS Subject Classification: 93A14, 93C10

1. INTRODUCTION

In multi-agent consensus applications, group of agents need to agree upon certain quantities of interest that depends on the state of all the agents [14], which is a kind of interesting collective phenomena in physics, nature and society [5]. In control community, many consensus protocols had been investigated in multi-agent systems with discrete-time or continuous-time dynamics [8, 10, 15, 17, 18, 21], which is a simplified Vicsek model [20]. Further, some researchers considered multi-agent consensus problems with directed and time-varying interconnection topologies [13, 17]. Recently, more researchers are interested in studying the robust consensus problems, where the information flows are subject to uncertainties such as bounded disturbances, noises and interconnection delays in multi-agent systems [6, 8, 15, 16, 19].

The consensus problems for multi-agent systems will become more complicated when some kinds of perturbations are exerted on the interconnections among the autonomous mobile agents. In [6], the authors proposed a distributed estimation protocol to achieve consensus for a leader-follower multi-agent system with bounded perturbations. In [19], consensus problems are considered for a group of agents with first-order dynamics and white noises. Time delays resulting from interconnection links have also been paid much attention to multi-agent systems because of the practical background (referring to [2, 11]). For example, a synchronous stability criterion was derived for a network, in which each oscillator receives delayed signals from selected other oscillators in [2]. In [15], a necessary and sufficient condition for a time-delay consensus problem was presented for the agents with first-order dynamics and undirected interconnection graph. Xiao et al. analyzed the consensus stability for discrete-time multi-agent systems with directed information flow in [21]. A consensus problem was resolved in leader-following coordination for a group of mobile agents in [8, 16]. These results suggest potential applications in areas including synchronization, flocking, distributed decision making and formation control.

The objective of this paper is to propose two robust consensus schemes for leaderless multi-agent systems with disturbances. In order to solve the consensus problem, local controller for each agent is neighbor-based as did in many references related to agent-based control systems. Different from some existing results about multi-agent systems with disturbances, the considered dynamics of each agent is second-order with directed interconnection graph, bounded disturbances and time-varying interconnection time delays. The convergence analysis of directed graphs (or digraph for short) is more challenging than that of undirected graphs due to the complexity of directed graphs. The analysis becomes even harder when disturbances and time delays are involved. For robust consensus for multi-agent systems modeled by delayed differential equations with disturbances, an effective way to deal with their convergence and stability problems is to use Lyapunov-based method, which was initiated by [4].

The paper is organized as follows. In Section 2, some preliminary knowledge related to graph theory and functional differential equations are presented, and then two robust consensus problems for multi-agent systems are formulated along with corresponding neighbor-based controllers in Section 3. With the proposed local control schemes, the consensus stability for each scheme is analyzed with both fixed and switched interconnection topologies, in Section 4 and Section 5, respectively. In the analysis, Lyapunov–Razumikhin functions are employed with help of the analysis of matrix inequalities. Finally, some concluding remarks are given in Section 6.

By convention, $\mathbb{R}$ (or $\mathbb{C}$) and $\mathbb{Z}^+$ represent the real (or complex) number set and the positive integer set, respectively; I_n is an $n \times n$ identity matrix; $0_{n\times m}$ is a $n \times m$ zero matrix; $\mathbf{1}_n = (1, \dots, 1)^T \in \mathbb{R}^n$ ($\mathbf{1}$ for short, when there is no confusion); x^T denotes is the transpose of vector x; H^* denotes the conjugate transpose of matrix H; $||\cdot||$ denotes Euclidean norm; col$(\cdot)$ denotes the concatenation; let $\mathcal{K} = \{a \in C(\bar{\mathbb{R}}^+, \bar{\mathbb{R}}^+) : a(s)$ is strictly increasing and $a(0) = 0\}$, where $\bar{\mathbb{R}}^+ = \mathbb{R}^+ \cup \{0\}$.

2. PRELIMINARIES

In this section, we first introduce some basic concepts and notations in graph theory that will be used [1, 3].

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ be a weighted digraph of order n with the set of nodes $\mathcal{V} = \{1, 2, \dots, n\}$, set of arcs $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ with nonnegative elements. The node indexes belong to a finite index set $\mathcal{I} = \{1, 2, \dots, n\}$. An *arc* of $\mathcal{G}$ is denoted by (i, j), which starts from i and ends on j. The element a_{ij} associated with the arc of the digraph is positive, i. e. $a_{ij} >$

$0 \Leftrightarrow (i,j) \in \mathcal{E}$. Moreover, we assume $a_{ii} = 0$ for all $i \in \mathcal{I}$. The set of neighbors of node i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (i,j) \in \mathcal{E}\}$. A *cluster* is any subset $\mathcal{J} \subset \mathcal{V}$ of the nodes of the digraph. The set of neighbors of a cluster $\mathcal{J}$ is defined by $\mathcal{N}_{\mathcal{J}} = \bigcup_{i\in\mathcal{J}} \mathcal{N}_i = \{j \in \mathcal{V} : i \in \mathcal{J}, (i,j) \in \mathcal{E}\}$. A *path* in a digraph is a sequence $i_0, i_1, \ldots, i_f$ of distinct nodes such that (i_{j-1}, i_j) is an arc for $j = 1, 2, \ldots, f, f \in \mathbb{Z}^+$. If there exists a path from node i to node j, we say that j is reachable from i. If a node i is reachable from every other node in $\mathcal{G}$, then we say it is *globally reachable*. A digraph $\mathcal{G}$ is *strongly connected* if any two distinct nodes are reachable from each other. A *strong component* of a digraph is an induced subgraph that is maximal, subject to being strongly connected. If $\sum_{j\in\mathcal{N}_i} a_{ij} = \sum_{j\in\mathcal{N}_i} a_{ji}$ for all $i = 1, \ldots, n$, the digraph $\mathcal{G}$ is called *balanced*.

The interconnection topology between n agents can be conveniently described by a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$, which is defined so that (i,j) defines one of the digraph's arcs in case agent j is a neighbor of agent i . A diagonal matrix $D = \mathrm{diag}\{d_1, \ldots, d_n\} \in \mathbb{R}^{n\times n}$ is a degree matrix of $\mathcal{G}$, whose diagonal elements $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$ for $i = 1, \ldots, n$. Then the Laplacian of the weighted digraph is defined as

$$L = D - A.$$

The next lemma shows an important property of Laplacian L associated with $\mathcal{G}$ ([8, 15]).

Lemma 2.1. *L has least one zero eigenvalue with $\mathbf{1} \in \mathbb{R}^n$ as its eigenvector, and all the nonzero eigenvalues of L have positive real parts. Laplacian L has a simple zero eigenvalue if and only if $\mathcal{G}$ has a globally reachable node.*

Let $S_1, S_2, \ldots, S_p$ be the strong components of $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ and $\mathcal{N}_{S_i}$ be the neighbor sets for $S_i, i = 1, \ldots, p, p > 1$. The following lemma is a revised version of a result reported in ([13]).

Lemma 2.2. *A digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ has a globally reachable node if and only if every pair of S_i, S_j satisfies $\mathcal{N}_{S_i} \bigcup \mathcal{N}_{S_j} \neq \emptyset$. Moreover, if the graph is strongly connected, then each node is globally reachable from every other node.*

Next, we introduce the stability of time-delay systems. Consider the following system with time delays [4]:

$$\begin{cases} \dot{x} = f(t, x_t), & t > t_0, \\ x(\theta) = \varphi(\theta), & \theta \in [-\tau, t_0], \end{cases} \tag{1}$$

where $x_t(\theta) = x(t+\theta)$, $\forall\theta \in [-\tau, t_0]$ and $f(t, 0) = 0$. In the sequel, suppose that $t_0 = 0$. Let $C([-\tau, 0], \mathbb{R}^n)$ be a Banach space of continuous function defined on an interval $[-\tau, 0]$, taking values in $\mathbb{R}^n$ with the topology of uniform convergence, and with a norm $||\varphi||_c = \max_{\theta\in[-\tau,0]} ||\varphi(\theta)||$. Assume that for any initial function $\varphi \in C([-\tau, 0], \mathbb{R}^n)$, there exists a unique solution $x(t, t_0, \varphi)$ of (1).

Definition 2.3. (Hale and Lunel [4]) Let $x(t) = x(t, t_0, \varphi)$ be any solution of system (1), system (1) is said to be

(i) uniformly bounded, if for any $\alpha > 0$, there exists $\beta = \beta(\alpha) > 0$ such that for any $t_0 \in \mathbb{R}$, $||\varphi||_c < \alpha$ implies $\|x(t)\| < \beta$ for all $t \geq t_0$;

(ii) uniformly ultimately bounded, if there exists a positive constant b, for any $\alpha > 0$ there exists a $T = T(\alpha) > 0$ such that for any $t_0 \in \mathbb{R}$, $||\varphi||_c < \alpha$ implies $\|x(t)\| < b$ for all $t \geq t_0 + T$.

The following lemma is given for the boundedness of system (1).

Lemma 2.4. (Hale and Lunel [4]) Assume that there exist a continuous function $V(t, x)$ and $u, v, w \in \mathcal{K}$ satisfying that

(i) $u(\|x\|) \leq V(t, x) \leq v(\|x\|)$, where $\lim\limits_{s \to +\infty} u(s) = +\infty$;

(ii) $\exists\ h > 0,\ l > 0$ and $p \in C(\bar{\mathbb{R}}^+, \bar{\mathbb{R}}^+),\ p(s) > s,\ s > 0$ such that

$$\dot{V}(t, x) \leq -w(\|x\|) + l,$$

whenever $\|x(t)\| > h$ and $V(t + \theta, x(t + \theta)) < p(V(t, x(t))), \quad \theta \in [-r, 0]$.

Then system (1) is uniformly bounded (UB) and also uniformly ultimately bounded (UUB).

Usually, $V(t, x)$ in Lemma 2.4 is called a Lyapunov–Razumikhin function.

In the stability of time-delay systems, the following result plays an important role [12].

Lemma 2.5. Assume that $\eta_1(\cdot) \in \mathbb{C}^{n_1}, \eta_2(\cdot) \in \mathbb{C}^{n_2}$ and $M(\cdot) \in \mathbb{C}^{n_1 \times n_2}$ are defined on an interval Ω. Then for any matrices $X \in \mathbb{C}^{n_1 \times n_1}, Y \in \mathbb{C}^{n_1 \times n_2}$ and $Z \in \mathbb{C}^{n_2 \times n_2}$, the following inequality holds:

$$-2 \int_\Omega \eta_1^*(s) M \eta_2(s)\, \mathrm{d}s \leq \int_\Omega \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}^* \begin{pmatrix} X & Y - M \\ Y^* - M^* & Z \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} \mathrm{d}s, \tag{2}$$

where

$$\begin{pmatrix} X & Y \\ Y^* & Z \end{pmatrix} \geq 0. \tag{3}$$

3. PROBLEM FORMULATION

In this paper, we consider a group of n identical agents move in an m-dimensional space and the agents are indexed by $1, \ldots, n$. A continuous-time model of the n agents is described as follows:

$$\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = u_i + \delta_i, \end{cases} \tag{4}$$

where $x_i \in \mathbb{R}^m$ can be the position (or angle) of agent i, $v_i \in \mathbb{R}^m$ its velocity (or angular velocity) and $u_i \in \mathbb{R}^m$ its interconnection control inputs for $i = 1, \ldots, n$. In addition, the feedback for the ith agent involves a perturbation $\delta_i(t)$ which is a periodic bounded function such that

$$\|\delta_i(t)\| \leq \hat{\delta}, \quad \left\| \int_0^t \delta_i(s)\,\mathrm{d}s \right\| \leq \varrho_1, \quad \left\| \int_0^t \int_0^s \delta_i(\omega)\,\mathrm{d}\omega \mathrm{d}s \right\| \leq \varrho_2,$$

where ϱ_1, ϱ_2 are some positive numbers. Without loss of generality, it is assumed further that $\delta_i(0) = 0$.

The design of neighbor-based feedback $u_i(t)$ usually depends on $x_j(t), v_j(t)$ for some $j \in \mathcal{N}_i$. However, in practice, there may be interconnection delays and perturbations, and each agent cannot instantly get the accurate information from others. Thus, the feedback $u_i(t)$ should be constructed based on the information of $x_j(t - r(t))$ and $v_j(t - r(t))$ for some $j \in \mathcal{N}_i$ and time-varying delay $r(t) > 0$, a continuously differentiable function satisfying:

$$0 < r(t) < \tau. \tag{5}$$

Remark 3.1. The assumptions on the perturbations $\delta_i(t)$ $(i = 1, \ldots, n)$ can be understood spontaneously since any function satisfying Dirichlet's condition can be expanded into a Fourier series.

Remark 3.2. When the perturbations $\delta_i(t)$ $(i = 1, \ldots, n)$ are white noises, we have to employ differential generator method in [9] to analyze the stochastic stability of system (4).

In this paper, two robust consensus problems, namely, free robust consensus and robust consensus with desired velocity, will be worked on for system (4).

- A free robust consensus problem of system (4) is solved if

  $$\|x_i - x_j\| \leq \sigma_1, \ \|v_i - v_j\| \leq \sigma_2$$

  for all $i, j \in \mathcal{I}$ and some nonnegative constants σ_1, σ_2. To deal with this consensus problem for system (4), we propose the following local control scheme

  $$\begin{aligned} u_i(t) =& k^2 \Big[\sum_{j \in \mathcal{N}_i(\sigma)} a_{ij}(x_j(t-r) - x_i(t-r)) \\ &+ k \sum_{j \in \mathcal{N}_i(\sigma)} a_{ij}(v_j(t-r) - v_i(t-r)) \Big], \quad k > 0. \end{aligned} \tag{6}$$

- A robust consensus problem with desired velocity v_0 is solved if

$$\|x_i - x_j\| \leq \sigma_3, \ \|v_i - v_0\| \leq \sigma_4$$

for all $i, j \in \mathcal{I}$ and some nonnegative constants σ_3, σ_4. To handle this problem for system (4), we propose the following local control scheme:

$$u_i(t) = \sum_{j \in \mathcal{N}_i(\sigma)} a_{ij}(x_j(t-r) - x_i(t-r)) + k(v_0 - v_i(t)), \quad k > 0. \tag{7}$$

The interconnection topologies will be discussed in two cases. A fixed topology, described by a digraph, is considered at first, and variable topologies described by balanced digraphs are analyzed. To study varying interconnection topology, we introduce a function $\sigma : [0, \infty) \to \mathcal{I}_\Gamma = \{1, \ldots, N\}$ (N denotes the total number of all possible digraphs), which is a switching signal to show the sequence of the switched interconnection topologies over time. The set $\Gamma = \{\mathcal{G}_1, \ldots, \mathcal{G}_N\}$ is a finite collection of graphs with a common node set $\mathcal{V}$. If σ is a constant function, then the corresponding interconnection topology is fixed. In addition, $\mathcal{N}_i(\sigma)$ is the index set of neighbors of agent i in the digraph $\mathcal{G}_\sigma$ while a_{ij} $(i, j = 1, \ldots, n)$ are elements of the adjacency matrix of $\mathcal{G}_\sigma$.

Denote x, v, u and δ as the concatenations of x_i, v_i, u_i and δ_i for $i = 1, \ldots, n$, respectively. Then, with the two control schemes (6) and (7), the closed-loop system (4) can be rewritten in the following respective forms:

$$\begin{cases} \dot{x} = v, \\ \dot{v} = u + \delta = -k^2(L_\sigma \otimes I_m)(x(t-r) + kv(t-r)) + \delta, \end{cases} \tag{8}$$

and,

$$\begin{cases} \dot{x} = v, \\ \dot{v} = u + \delta = -(L_\sigma \otimes I_m)x(t-r) - k(v - \mathbf{1} \otimes v_0) + \delta, \end{cases} \tag{9}$$

where $\otimes$ denotes Kronecker product [7].

The stability analysis of multi-agent systems with time delays under controller (6) or (7), that is, (8) and (9), will be studied in the following sections.

4. FIXED INTERCONNECTION TOPOLOGY

In this section, we will focus on the convergence analysis of the system (8) or (9) when the switching signal is constant (or equivalently, the interconnection topology is fixed). Then the subscript σ is dropped for simplicity and the system (8) or (9) can be expressed with the following linear delayed differential equations:

$$\begin{cases} \dot{x} = v, \\ \dot{v} = -k^2(L \otimes I_m)(x(t-r) + kv(t-r)) + \delta, \end{cases} \tag{10}$$

or,

$$\begin{cases} \dot{x} = v, \\ \dot{v} = -(L \otimes I_m)x(t-r) - k(v - v_0\mathbf{1}) + \delta. \end{cases} \tag{11}$$

We will focus on the two consensus problems with fixed topologies, corresponding to the two time-delay systems (10) and (11), in the following two respective subsections.

4.1. Free robust consensus problem

To solve the free consensus problem of the system (10), we first give a lemma. Its proof is quite obvious and omitted here.

Lemma 4.1. For Laplacian L associated with digraph $\mathcal{G}$, then there exists a nonsingular matrix

$$U = \begin{pmatrix} 1 & * & \dots & * \\ 1 & * & \dots & * \\ \vdots & \vdots & & \vdots \\ 1 & * & \dots & * \end{pmatrix} \in \mathbb{R}^{n \times n} \tag{12}$$

such that

$$U^{-1}LU = \begin{pmatrix} 0 & \alpha^T \\ 0_{n-1} & H \end{pmatrix} = \Lambda \in \mathbb{R}^{n\times n},\ \alpha \in \mathbb{R}^{n-1},\ H \in \mathbb{R}^{(n-1)\times(n-1)}. \tag{13}$$

According to Lemma 4.1, with a coordinate transformation

$$\bar{x} = (U^{-1} \otimes I_m)x,\ \bar{v} = (U^{-1} \otimes I_m)v,\ \bar{\delta} = (U^{-1} \otimes I_m)\delta, \tag{14}$$

system (10) becomes

$$\begin{cases} \dot{\bar{x}} = \bar{v}, \\ \dot{\bar{v}} = -k^3(\Lambda \otimes I_m)\bar{v}(t-r) - k^2(\Lambda \otimes I_m)\bar{x}(t-r) + \bar{\delta}, \end{cases}$$

or equivalently,

$$\begin{cases} \dot{\bar{x}}_1 = \bar{v}_1, \\ \dot{\bar{v}}_1 = -k^3(\alpha^T \otimes I_m)\bar{v}_2(t-r) - k^2(\alpha^T \otimes I_m)\bar{x}_2(t-r) + \bar{\delta}_1, \end{cases} \tag{15}$$

and

$$\begin{cases} \dot{\bar{x}}_2 = \bar{v}_2, \\ \dot{\bar{v}}_2 = -k^3(H \otimes I_m)\bar{v}_2(t-r) - k^2(H \otimes I_m)\bar{x}_2(t-r) + \bar{\delta}_2, \end{cases} \tag{16}$$

where

$$\bar{x} = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix},\ \bar{v} = \begin{pmatrix} \bar{v}_1 \\ \bar{v}_2 \end{pmatrix},\ \bar{\delta} = \begin{pmatrix} \bar{\delta}_1 \\ \bar{\delta}_2 \end{pmatrix}\ \bar{x}_1, \bar{v}_1, \bar{\delta}_1 \in \mathbb{R}^m, \bar{x}_2, \bar{v}_2, \bar{\delta}_2 \in \mathbb{R}^{m(n-1)}.$$

Remark 4.2. From Lemmas 2.1 and 4.1, if graph $\mathcal{G}$ has a globally reachable node, the real parts of all the eigenvalues of $H \in \mathbb{R}^{(n-1)\times(n-1)}$ are positive, or equivalently, $-H$ is Hurwitz stable. Therefore, there exist a positive definite matrix $\bar{P} \in \mathbb{R}^{(n-1)\times(n-1)}$ such that

$$\bar{P}H + H^T\bar{P} = I_{n-1}. \tag{17}$$

Let $\bar{\lambda}$ (or λ) denote the minimum (or maximum) eigenvalue of $\bar{P}$ and μ the maximum eigenvalue of $H^T\bar{P}\bar{P}H$. Then a result can be obtained for system (10).

Theorem 4.3. For system (10), take

$$k > k_1^\star = \max\left\{\sqrt{\frac{1}{2}\lambda + 1}, \frac{\mu}{\bar{\lambda}} + 1\right\}. \tag{18}$$

If $\mathcal{G}$ has a globally reachable node and τ is sufficiently small, the free robust consensus problem of the system (10) is solved.

Proof. Since $\mathcal{G}$ has a globally reachable node, zero is a simple eigenvalue of Laplacian L while other eigenvalues have positive real parts (from Lemma 2.1). By Lemma 4.1, there exists a nonsingular matrix U given in (12) such that L can be transformed to (13), where H has eigenvalues with positive real parts. Then there is a positive definite matrix $\bar{P}$ satisfying (17).

For the subsystem (16), let $\varepsilon = \mathrm{col}(\bar{x}_2, \bar{v}_2) \in \mathbb{R}^{2m(n-1)}, \tilde{\delta}_2 = \mathrm{col}(0, \bar{\delta}_2)$. Then we have a compact form:

$$\dot{\varepsilon} = B\,\varepsilon(t) + E\varepsilon(t-r) + \tilde{\delta}_2, \tag{19}$$

where

$$B = \begin{pmatrix} 0_{(n-1)\times(n-1)} & I_{n-1} \\ 0_{(n-1)\times(n-1)} & 0_{(n-1)\times(n-1)} \end{pmatrix} \otimes I_m,\ E = \begin{pmatrix} 0_{(n-1)\times(n-1)} & 0_{(n-1)\times(n-1)} \\ -k^2H & -k^3H \end{pmatrix} \otimes I_m.$$

Take a Lyapunov–Razumikhin function

$$V(\varepsilon) = \varepsilon^T P \varepsilon, \tag{20}$$

where

$$P = \begin{pmatrix} k\bar{P} & \bar{P} \\ \bar{P} & k\bar{P} \end{pmatrix} \otimes I_m$$

is positive definite since $k > 1$.

Furthermore, by Leibniz–Newton formula,

$$\begin{aligned} \varepsilon(t-r) &= \varepsilon(t) - \int_{-r}^{0} \dot{\varepsilon}(t+s)\,\mathrm{d}s \\ &= \varepsilon(t) - B\int_{-r}^{0} \varepsilon(t+s)\,\mathrm{d}s - E\int_{-r}^{0} \varepsilon(t-r+s)\,\mathrm{d}s - \int_{-r}^{0} \tilde{\delta}_2(t+s)\,\mathrm{d}s. \end{aligned}$$

Therefore, (19) can be rewritten as

$$\begin{aligned} \dot{\varepsilon} \;&=\; F\varepsilon - EB\int_{-r}^{0}\varepsilon(t+s)\,\mathrm{d}s - E^2\int_{-r}^{0}\varepsilon(t-r+s)\,\mathrm{d}s \\ &\quad - E\int_{-r}^{0}\tilde{\delta}_2(t+s)\,\mathrm{d}, \quad F = B+E, \end{aligned} \tag{21}$$

for arbitrary initial function on $[-2\tau, 0]$. If the zero solution of (21) is UUB, then the zero solution of (19) is UUB since (19) is a special case of (21) with continuous initial function $\tilde{\varphi}(s)$ given by $\tilde{\varphi}(s)$ arbitrary for $s \in [-2\tau, -\tau - r(0)]$, $\tilde{\varphi}(s) = \varphi(s + r(0)), -\tau - r(0) \le s \le -r(0)$, and $\tilde{\varphi}(s) = \varepsilon(t+s), -r(0) \le s \le 0$ where $\varepsilon(t)$ is the solution of (19) with initial function φ on $[-\tau, 0]$.

Set $\eta_1 = \varepsilon(t), \eta_2 = \varepsilon(t+s), \eta_3 = {E^2}^T P\varepsilon(t-r+s), \eta_4 = \varepsilon(t-r+s), M_1 = PEB = Y_1, M_2 = I_{2m(n-1)} = Y_2, X_1 = k^4(1+k^2)I_{2m(n-1)}, X_2 = P^{-1}$ and $Z_1 = Z_2 = P$. Invoking Lemma 2.5 gives (3) with k given in (18), and leads to

$$\begin{aligned} \dot{V} =& \varepsilon^T(F^TP + PF)\varepsilon - 2\int_{-r}^{0}\varepsilon^T PEB\varepsilon(t+s)\,\mathrm{d}s - 2\int_{-r}^{0}\varepsilon^T PE^2\varepsilon(t-r+s)\,\mathrm{d}s \\ &- 2\int_{-r}^{0}\varepsilon^T PE\tilde{\delta}_2(t+s)\,\mathrm{d}s \\ \le& \varepsilon^T(F^TP + PF)\varepsilon + rk^4(1+k^2)\varepsilon^T\varepsilon + \int_{-r}^{0}\varepsilon^T(t+s)P\varepsilon(t+s)\,\mathrm{d}s \\ &+ r\varepsilon^T PE^2P^{-1}{E^2}^T P\varepsilon + \int_{-r}^{0}\varepsilon^T(t-r+s)P\varepsilon(t-r+s)\,\mathrm{d}s \\ &+ r\varepsilon^T PEE^TP\varepsilon + \tau\check{\delta}^2, \end{aligned}$$

where $\check{\delta} = \|U\|\hat{\delta}$.

Take $\phi(s) = qs$ for some constant $q > 1$. In the case of

$$V(\varepsilon(t+\theta)) < qV(\varepsilon(t)), \; -2\tau \le \theta \le 0, \tag{22}$$

we have, with Remark 4.2,

$$\dot{V} \le -\varepsilon^T Q\varepsilon + r\varepsilon^T((k^4+k^6)I_{2m(n-1)} + PE^2P^{-1}{E^2}^TP + PEE^TP + 2qP)\varepsilon + \tau\check{\delta}^2,$$

where

$$Q = -(F^TP + PF) = \begin{pmatrix} k^2 I_{n-1} & k^3 I_{n-1} - k\bar{P} \\ k^3 I_{n-1} - k\bar{P} & k^4 I_{n-1} - 2\bar{P} \end{pmatrix} \otimes I_m.$$

Q is positive definite if k satisfies (18), according to Schur complements theorem ([7]). Let $\lambda_{\min}$ denote the minimum eigenvalues of Q. If we take

$$r < \tau = \frac{\lambda_{\min}}{k^4 + k^6 + \|PE^2P^{-1}{E^2}^TP\| + \|PEE^TP\| + 2q(k+1)\lambda},$$

then $\dot{V}(\varepsilon) \leq -\eta\varepsilon^T\varepsilon + \tau\check{\delta}^2$ for some $\eta > 0$. By Lemma 2.4, we conclude that

$$\|\varepsilon\| \leq \sqrt{\frac{(k_1^\star + 1)\tau\lambda}{(k_1^\star - 1)\eta\bar{\lambda}}}\check{\delta}.$$

On the other hand, for the system (15), let $\bar{x}_1(0), \bar{v}_1(0)$ be the initial values of $\bar{x}_1(t), \bar{v}_1(t)$ and take a variable of change $\tilde{x}_1 = \bar{x}_1 - (\bar{v}_1(0)t + \bar{x}_1(0)) - \int_0^t \int_0^s \bar{\delta}_1(\omega)\, \mathrm{d}\omega \mathrm{d}s, \tilde{v}_1 = \bar{v}_1 - \bar{v}_1(0) - \int_0^t \bar{\delta}_1(s)\, \mathrm{d}s$. Then according to Lemma 5.3 of [4], it is not difficult to have

$$\|\bar{x}_1 - (\bar{v}_1(0)t + \bar{x}_1(0))\| \leq \kappa_U \varrho_1, \|\bar{v}_1 - \bar{v}_1(0)\| \leq \kappa_U \varrho_2,$$

where κ_U is a positive constant depending on the matrix U. From transformation (14), we have

$$\begin{aligned}\left\| x - (U \otimes I_m) \begin{pmatrix} \bar{v}_1(0)t + \bar{x}_1(0) \\ 0_{m(n-1)} \end{pmatrix} \right\| &= \|x - \mathbf{1} \otimes (\bar{v}_1(0)t + \bar{x}_1(0))\| \\ &\leq \|U\| \min\left\{ \sqrt{\frac{(k_1^\star + 1)\tau\lambda}{(k_1^\star - 1)\eta\bar{\lambda}}}\check{\delta}, \kappa_U \varrho_1 \right\},\end{aligned}$$

$$\left\| v - (U \otimes I_m) \begin{pmatrix} \bar{v}_1(0) \\ 0_{m(n-1)} \end{pmatrix} \right\| = \|v - \mathbf{1} \otimes \bar{v}_1(0)\| \leq \|U\| \min\left\{ \sqrt{\frac{(k_1^\star + 1)\tau\lambda}{(k_1^\star - 1)\eta\bar{\lambda}}}\check{\delta}, \kappa_U \varrho_2 \right\}.$$

Therefore, the free robust consensus can be achieved. The conclusion follows. □

Further, by using contradiction method, it can be shown that the condition that $\mathcal{G}$ has a globally reachable node is also a necessary condition to solve the free robust consensus problem of system (10) when there is no perturbation δ, i. e., $\hat{\delta} = 0,\ \varrho = 0$. Then we describe the result as follows:

Corollary 4.4. With k and τ given in Theorem 4.3, if $\hat{\delta} = 0$, the free consensus problem (i. e. $\sigma_1 = 0, \sigma_2 = 0$) can be solved if and only if $\mathcal{G}$ has a globally reachable node.

4.2. Robust consensus problem with desired velocity

In this subsection, we analyze the consensus problem of the system (11) with the controller (7).

Take

$$\tilde{x} = x - \mathbf{1} \otimes v_0 t,\ \ \tilde{v} = v - \mathbf{1} \otimes v_0 \tag{23}$$

and system (11) can be expressed in the following form:

$$\begin{cases} \dot{\tilde{x}} = \tilde{v}, \\ \dot{\tilde{v}} = -(L \otimes I_m)\tilde{x}(t - r) - k\tilde{v} + \delta. \end{cases}$$

Select a coordinate transformation $\bar{x} = (U^{-1} \otimes I_m)\tilde{x}$, $\bar{v} = (U^{-1} \otimes I_m)\tilde{v}$ with the matrix U given in (12), and then system (11) becomes

$$\begin{cases} \dot{\bar{x}}_1 = \bar{v}_1, \\ \dot{\bar{v}}_1 = -k\bar{v}_1 - (\alpha^T \otimes I_m)\bar{x}_2(t-r) + \bar{\delta}_1, \end{cases} \tag{24}$$

$$\begin{cases} \dot{\bar{x}}_2 = \bar{v}_2, \\ \dot{\bar{v}}_2 = -k\bar{v}_2 - (H \otimes I_m)\bar{x}_2(t-r) + \bar{\delta}_2, \end{cases} \tag{25}$$

where $\bar{x}_1, \bar{v}_1 \in \mathbb{R}^m, \bar{x}_2, \bar{v}_2 \in \mathbb{R}^{m(n-1)}$.

Then the following result for system (11) can be derived.

Theorem 4.5. For system (11), take

$$k > k_2^\star = \frac{\mu}{\bar{\lambda}} + 1, \tag{26}$$

with μ and $\bar{\lambda}$ as defined in Theorem 4.3. If $\mathcal{G}$ has a globally reachable node and τ is sufficiently small, the robust consensus problem with desired velocity v_0 of the system (11) is solved.

Proof. The proof is similar to that of Theorem 4.3. Here, we just give a proof outline. Note that subsystem (25) can be written as:

$$\dot{\varepsilon} = \bar{B}\ \varepsilon(t) + \bar{E}\varepsilon(t-r) + \tilde{\delta}_2,$$

where

$$\bar{B} = \begin{pmatrix} 0_{(n-1)\times(n-1)} & I_{n-1} \\ 0_{(n-1)\times(n-1)} & -kI_{n-1} \end{pmatrix} \otimes I_m,\ \bar{E} = \begin{pmatrix} 0_{(n-1)\times(n-1)} & 0_{(n-1)\times(n-1)} \\ -H & 0_{(n-1)\times(n-1)} \end{pmatrix} \otimes I_m.$$

As discussed above, set $\bar{F} = \bar{B} + \bar{E}$ for simplicity.

To analyze the stability of the subsystem (25), a Lyapunov–Razumikhin function can be taken as

$$V(\varepsilon) = \varepsilon^T \hat{P}\varepsilon, \tag{27}$$

where

$$\hat{P} = \begin{pmatrix} k\bar{P} & \bar{P} \\ \bar{P} & \bar{P} \end{pmatrix} \otimes I_m$$

is positive definite for $k > 1$.

Invoking Lemma 2.5, when $X = (k + 1 + \sqrt{k^2 - 2k + 5})I_{2m(n-1)}, Z = \hat{P}$ and k satisfies (26), we can get

$$0 < r \leq \tau = \frac{\bar{\lambda}_{\min}}{(k + 1 + \sqrt{k^2 - 2k + 5})(1 + \frac{1}{2}q\lambda) + \|\hat{P}\bar{E}\bar{E}^T\hat{P}\|},$$

where $\bar{\lambda}_{\min}$ denotes the minimum eigenvalue of the matrix

$$\bar{Q} = -(\bar{F}^T \hat{P} + \hat{P}\bar{F}) = \begin{pmatrix} I_{n-1} & H^T \bar{P} \\ \bar{P}H & 2(k-1)\bar{P} \end{pmatrix} \otimes I_m.$$

Then it can be shown that system (11) is UUB. Hence, the conclusion follows. □

We also have

Corollary 4.6. With k and τ given in Theorem 4.5, if $\hat{\delta} = 0$, the consensus problem with desired velocity v_0 (i. e. $\sigma_3 = 0, \sigma_4 = 0$) can be solved if and only if $\mathcal{G}$ has a globally reachable node.

5. SWITCHED INTERCONNECTION TOPOLOGY

In this section, we consider the convergence of time-delay systems (8) or (9) for the switched interconnection topology. It is hard to do this for switched interconnection topologies described by general digraphs. Here a special class of digraphs, that is, balanced digraphs, are considered in the following stability analysis.

As we did in the preceding section, the two consensus problems, corresponding to systems (8) and (9), are considered in the following two respective subsections.

5.1. Free robust consensus problem

At first, a lemma about Laplacian L associated with a balanced digraph $\mathcal{G}$ is given.

Lemma 5.1. If $\mathcal{G}$ is balanced, then there exists a unitary matrix

$$V = \begin{pmatrix} \frac{1}{\sqrt{n}} & * & \dots & * \\ \frac{1}{\sqrt{n}} & * & \dots & * \\ \vdots & \vdots & & \vdots \\ \frac{1}{\sqrt{n}} & * & \dots & * \end{pmatrix} \in \mathbb{C}^{n\times n} \tag{28}$$

such that

$$V^* L V = \begin{pmatrix} 0 & \\ & H \end{pmatrix} = \Lambda \in \mathbb{C}^{n\times n}, \ H \in \mathbb{C}^{(n-1)\times(n-1)}. \tag{29}$$

Moreover, if $\mathcal{G}$ has a globally reachable node, $H + H^*$ is positive definite.

Proof. Let $V = [\zeta_1, \zeta_2, \dots, \zeta_n]$ be a unitary matrix where $\zeta_i \in \mathbb{C}^n$ $(i = 1, \dots, n)$ are the column vectors of V and $\zeta_1 = \frac{1}{\sqrt{n}}\mathbf{1} = (\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}, \dots, \frac{1}{\sqrt{n}})^T$. Notice that if $\mathcal{G}$

is balanced, it implies that $\zeta_1^* L = 0$. Then we have

$$V^*LV = V^*L[\zeta_1, \zeta_2, \ldots, \zeta_n]$$
$$= \begin{pmatrix} \zeta_1^* \\ \zeta_2^* \\ \vdots \\ \zeta_n^* \end{pmatrix} [0_n, L\zeta_2, \ldots, L\zeta_n]$$
$$= \begin{pmatrix} 0 & 0_{n-2}^T \\ 0_{n-2} & H \end{pmatrix}.$$

Furthermore, if $\mathcal{G}$ has a globally reachable node, then $L + L^T$ is positive semidefinite (Theorem 7, [15]). Hence, $V^*(L + L^T)V$ is also positive semidefinite. From Lemma 2.1, zero is a simple eigenvalue of L and, therefore, $H + H^*$ is positive definite. □

With a coordinate transformation

$$\bar{x} = (V^* \otimes I_m)x, \ \ \bar{v} = (V^* \otimes I_m)v, \ \ \bar{\delta} = (V^* \otimes I_m)\delta, \tag{30}$$

the system (8) becomes

$$\begin{cases} \dot{\bar{x}}_1 = \bar{v}_1, \\ \dot{\bar{v}}_1 = \bar{\delta}_1, \end{cases} \tag{31}$$

$$\begin{cases} \dot{\bar{x}}_2 = \bar{v}_2, \\ \dot{\bar{v}}_2 = -k^3(H_\sigma \otimes I_m)\bar{v}_2(t-r) - k^2(H_\sigma \otimes I_m)\bar{x}_2(t-r) + \bar{\delta}_2, \end{cases} \tag{32}$$

where $\bar{x}_1, \bar{v}_1, \bar{\delta}_1 \in \mathbb{C}^m, \bar{x}_2, \bar{v}_2, \bar{\delta}_2 \in \mathbb{C}^{m(n-1)}$.

Based on Lemma 5.1 and the fact that the set $\mathcal{I}_\Gamma$ is finite, if the balanced digraph $\mathcal{G}_\sigma$ has a globally reachable node,

$$\tilde{\lambda} = \min\{\text{eigenvalues of } H_\sigma + H_\sigma^*\} > 0$$

$$\tilde{\mu} = \max\{\text{eigenvalues of } H_\sigma H_\sigma^*\} > 0$$

can be well defined. Then a result of the switched system (8) with time-varying delay is given as follows.

Theorem 5.2. For system (8) with balanced interconnection topology $\mathcal{G}_\sigma$, take

$$k > k_3^\star = \max\{\sqrt{\frac{1}{2\tilde{\lambda}} + 1}, \ \tilde{\mu} + 1\}. \tag{33}$$

If $\mathcal{G}_\sigma$ has a globally reachable node and τ is sufficiently small, then the free robust consensus problem of system (8) is solved.

P r o o f. To get the result, we first consider (32), or equivalently,

$$\dot{\varepsilon} = B\varepsilon(t) + E_\sigma \varepsilon(t-r) + \tilde{\delta}_2, \tag{34}$$

where

$$\begin{aligned} B &= \begin{pmatrix} 0_{(n-1)\times(n-1)} & I_{n-1} \\ 0_{(n-1)\times(n-1)} & 0_{(n-1)\times(n-1)} \end{pmatrix} \otimes I_m, \\ E_\sigma &= \begin{pmatrix} 0_{(n-1)\times(n-1)} & 0_{(n-1)\times(n-1)} \\ -k^2 H_\sigma & -k^3 H_\sigma \end{pmatrix} \otimes I_m. \end{aligned}$$

Take a Lyapunov–Razumikhin function

$$V(\varepsilon) = \varepsilon^* \tilde{P} \varepsilon, \tag{35}$$

where

$$\tilde{P} = \begin{pmatrix} kI_{n-1} & I_{n-1} \\ I_{n-1} & kI_{n-1} \end{pmatrix} \otimes I_m$$

is positive definite for $k > 1$.

Similar to the proof of Theorem 4.3, we can obtain

$$\begin{aligned} \dot{V}|_{(34)} &= \varepsilon^T (F_\sigma^T \tilde{P} + \tilde{P} F_\sigma)\varepsilon - 2\varepsilon^T \tilde{P} E_\sigma B \int_{-r}^{0} \varepsilon(t+s)\,\mathrm{d}s \\ &\quad - 2\varepsilon^T \tilde{P} E_\sigma^2 \int_{-r}^{0} \varepsilon(t-r+s)\,\mathrm{d}s - 2\varepsilon^T \tilde{P} E_\sigma \int_{-r}^{0} \tilde{\delta}_2(t+s)\,\mathrm{d}s, \end{aligned}$$

where $F_\sigma = B + E_\sigma$.

Set $\phi(s) = qs$ for some constant $q > 1$. In the case of

$$V(\epsilon(t+\theta)) < qV(\epsilon(t)), \quad -2\tau \le \theta \le 0, \tag{36}$$

using Lemma 2.5, we have

$$\dot{V} \le -\varepsilon^T Q_\sigma \varepsilon + r\varepsilon^T ((k^4 + k^6) I_{2m(n-1)} + \tilde{P} E_\sigma^2 \tilde{P}^{-1} E_\sigma^{2\,T} \tilde{P} + \tilde{P} E_\sigma E_\sigma^T \tilde{P} + 2q\tilde{P})\varepsilon + \tau\hat{\delta}^2,$$

where

$$Q_\sigma = -(F_\sigma^T \Phi + \Phi F_\sigma) = \begin{pmatrix} k^2(H_\sigma^* + H_\sigma) & k^3(H_\sigma^* + H_\sigma) - kI_{n-1} \\ k^3(H_\sigma^* + H_\sigma) - kI_{n-1} & k^4(H_\sigma^* + H_\sigma) - 2I_n \end{pmatrix} \otimes I_m.$$

Clearly, Q_σ is positive definite and then $\dot{V}(\epsilon)$ is negative definite if k is taken as (33) and

$$r < \tau = \frac{\lambda_{\min}}{k^4 + k^6 + (k^{13} + k^{11})\tilde{\mu}^2 + k^4(k^2+1)^2\tilde{\mu} + 2q(k+1)},$$

where $\lambda_{\min}$ denotes the minimum eigenvalue of all possible Q_σ. Thus, the subsystem (32) is UUB, i. e.,

$$\|\varepsilon\| \le \sqrt{\frac{(k_3^\star + 1)\tau}{(k_3^\star - 1)\bar{\eta}}}\,\hat{\delta}.$$

On the other hand, for the system (31), Let $\tilde{x}_1 = \bar{x}_1 - \bar{v}_1(0)t - \bar{x}_1(0) - \int_0^t \int_0^s \bar{\delta}_1(\omega)\,\mathrm{d}\omega\mathrm{d}s$, $\tilde{v}_1 = \bar{v}_1 - \bar{v}_1(0) - \int_0^t \bar{\delta}_1(s)\,\mathrm{d}s$ with $\bar{x}_1(0) = \frac{1}{\sqrt{n}}\sum_{i=1}^n x_i(0), \bar{v}_1(0) = \frac{1}{\sqrt{n}}\sum_{i=1}^n v_i(0)$. Then with the transformation (30), we have

$$\left\|x - (V \otimes I_m)\begin{pmatrix} \bar{v}_1(0)t + \bar{x}_1(0) \\ 0_{m(n-1)} \end{pmatrix}\right\| = \left\|x - \mathbf{1} \otimes \frac{1}{n}\sum_{i=1}^n x_i(0)\right\| \tag{37}$$

$$\leq \min\left\{\varrho_1, \sqrt{\frac{(k_3^\star + 1)\tau}{(k_3^\star - 1)\bar{\eta}}}\hat{\delta}\right\},$$

$$\left\|v - \mathbf{1} \otimes \frac{1}{n}\sum_{i=1}^n v_i(0)\right\| \leq \min\left\{\varrho_2, \sqrt{\frac{(k_3^\star + 1)\tau}{(k_3^\star - 1)\bar{\eta}}}\hat{\delta}\right\}. \tag{38}$$

In this way, the conclusion follows. □

Remark 5.3. In fact, if the is no perturbation $\delta_i(t)$ and $\mathcal{G}_\sigma$ is balanced graph, the position x_i and the velocity v_i of agent i $(i = 1, \ldots, n)$ in the considered multi-agent system converge to the average values of initial positions (i. e., $\frac{1}{n}\sum_{i=1}^n x_i(0)$) and initial velocities (i. e., $\frac{1}{n}\sum_{i=1}^n v_i(0)$), respectively, due to (37) and (38).

Remark 5.4. Theorem 5.2 can be easily extended to the leader-following case with time-varying delays and bounded disturbances under the frameworks in [18] and [6].

5.2. Consensus problem with desired velocity

The consensus problem with desired velocity for system (9) is considered in this section. Applying the changes of variable (23) yields

$$\begin{cases} \dot{\tilde{x}} = \tilde{v}, \\ \dot{\tilde{v}} = -(L_\sigma \otimes I_m)\tilde{x}(t-r) - k\tilde{v} + \delta. \end{cases}$$

Take a coordinate transformation

$$\bar{x} = (V^* \otimes I_m)\tilde{x},\ \ \bar{v} = (V^* \otimes I_m)\tilde{v}, \bar{\delta} = (V^* \otimes I_m)\delta, \tag{39}$$

with the matrix V given in (28) and we have the following two subsystems:

$$\begin{cases} \dot{\bar{x}}_1 = \bar{v}_1, \\ \dot{\bar{v}}_1 = -k\bar{v}_1 + \bar{\delta}_1, \end{cases} \tag{40}$$

$$\begin{cases} \dot{\bar{x}}_2 = \bar{v}_2, \\ \dot{\bar{v}}_2 = -k\bar{v}_2 - (H_\sigma \otimes I_m)\bar{x}_2(t-r) + \bar{\delta}_2, \end{cases} \tag{41}$$

where $\bar{x}_1, \bar{v}_1 \in \mathbb{C}^m, \bar{x}_2, \bar{v}_2 \in \mathbb{C}^{m(n-1)}$.

Employing the same technique in the proof of Theorem 5.2, choose a Lyapunov–Razumikhin function

$$V(\varepsilon) = \varepsilon^* \check{P} \varepsilon, \tag{42}$$

where

$$\check{P} = \begin{pmatrix} kI_{n-1} & I_{n-1} \\ I_{n-1} & I_{n-1} \end{pmatrix} \otimes I_m$$

is positive definite for $k > 1$. Then a main result for system (9) with the switched balanced topology can be obtained as followed:

Theorem 5.5. For system (9) with balanced interconnection topology $\mathcal{G}_\sigma$, take

$$k > k_4^\star = \max\left\{ \frac{\tilde{\mu}}{2\tilde{\lambda}} + 1, \tilde{\mu} + 1 \right\}, \tag{43}$$

with $\tilde{\mu}$ and $\tilde{\lambda}$ as defined in Theorem 5.2. If $\mathcal{G}_\sigma$ has a globally reachable node and τ is sufficiently small, then the robust consensus problem with desired velocity of the system (9) is solved.

6. CONCLUSIONS

The paper addressed two robust consensus problems of a group of multiple mobile agents, with two respective neighbor-based rules. The robust consensus stability was guaranteed with both fixed and switched interconnection topologies of the considered multi-agent system. The dynamics of each agent was second-order with time-varying delays and bounded disturbances and Lyapunov–Razumikhin function (rather than the stochastic matrix related to graph theory) was employed in the stability analysis.

(Received December 2, 2008.)

REFERENCES

[1] J. Bang-Jensen and G. Gutin: Digraphs Theory, Algorithms and Applications, Springer-Verlag, New York 2002.

[2] M. G. Earl and S. H. Strogatz: Synchronization in oscillator networks with delayed coupling: A stability criterion. Phys. Rev. *E 67* (2003), 036204.1–036204.4.

[3] C. Godsil and G. Royle: Algebraic Graph Theory. Springer-Verlag, New York 2001.

[4] J. K. Hale and S. M. V. Lunel: Introduction to the Theory of Functional Differential Equations. Springer-Verlag, New York 1991.

[5] I. M. Havel: Sixty years of cybernetics: cybernetics still alive. Kybernetika *44* (2008), 314–327.

[6] Y. Hong, G. Chen, and L. Bushnell: Distributed observers design for leader-following control of multi-agent networks. Automatica *44* (2008), 846–850.

[7] R. Horn and C. Johnson: Matrix Analysis. Cambridge Univ. Press, New York 1985.

[8] J. Hu and Y. Hong: Leader-following coordination of multi-agent systems with coupling time delays. Physica *A 374* (2007), 853–863.

[9] J. Hu and X. Hu: Optimal target trajectory estimation and filtering using networked sensors. J. Systems Sci. Complexity *21* (2008), 325–336.

[10] A. Jadbabaie, J. Lin, and A. S. Morse: Coordination of groups of mobile agents using nearest neighbor rules. IEEE Trans. Automat. Control *48* (2003), 988–1001.

[11] S. Low, F. Paganini, and J. Doyle: Internet congestion control. IEEE Control Systems Magazine *32* (2002), 28–43.

[12] Y. S. Moon, P. Park, W. H. Kwon, and Y. S. Lee: Delay-dependent robust stabilization of uncertain state-delayed systems. Internat. J. Control *74* (2001), 1447–1455.

[13] L. Moreau: Stability of multiagent systems with time-dependent communication links. IEEE Trans. Automat. Control *50* (2005), 169–182.

[14] R. Olfati-Saber, J. A. Fax, and R. M. Murray: Consensus and cooperation in networked multi-agent systems. Proc. IEEE *95* (2007), 215–233.

[15] R. Olfati-Saber and R. M. Murray: Consensus problems in networks of agents with switching topology and time-delays. IEEE Trans. Automat. Control *49* (2004), 1520–1533.

[16] K. Peng and Y. Yang: Leader-following consensus problem with a varying-velocity leader and time-varying delays. Physica *A 388* (2009), 193–208.

[17] W. Ren and R. W. Beard: Consensus seeking in multiagent systems under dynamically changing interaction topologies. IEEE Trans. Automat. Control *50* (2005), 655–661.

[18] G. Shi and Y. Hong: Global target aggregation and state agreement of nonlinear multi-agent systems with switching topologies. Automatica *45* (2009), 1165–1175.

[19] Y. Sun and J. Ruan: Consensus problems of multi-agent systems with noise perturbation. Chinese Phys. *B 17* (2008), 4137–4141.

[20] T. Vicsek, A. Czirok, and E. B. Jacob, I. Cohen, and O. Schochet: Novel type of phase transitions in a system of self-driven particles. Phys. Rev. Lett. *75* (1995), 1226–1229.

[21] F. Xiao and L. Wang: Consensus protocols for discrete-time multi-agent systems with time-varying delays. Automatica *44* (2008), 2577–2582.

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