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THE DYNAMICS OF WEAKLY INTERACTING FRONTS IN AN ADSORBATE–INDUCED PHASE TRANSITION MODEL

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Dedicated to Professor Kenji Tomoeda on the occasion of his sixtieth birthday.

Hildebrand et al. [5, 7] proposed an adsorbate-induced phase transition model. For this model, Takei et al. [10] found several stationary and evolutionary patterns by numerical simulations. Due to bistability of the system, there appears a phase separation phenomenon and an interface separating these phases. In this paper, we introduce the equation describing the motion of two interfaces in $\mathbb{R}^2$ and discuss an application. Moreover, we prove the existence of the traveling front solution which approximates the shape of the solution in the neighborhood of the interface.

Keywords: reaction-diffusion system, interaction of fronts, phase transition model

AMS Subject Classification: 35B25, 35B40, 35K57

1. INTRODUCTION

Several people [3, 5, 6, 7, 9] proposed models which describe the process of pattern formation in the catalytic oxidation of CO molecules on a platinum surface. Here, we consider the model given in [5] as follows:

$$\begin{cases} u_t = d_u \Delta u + f(u, v), \\ \tau v_t = d_v \Delta v + \gamma \nabla \{v(1-v)\nabla \chi(u)\} + g(u, v), \end{cases} \tag{1}$$

where $f(u, v) = u(u + v - 1)(1 - u)$, $g(u, v) = c(1 - v) - (ae^{\beta\chi(u)} + b)v$ and $\tau, d_u, d_v, a, b, c, \beta, \gamma$ are positive constants. The unknown functions $u = u(x, t)$ and $v = v(x, t)$ denote the structural state of surface and the adsorbate coverage rate of the surface by CO molecules, respectively. The function $\chi(u)$ is defined by

$$\chi(u) = u^2(2u - 3). \tag{2}$$

As shown in Tsujikawa and Yagi [13], Takei et al. [11] and [10], there exists a unique global solution of (1) in a bounded domain of $\mathbb{R}^2$ with Neumann boundary condition and an exponential attractor of the corresponding dynamical system.

From the view point of pattern formation, the existence of stationary spot solutions was shown and its stability of (1) in $\mathbb{R}$ and $\mathbb{R}^2$ was considered by using the singular perturbation method [6, 7]. On the other hand, various types of stationary patterns are observed by numerical simulations in [9, 10]. They are stationary stripe, square and hexagonal patterns on the surface and their existence is proved by the bifurcation method [8]. Here, we mainly consider the dynamics of stripe and snaky patterns. For the model, the phase transition phenomenon appears due to the bistable system and we call the boundary of two phases an interface. When d_u is sufficiently small, it is expected that the width of the domain separating two phases is of order $O(d_u)$. Therefore, it is enough to consider the dynamics of the movement of the interface for the understanding of these pattern formations. In particular, we are concerned with the interactive dynamics of two interfaces far from equilibrium in $\mathbb{R}^2$. To do so, we introduce the equation which describes the motion of the interface and discuss the interactive dynamics between two interfaces. Next, the equation is applied in order to understand the motion of two fronts in $\mathbb{R}$. Finally, we show the existence of the traveling front solution to obtain the equation, which approximates the shape of the solution in the neighborhood of the interface.

2. INTERACTIVE DYNAMICS OF TWO INTERFACES

In this section, we formally introduce the equation describing the dynamics of the interfaces with weakly interaction in $\mathbb{R}^2$. To do so, we rewrite (1) as

$$\Pi \mathbf{u}_t = D\Delta \mathbf{u} + \gamma \mathbf{K}_2(\mathbf{u}) + F(\mathbf{u}), \tag{3}$$

where
$$\Pi = \begin{pmatrix} 1 & 0 \\ 0 & \tau \end{pmatrix}, \qquad D = \begin{pmatrix} d_u & 0 \\ 0 & d_v \end{pmatrix},$$

and
$$\mathbf{K}_2(\mathbf{u}) = \mathrm{div}(0, v(1-v)\nabla\chi(u))^{\mathrm{T}}, \qquad F(\mathbf{u}) = (f(u,v), g(u,v))^{\mathrm{T}},$$

for $\mathbf{u} = (u,v)^{\mathrm{T}}$.

When $\Gamma(\sigma)$ is a curve corresponding to the interface with a parameter σ, the local coordinate is given by $(x,y) = \Gamma(\sigma) + \lambda\nu(\sigma), (\lambda,\sigma) = (\Lambda(x,y), \Sigma(x,y))$ with the normal vector $\nu(\sigma)$ at $\Gamma(\sigma)$. Then, we note that $\nabla\Lambda = \nu,\ \nabla\Sigma = \frac{1}{1-\hat{\kappa}\lambda}\Gamma_\sigma,\ |\nabla\Lambda|^2 = 1,$ $|\Gamma_\sigma|^2 = 1,\ <\nabla\Lambda,\ \nabla\Sigma> = 0,\ \Delta\Lambda = -\frac{\hat{\kappa}}{1-\hat{\kappa}\lambda},\ \Delta\Sigma = \frac{\hat{\kappa}_\sigma\lambda}{(1-\hat{\kappa}\lambda)^3}$ where $\hat{\kappa}$ is the curvature of $\Gamma(\sigma)$ and $\langle\cdot,\cdot\rangle$ means the inner product.

Let

$$\begin{aligned}
\hat{K}(u,v) &= \frac{(1-2v)\chi'(u)}{(1-\hat{\kappa}\lambda)^2} u_\sigma v_\sigma \\
&\quad + v(1-v)\left\{\frac{\chi''(u)}{(1-\hat{\kappa}\lambda)^2}u_\sigma^2 + \chi'(u)\left(-\frac{\hat{\kappa}}{1-\hat{\kappa}\lambda}u_\lambda + \frac{1}{1-\hat{\kappa}\lambda}(\frac{1}{1-\hat{\kappa}\lambda}u_\sigma)_\sigma\right)\right\}, \\
K_1(u,v) &= \{v(1-v)\chi'(u)u_\lambda\}_\lambda, \\
K_2(u,v) &= \mathrm{div}(v(1-v)\nabla\chi(u)) = K_1(u,v) + \hat{K}(u,v).
\end{aligned}$$

Since $\mathbf{K}_2 = \mathbf{K}_1 + \hat{\mathbf{K}}$ with $\mathbf{K}_1(\mathbf{u}) = (0, K_1(u,v))^{\mathrm{T}}$, $\mathbf{K}_2(\mathbf{u}) = (0, K_2(u,v))^{\mathrm{T}}$, $\hat{\mathbf{K}}(\mathbf{u}) = (0, \hat{K}(u,v))^{\mathrm{T}}$, (3) becomes

$$\Pi \mathbf{u}_t = L_1(\mathbf{u}) + D\mathbf{K}\mathbf{u} + \gamma \mathbf{K}_1(\mathbf{u}) + \gamma \hat{\mathbf{K}}(\mathbf{u}), \tag{4}$$

where $L_1(\mathbf{u}) = D\mathbf{u}_{\lambda\lambda} + F(\mathbf{u})$ and $\mathbf{K}\mathbf{u} = -\frac{\hat{\kappa}}{1-\hat{\kappa}\lambda}\mathbf{u}_\lambda + \frac{1}{1-\hat{\kappa}\lambda}(\frac{1}{1-\hat{\kappa}\lambda}\mathbf{u}_\sigma)_\sigma$.

Next, we consider the profile of the solution in the neighborhood of the interface. To do so, we first treat the traveling front solution connecting the two equilibrium points in 1-dimensional domain $\mathbb{R}$. Then, the equation corresponding to (4) is given by

$$\Pi \mathbf{u}_t = D\mathbf{u}_{xx} + \gamma \hat{\mathbf{K}}_1(\mathbf{u}) + F(\mathbf{u}), \quad x \in \mathbb{R}, \tag{5}$$

where $\hat{\mathbf{K}}_1(\mathbf{u}) = (0, (1-2v)\chi'(u)u_x v_x + v(1-v)\{\chi'(u)u_x\}_x)^{\mathrm{T}}$.
We remark that there are three roots $P_- = (0, v_-)^{\mathrm{T}}$, $P_0 = (u_0, v_0)^{\mathrm{T}}$ $P_+ = (1, v_+)^{\mathrm{T}}$ of $F(\mathbf{u}) = 0$ where $v_- = \frac{c}{a+b+c} < v_0 < v_+ = \frac{c}{a^{-\beta}+b+c}$. Then, we set the boundary conditions of (4) as follows:

$$\lim_{x\to-\infty} \mathbf{u}(t,x) = P_-, \ \lim_{x\to\infty} \mathbf{u}(t,x) = P_+. \tag{6}$$

Assumption 1. Let $\lambda = x + ct$. There exists a traveling front solution of (5), (6) with a velocity c, that is, a solution $S(\lambda) = (\Phi(\lambda), \Psi(\lambda))^{\mathrm{T}}$ satisfying

$$\begin{cases} 0 = DS_{\lambda\lambda} - c\Pi S_\lambda + \gamma \mathbf{K}_1(S) + F(S), \ \lambda \in \mathbb{R}, \\ S(\pm\infty) = P_\pm, \end{cases} \tag{7}$$

where $\mathbf{K}_1(S) = (0, (1-2\Psi)\chi'(\Phi)\Phi_\lambda\Psi_\lambda + \Psi(1-\Psi)\{\chi'(\Phi)\Phi_\lambda\}_\lambda)^{\mathrm{T}}$.

Remark 1. For suitable constants d_u, d_v and τ, there exists a traveling front solution of (7) (see the Appendix).

We treat two interface curves Γ_i $(i = 1, 2)$ which have not any common point. Let $(x, y) = \Gamma_i(\sigma_i) + \lambda_i \nu_i(\sigma_i)$ be local coordinates in the neighborhood of each curve Γ_i. Then, we assume that the solution $\mathbf{u}(t, x, y)$ is expanded as

$$\mathbf{u}(t,x,y) = S(\Lambda_1(t,x,y)) + S(-\Lambda_2(t,x,y)) - P_+ + \mathbf{w}(t,x,y). \tag{8}$$

Assumption 2. Let ε be a small parameter. Then, it holds that for each curve Γ_i $(i = 1, 2)$ curvatures $\hat{\kappa}_i$ of Γ_i and σ_i are of order $O(\varepsilon)$ and $\mathbf{w}$ is approximately represented by $\mathbf{w}(t, \lambda_i, \sigma_i)$ in the neighborhood of Γ_i. Let $\hat{\kappa}_i = \varepsilon\kappa_i$ and $\sigma_i = \varepsilon\ell_i$.

Substituting (8) into (4), the left hand side is rewritten as

$$\mathbf{u}_t = \Lambda_{1t}S_\lambda(\Lambda_1) - \Lambda_{2t}S_\lambda(-\Lambda_2) + \mathbf{w}_t + \Lambda_{it}\mathbf{w}_{\lambda_i} + \varepsilon\Sigma_{it}\mathbf{w}_{\ell_i} \quad \text{in the neighborhood of } \Gamma_i. \tag{9}$$

First, we consider the problem in the neighborhood of Γ_1. Then, $S(-\Lambda_2) - P_+ + \mathbf{w}$ is the remainder term in the neighborhood.

For simplicity, let $\lambda_1 \to \lambda$, $\ell_1 \to \ell$, $\kappa_1 \to \kappa$, $\Lambda_1 \to \Lambda$, $\Sigma_1 \to \Sigma$. Then, the right hand side approximately becomes

$$\begin{aligned} L_1(\mathbf{u}) + D\mathbf{K}\mathbf{u} + \gamma\mathbf{K}_1(\mathbf{u}) + \gamma\hat{\mathbf{K}}(\mathbf{u}) \sim L_1(S) &+ \hat{L}(S(-\Lambda_2) - P_+ + \mathbf{w}) + D\mathbf{K}S \\ &+ \gamma\mathbf{K}_1(S) + \gamma\hat{\mathbf{K}}(S) + D\mathbf{K}(S(-\Lambda_2) - P_+) \\ &+ D\mathbf{K}\mathbf{w} + \gamma\mathbf{K}_1'(S)(S(-\Lambda_2) - P_+ + \mathbf{w}) \\ &+ \gamma\hat{\mathbf{K}}'(S)(S(-\Lambda_2) - P_+ + \mathbf{w}), \end{aligned} \tag{10}$$

where $\hat{L} = L_1'(S) = D\,\mathrm{d}^2/\mathrm{d}\lambda^2 + F'(S)$. Here, higher order terms with respect to small ε and $\mathbf{w}$ are neglected.

Assumption 3. For the solution $S(\lambda)$ of (7), there is a positive constant α and vector $\mathbf{a}_+ = (p, q)^{\mathrm{T}}$ such that

$$S(\lambda) - P_+ \sim e^{-\alpha\lambda}\mathbf{a}_+ \text{ as } \lambda \to \infty. \tag{11}$$

Then, we note that $D\mathbf{K}(S(-\Lambda_2) - P_+) = O(\varepsilon\kappa e^{\alpha\Lambda_2})$, $\Lambda_t\mathbf{w}_\lambda$ and $\varepsilon\Sigma_t\mathbf{w}_\ell$ are small when ε is small. Since $0 = L_1(S) - cS_\lambda + \gamma\mathbf{K}_1(S)$ and $D\mathbf{K}\mathbf{w} = O(\varepsilon^2 + \varepsilon\mathbf{w}_\lambda)$, it follows that

$$\begin{aligned} \Pi\mathbf{w}_t + \Pi\Lambda_t S_\lambda \sim c\Pi S_\lambda &+ \hat{L}\mathbf{w} + \hat{L}(e^{\alpha\Lambda_2}\mathbf{a}_+) + D\mathbf{K}S - \hat{\mathbf{K}}(S) \\ &+ \gamma\mathbf{K}_1'(S)(e^{\alpha\Lambda_2}\mathbf{a}_+ + \mathbf{w}) + \gamma\hat{\mathbf{K}}'(S)(e^{\alpha\Lambda_2}\mathbf{a}_+ + \mathbf{w}). \end{aligned}$$

By (7), (11), it holds that

$$0 \sim \alpha^2 D e^{-\alpha\lambda}\mathbf{a}_+ + F'(P_+)e^{-\alpha\lambda}\mathbf{a}_+ + \alpha c\Pi e^{-\alpha\lambda}\mathbf{a}_+ + \gamma\mathbf{K}_1'(P_+)e^{-\alpha\lambda}\mathbf{a}_+. \tag{12}$$

Therefore, (12) implies

$$\hat{L}(e^{-\alpha\lambda}\mathbf{a}_+) \sim (F'(S) - F'(P_+) - \alpha c\Pi - \gamma\mathbf{K}_1'(P_+))e^{-\alpha\lambda}\mathbf{a}_+ \tag{13}$$

by $\hat{L}(e^{-\alpha\lambda}\mathbf{a}_+) = (\alpha^2 D + F'(S))e^{-\alpha\lambda}\mathbf{a}_+$.

Since

$$\begin{aligned} &\hat{\mathbf{K}}'(S)(e^{\alpha\Lambda_2}\mathbf{a}_+ + \mathbf{w}) \sim 0, \\ &\mathbf{K}S = -\frac{\varepsilon\kappa}{1 - \varepsilon\kappa\lambda}S_\lambda \sim -\varepsilon\kappa S_\lambda, \\ &\hat{\mathbf{K}}(S) \sim (0, -\varepsilon\kappa\Psi(1 - \Psi)\chi(\Phi)_\lambda)^{\mathrm{T}}, \end{aligned}$$

it follows from (10), (13) that

$$\begin{aligned} \Pi\mathbf{w}_t + \Lambda_t\Pi S_\lambda \sim (\hat{L} - \mathbf{K}_1'(S))\mathbf{w} - \varepsilon\kappa D S_\lambda + c\Pi S_\lambda - \varepsilon\kappa\gamma \begin{pmatrix} 0 \\ \Psi(1 - \Psi)\chi(\Phi)_\lambda \end{pmatrix} \\ + (F'(S) - F'(P_+) - \gamma\mathbf{K}_1'(P_+) + \gamma\mathbf{K}_1'(S) - \alpha c\Pi)e^{\alpha\Lambda_2}\mathbf{a}_+. \end{aligned} \tag{14}$$

Here, we note that

$$\mathbf{K}_1'(S)e^{-\alpha\lambda}\mathbf{a}_+ = \begin{pmatrix} 0 & 0 \\ \Theta_{21} & \Theta_{22} \end{pmatrix} e^{-\alpha\lambda}\mathbf{a}_+,$$

and

$$\mathbf{K}_1'(P_+)e^{-\alpha\lambda}\mathbf{a}_+ = \begin{pmatrix} 0 \\ w_1(1-w_1)\chi'(1)\alpha^2 p \end{pmatrix} e^{-\alpha\lambda},$$

where

$$\begin{aligned}\Theta_{21} &= (1-2\Psi)\Psi_\lambda\{\chi''(\Phi)\Phi_\lambda - \alpha\chi'(\Phi)\} \\ &\qquad + \Psi(1-2\Psi)\{-2\alpha\chi''(\Phi)\Phi_\lambda + \{\chi''(\Phi)\Phi_\lambda\}_\lambda + \alpha^2\chi'(\Phi)\}, \\ \Theta_{22} &= -2\chi'(\Phi)\Phi_\lambda\Psi_\lambda + (1-2\Psi)\{-\alpha\chi'(\Phi)\Phi_\lambda + \{\chi'(\Phi)\Phi_\lambda\}_\lambda\}.\end{aligned}$$

Next, we will compute the outward normal velocity V of the interface. Let $\varphi^* = (\varphi_1^*, \varphi_2^*)$ be an eigenfunction corresponding to the eigenvalue 0 of the adjoint operator $(\hat{L} - \mathbf{K}_1'(S))^*\Pi^{-1}$ of $\hat{L} - \mathbf{K}_1'(S)\Pi^{-1}$ normalized by $\langle \Pi S_\lambda, \varphi^* \rangle_{L^2} = 1$ where $\langle \cdot, \cdot \rangle_{L^2}$ means the $L^2(\mathbb{R})$ inner product. Then, it follows from the solvability condition (e. g. [2]) and (14) that

$$\begin{aligned}\Lambda_t = \ & -\varepsilon\kappa\langle DS_\lambda, \varphi^*\rangle_{L^2} + \varepsilon\kappa\langle \Psi(1-2\Psi)\chi(\Phi)\Phi_\lambda, \varphi_1^*\rangle_{L^2} + c\Pi \\ & + \int_{-\infty}^{\infty} e^{\alpha\Lambda_2(x(\lambda),y(\lambda))}\langle (G(S(\lambda)) - G(P_+) - \alpha c\Pi)\mathbf{a}_+, \varphi^*(\lambda)\rangle \, \mathrm{d}\lambda,\end{aligned}$$

(see Figure 1) where $G(X) = F'(X) + \gamma\mathbf{K}_1'(X)$ and we assume $\langle \Pi\mathbf{w}, \varphi^* \rangle_{L^2} = 0$.

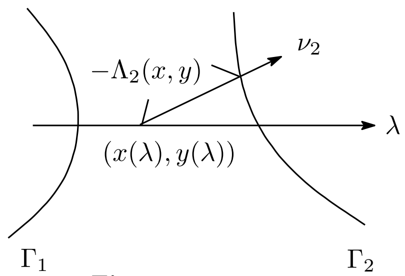

Fig. 1.

From $\Lambda_t = -V$, the velocity V_1 of Γ_1 is given by

$$V_1 = \varepsilon\kappa_1(\langle DS_\lambda, \varphi^*\rangle_{L^2} - \langle \Psi(1-2\Psi)\chi(\Phi)\Phi_\lambda, \varphi_1^*\rangle_{L^2}) - c\Pi - g_1, \tag{15}$$

where

$$g_1 = \int_{-\infty}^{\infty} e^{\alpha\Lambda_2(x(\lambda),y(\lambda))}\langle (G(S(\lambda)) - G(P_+) - \alpha c\Pi)\mathbf{a}_+, \varphi^*(\lambda)\rangle \, \mathrm{d}\lambda.$$

On the other hand, let $\hat{S}(\lambda) = S(-\lambda)$, $\hat{\varphi}^*(\lambda) = \varphi^*(-\lambda)$ and so on. Then, the normal velocity $V_2 = \Lambda_{2t}$ of the interface Γ_2 is represented by

$$V_2 = -\varepsilon\kappa_2(\langle D\hat{S}_\lambda, \hat{\varphi}^*\rangle_{L^2} - \langle \hat{\Psi}(1-2\hat{\Psi})\chi(\hat{\Phi})\hat{\Phi}_\lambda, \hat{\varphi}_1^*\rangle_{L^2}) + c\Pi + g_2, \qquad (16)$$

where

$$g_2 = \int_{-\infty}^{\infty} e^{-\alpha\Lambda_1(x(\lambda),y(\lambda))}\langle (G(\hat{S}(\lambda)) - G(P_+) - \alpha c\Pi)\mathbf{a}_+, \hat{\varphi}^*(\lambda)\rangle \, \mathrm{d}\lambda.$$

Therefore, V_1 and V_2 in (15) and (16) represent the normal velocity of the weakly interacting interfaces. Although we do not estimate g_1 and g_2, it is presumed that these terms are small as the distance between two interfaces is very large (see the Appendix). If the velocity c of the traveling front solution of (7) is of order ε, then the velocity of the interface also depends on the curvature.

3. APPLICATION (1–DIMENSIONAL PROBLEM)

In this section, we apply the result in Section 2 to the 1-dimensional problem. Let $\ell_1(t)$, $\ell_2(t)$ ($\ell_1(t) < \ell_2(t)$) be interface positions in the line, that is, the shape of the solution looks like a trapezoid. It follows from (15), (16) that

$$\begin{cases} \ell_{1t}(t) = V_1 = -c - e^{\ell_1(t)-\ell_2(t)}H, \\ \ell_{2t}(t) = V_2 = c + e^{\ell_2(t)-\ell_1(t)}H, \end{cases}$$

where $H = \displaystyle\int_{-\infty}^{\infty} e^{\alpha\lambda}\langle (G(S(\lambda)) - G(P_+) - \alpha c)\mathbf{a}_+, \varphi^*(\lambda)\rangle \, \mathrm{d}\lambda.$

As $|\ell_1(t) - \ell_2(t)| \gg 1$, these equations imply that

$$\begin{cases} c < 0 \Longrightarrow \text{two interfaces are attractive,} \\ c > 0 \Longrightarrow \text{two interfaces are repulsive.} \end{cases}$$

As $c = 0$, it holds that

$$\begin{cases} H^* < 0 \Longrightarrow \text{two interfaces are attractive,} \\ H^* > 0 \Longrightarrow \text{two interfaces are repulsive,} \end{cases}$$

where $H^* = \displaystyle\int_{-\infty}^{\infty} e^{\alpha\lambda}\langle (G(S(\lambda)) - G(P_+))\mathbf{a}_+, \varphi^*(\lambda)\rangle \, \mathrm{d}\lambda.$

APPENDIX

By using a singular perturbation method similar to the one in [4], we can prove the following theorem.

Theorem. (Tsujikawa [12]) Let $\hat{\delta} = \exp(-\gamma/d_v)$ and $\tau = O(d_u)$. If

$$\frac{\sqrt{v_-}(1-v_+)}{\sqrt{v_+}(1-v_-)} > \hat{\delta},$$

there exists a traveling front solution $(\Phi(\lambda, d_u), \Psi(\lambda, d_u))$ of (7) with the velocity $c = \hat{c}d_u + o(1)$ for small $d_u > 0$ such that

$$\lim_{d_u \to 0} (\Phi(\lambda, d_u), \Psi(\lambda, d_u)) = (\Phi_0(\lambda), \Psi_0(\lambda)),$$

where $\Phi_0(\lambda)$ is the Heaviside function and $\Psi_0(\lambda)$ satisfies

$$\begin{cases} 0 = d_v \Psi_{0\lambda\lambda} + g(\Phi_0, \Psi_0), \quad \lambda \in \mathbb{R}, \\ \dfrac{\Psi_{0\lambda}(-0)}{\Psi_{0\lambda}(-0)(1-\Psi_{0\lambda}(-0))} = \dfrac{\Psi_{0\lambda}(+0)}{\Psi_{0\lambda}(+0)(1-\Psi_{0\lambda}(+0))}, \\ \Psi_0(-\infty) = v_-, \ \Psi_0(\infty) = v_+. \end{cases}$$

Let $\Psi(0, d_u) = 1/(1+N)$. It holds that

$$0 \lesseqqgtr \hat{c} \text{ if and only if } \sqrt{\hat{\delta}} \lesseqqgtr N. \tag{17}$$

Moreover, if

$$\left| \frac{\sqrt{v_-}(1-v_+)}{\sqrt{v_+}(1-v_-)} - \hat{\delta} \right| \ll 1, \qquad 0 < \frac{\sqrt{v_-}(1-v_+)}{\sqrt{v_+}(1-v_-)} < \hat{\delta},$$

then there exist at least two traveling front solutions for small $d_u > 0$.

Proof. Here we only prove (17). The solution $\Phi_0(\lambda)$ satisfies

$$\begin{cases} 0 = \Phi_{0\lambda\lambda} - \hat{c}\Phi_{0\lambda} + f(\Phi_0, W(\Phi_0)), \quad \lambda \in \mathbb{R}, \\ \Phi_0(-\infty) = 0, \ \Phi_0(\infty) = 1, \end{cases}$$

where $W(\Phi_0) = 1/(1 + N\exp(\gamma\chi(\Phi_0)/d_v))$ and $f(U) = U(U + W(U) - 1)(1-U)$.

Since $\Phi_0(\lambda)$ satisfies $\Phi_{0\lambda}(-\infty) = 0 = \Phi_{0\lambda}(+\infty)$, it holds that

$$\hat{c}\int_{-\infty}^{+\infty} \Phi_{0\lambda}^2 \, d\lambda = \int_0^1 f(U, W(U)) \, dU = \frac{1}{12}\left\{1 + \frac{2d_v}{\gamma}\log\left|\frac{1+N\exp(-\frac{\gamma}{d_v})}{1+N}\right|\right\}.$$

Therefore, we have (17). □

For $v_- = 0.2$ and $v_+ = 0.8$, the curve in Figure 2 corresponds to the traveling front solutions depending on the parameter $\hat{\delta} > 0$ as $d_u \downarrow 0$ and vertical axis means the value of $\Psi(0,0)$. The dot and solid parts in the curve means the negative and positive velocity of traveling front solutions.

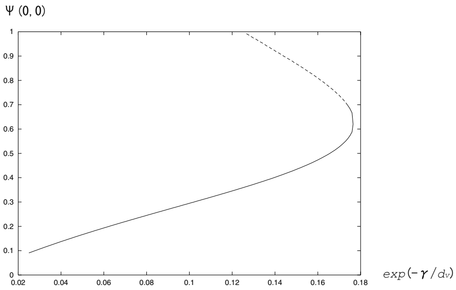

Fig. 2.

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