Kybernetika

VOLUME *45* (2009), NUMBER 1

The Journal of the Czech Society for Cybernetics and Information Sciences

Published by:

Institute of Information Theory and Automation of the AS CR

Editorial Office:

Pod Vodárenskou věží 4, 182 08 Praha 8

Kybernetika is a bi-monthly international journal dedicated for rapid publication of high-quality, peer-reviewed research articles in fields covered by its title.

Kybernetika traditionally publishes research results in the fields of Control Sciences, Information Sciences, System Sciences, Statistical Decision Making, Applied Probability Theory, Random Processes, Fuzziness and Uncertainty Theories, Operations Research and Theoretical Computer Science, as well as in the topics closely related to the above fields.

The Journal has been monitored in the Science Citation Index since 1977 and it is abstracted/indexed in databases of Mathematical Reviews, Zentralblatt für Mathematik, Current Mathematical Publications, Current Contents ISI Engineering and Computing Technology.

K y b e r n e t i k a . Volume *45* (2009) ISSN 0023-5954, MK ČR E 4902.

Published bimonthly by the Institute of Information Theory and Automation of the Academy of Sciences of the Czech Republic, Pod Vodárenskou věží 4, 182 08 Praha 8. — Address of the Editor: P. O. Box 18, 182 08 Prague 8, e-mail: kybernetika@utia.cas.cz. — Printed by PV Press, Pod vrstevnicí 5, 140 00 Prague 4. — Orders and subscriptions should be placed with: MYRIS TRADE Ltd., P. O. Box 2, V Štíhlách 1311, 142 01 Prague 4, Czech Republic, e-mail: myris@myris.cz. — Sole agent for all “western” countries: Kubon & Sagner, P. O. Box 34 01 08, D-8 000 München 34, F.R.G.

Published in February 2009.

KYBERNETIKA — VOLUME 45 (2009), NUMBER 1, PAGES 33 – 48

DECENTRALIZED OUTPUT REGULATION OF LARGE SCALE NONLINEAR SYSTEMS WITH DELAY

Zhengtao Ding

This paper deals with output regulation of a class of large-scale nonlinear systems with delays. Each of the subsystems is in the output feedback form, with nonlinear functions of the subsystem output and the outputs of other subsystems. The system outputs are subject to unknown constant delays. Both the system dynamics and the measurements are subject to unknown disturbances generated from unknown linear exosystems. Decentralized control design approach is adopted to design local controllers using measurements or regulated errors in each subsystems. It is shown in this paper that delays in the outputs of subsystems do not affect the existence of desired feedforward control input, and the invariant manifolds and the desired feedforward inputs always exist if the nonlinear functions are polynomials. Through a special parameterization of an augmented exosystem, an internal model can be designed for each subsystem, without the involvements of the uncertain parameters. The uncertain parameters affected by the uncertainty of the exosystem are estimated using adaptive control laws, and adaptive coefficients in the control inputs are used to suppress other uncertainties. The proposed decentralized adaptive control strategy ensures the global stability of the entire system, and the convergence to zero of the regulated errors. An example is included to demonstrate the proposed control strategy.

Keywords: decentralized control, output regulation, nonlinear systems, time delay
AMS Subject Classification: 93A14, 93D15, 93D21, 93D05, 93C10, 37C27

1. INTRODUCTION

Many industrial control systems can be modeled as large-scale nonlinear systems, and decentralized control has been a successful strategy in dealing with control design based on local information, see [8, 9, 10, 13, 15] and references therein. While most of results of decentralized control focus on stabilization and tracking, output regulation has been considered in [13, 15], where output tracking and disturbance rejection can be uniformly formulated and solved in the control design. These results on output regulation are obtained under the assumption that there is not time delay in the system, and there exists certain immersion of the exosystem for internal model design. As time delay is a realistic problem in control systems, especially in large scale control systems, we devote this paper to address decentralized control design for output regulation of large scale nonlinear systems with time delay.

Output regulation deals with tracking and rejecting periodic signals while maintaining the stability of the closed loop control system. Local results for output regulation for nonlinear systems are reported in [6, 7]. More recently, global output regulation has been addressed in the literature for nonlinear systems in the output feedback form [2, 12], and the results have been extended to deal with unknown linear exosystems with adaptive control techniques and nonlinear exosystems [3, 11, 14]. On the other hand, time delay has been a control design problem for even longer time with vast amount of results. With a big contrast to the large amount of results on control design for systems with time delay, there are much fewer results on output regulation for systems with time delay. A result on output regulation of linear systems is reported in [5], where a transcend regulator equation is proposed. For output regulation of nonlinear systems with time delay, it is shown that the problem is solvable for a special class of nonlinear systems if and only if an integral equation is solvable [4]. Those methods cannot be directly extended to solve the output regulation considered in this paper. In one of our recent results [1], time delay has been tackled in a different way for a class of single input and single output nonlinear systems with time delay. The key idea is to convert the time delay in the state for the exosystem to a linear transformation. This idea is further explored in this paper to establish the existence of the invariant manifolds and desired feedforward control input for each subsystems.

In this paper, we present a systematic design method for decentralized output regulation of a class of large scale nonlinear systems with unknown time delays. The nonlinear systems considered are allowed to have unknown parameters for almost all the coefficients, with the exception of the signs of the high frequency gains of the subsystems. The exosystem is assumed linear with a known order, otherwise completely unknown. The nonlinear functions are functions of system outputs and unknown disturbances. The control design only requires the structures of nonlinear functions, to determine the structure of the internal models. When assuming all the nonlinear functions are polynomials, the existence of the augmented linear exosystems will be shown to produce the desired feedforward inputs and internal models are then designed for every subsystems. Based on the structure of augmented exosystems and adaptive control techniques, a decentralized control is designed for each of the subsystems to ensure the global stability of the overall large scale nonlinear system and the convergence to zero of the regulated errors or the measurements of the system. A design example is included to demonstrate the proposed decentralized control strategy and the simulation results are also included in the paper.

2. PROBLEM FORMULATION

Consider a class of large-scale nonlinear systems which can be transformed into the output feedback form

$$\begin{cases} \dot{x}_i(t) &=& A_i(a)x_i(t) + \bar{\phi}_i(y_1(t), \dots, y_N(t), y_1(t-d_1), \dots \\ & & \quad \dots, y_N(t-d_N), w, a) + b_i(a)u_i(t), \\ y_i(t) &=& C_i(a)x_i(t) \\ e_i(t) &=& y_i(t) - q_i(w(t), a), \qquad i = 1, \dots, N, \\ x_i(\theta_i) &=& \delta_i(\theta_i), \theta_i \in [-\bar{d}_i, 0] \end{cases} \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}, y_i \in \mathbb{R}, u_i \in \mathbb{R}, t \in \mathbb{R}$ and $e_i \in \mathbb{R}$ are respectively, the state, output, input, and the regulated error of the ith subsystem; and $a \in \mathbb{R}^q$ is a vector of unknown parameters; $\bar{\phi}_i$, and q_i are known polynomials of their variables, d_i are constant but unknown delays in the system outputs, with $\bar{d}_i$ as the upper bounds, $\delta_i : \mathbb{R} \to \mathbb{R}^{n_i}$ are bounded functions denoting the initial conditions, and, $w \in \mathbb{R}^m$ are disturbances which are generated by a linear exosystem

$$\dot{w} = Sw \tag{2}$$

where S is an unknown matrix.

Remark 1. The nonlinear functions $\bar{\phi}_i$ and q_i can be more general nonlinear functions, as shown in our previous result [13]. They are assumed to be polynomials here for the convenience of presentation. Multiple delays in each of y_i can be considered as well. For the same reason, we only consider one delay for each y_i in this paper.

Assumption 1. The eigenvalues of S are with zero real parts, and are distinct.

Assumption 2. Linear system characterized by $\{A_i(a), b_i(a), C_i(a)\}$, for $i = 1, \ldots, N$, are of minimum phase, controllable and observable, and with relative degree one, and with known signs of the high frequency gains.

Without lost of generality, we further assume the sign of high frequency gain is positive.

Remark 2. Systems with higher relative degrees than one can be dealt with by backstepping technique. Here, we assume that the subsystems are with relative degree one, again, for the convenience of presentation.

From Assumption 2, we can assume, without lost of generality, that

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ 0 & 0 & 0 & \ldots & 0 \end{bmatrix}, \quad b_i = \begin{bmatrix} b_{i,1}(a) \\ \vdots \\ b_{i,n_i}(a) \end{bmatrix}, \quad C_i = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T.$$

Remark 3. Since each subsystem is observable, there exists a transformation for $\{A_i(a), b_i(a), C_i(a)\}$ to the observer canonical form. We can apply the same transformation to the subsystem and absorb all the output related terms in $\bar{\phi}_i$.

The decentralized output regulation problem that we are going to solve is to find, for each subsystem, a finite dimensional system

$$\begin{cases} \dot{\mu}_i = \nu_i(\mu_i, e_i(t), u_i), & \mu_i \in \mathbb{R}^{s_i} \\ u_i = u_i(\mu_i, e_i(t)), & i = 1, \ldots, N \end{cases}$$

such that for every initial condition $\delta_i(\theta_i) \in \mathbb{R}^{n_i}, w(0) \in \Omega \subset \mathbb{R}^m, x_i(t), \mu_i(t)$ and $u_i(t)$ are bounded $\forall t \geq 0$, and $\lim_{t\to\infty} e_i(t) = 0$.

We introduce a state transformation for each subsystem as

$$\begin{cases} z_i = x_{i,2:n_i} - h_i^{-1}(a) b_{i,2:n_i}(a) x_{i,1} \\ y_i = x_{i,1} \end{cases} \tag{3}$$

with subscript $2 : n_i$ denoting the 2 to n_i elements of a vector, and $h_i = b_{i,1}$, which transforms the subsystem into

$$\begin{cases} \dot{z}_i(t) = B_i(a) z_i + \phi_i(y_1(t), \ldots, y_N(t), y_1(t-d_1), \ldots, y_N(t-d_N), w(t), a) \\ \dot{y}_i(t) = H_i(a) z_i + \psi_i(y_1(t), \ldots, y_N(t), y_1(t-d_1), \ldots, y_N(t-d_N), w(t), a) \\ \qquad\qquad + h_i(a) u_i(t) \end{cases}$$

where

$$\begin{aligned} B_i &= \begin{bmatrix} -b_{i,2}/b_{i,1} & 1 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -b_{i,n_i-1}/b_{i,1} & 0 & 0 & \ldots & 1 \\ -b_{i,n_i}/b_{i,1} & 0 & 0 & \ldots & 0 \end{bmatrix}, \\ \phi_i &= \bar{\phi}_{i,2:n} - h_i^{-1} b_{i,2:n_i} \bar{\phi}_{i,1}, \\ \psi_i &= \phi_{i,1} + h_i^{-1} b_{i,2} y, \\ H_i &= [1, \ldots, 0] \end{aligned}$$

with $H_i \in \mathbb{R}^{n_i - 1}$.

3. INVARIANT MANIFOLD AND INTERNAL MODEL DESIGN

To deal with delays in the system outputs, we have the following lemma.

Lemma 1. Given the exosystem (2) and a constant delay d, there exists a constant matrix $T(d)$ such that $w(t-d) = T(d)w(t)$.

P r o o f. From Assumption 1, we know that the eigenvalues of S are distinct with zero real parts, and therefore the eigenvalues can only take the values

$$\{0, \pm j\omega_1, \ldots, \pm j\omega_{(m-1)/2}\}$$

or

$$\{\pm j\omega_1, \ldots, \pm j\omega_{m/2}\}$$

for some positive values ω_i, $i = 1, \ldots, (m-1)/2$ or ω_i, $i = 1, \ldots, m/2$. When there is a zero, the corresponding disturbance will be a constant, and it will not be affected by the time delay. Therefore, we consider the case with no constant bias, that is, the eigenvalues of S are $\{\pm j\omega_1, \ldots, \pm j\omega_{m/2}\}$ with m as an even number. In this case, we have

$$S = D^{-1} \Sigma D$$

where $D \in \mathbb{R}^{m\times m}$ is a constant matrix, and

$$\Sigma = \mathrm{diag}\{\Sigma_1, \ldots, \Sigma_{m/2}\}$$

with

$$\Sigma_i = \begin{bmatrix} 0 & \omega_i \\ -\omega_i & 0 \end{bmatrix}.$$

From the exosystem, we have

$$w(t) = e^{Sd} w(t-d).$$

Therefore we have

$$T(d) = e^{-Sd} = D^{-1} e^{-\Sigma d} D = D^{-1} \mathrm{diag}\{e^{-\Sigma_1 d}, \ldots, e^{-\Sigma_{m/2} d}\} D$$

with

$$e^{-\Sigma_i d} = \begin{bmatrix} \cos\omega_i d & -\sin\omega_i d \\ \sin\omega_i d & \cos\omega_i d \end{bmatrix}.$$

This completes the proof. □

With the result shown in Lemma 1, we can express each $q_i(w(t-d_i), a)$ as a function of $w(t)$

$$\bar{q}_i(w, a) := q_i(w(t-d_i), a) = q_i(T(d_i)w(t), a)$$

and define

$$\begin{aligned} \phi_i^{[1]}(w,a) &:= \phi_i(q_1(w,a), \ldots, q_N(w,a), q_1(T(d_1)w(t),a), \ldots, q_N(T(d_N)w(t),a), w, a) \\ &= \phi_i(q_1, \ldots, q_N, \bar{q}_1, \ldots, \bar{q}_N, w, a), \\ \psi_i^{[1]}(w,a) &:= \psi_i(q_1(w,a), \ldots, q_N(w,a), q_1(T(d_1)w(t),a), \ldots, q_N(T(d_N)w(t),a), w, a) \\ &= \psi_i(q_1, \ldots, q_N, \bar{q}_1, \ldots, \bar{q}_N, w, a). \end{aligned}$$

We have the following lemma for the invariant manifolds and the desired feedforward control inputs.

Lemma 2. There exists an augmented exosystem

$$\dot{\eta}(t) = \bar{S}\eta(t) \tag{4}$$

where $\eta \in \mathbb{R}^{\bar{m}}$ with $\bar{m} \geq m$ such that, each $q_i(w,a)$, $\phi_i^{[1]}$ and $\psi_i^{[1]}$, $i = 1, \ldots, N$, can be expressed as

$$\begin{aligned} q_i(w(t),a) &= Q_i(a)\eta(t), & (5)\\ \phi_i^{[1]}(w(t),a) &= \Phi_i(a)\eta(t), & (6)\\ \psi_i^{[1]}(w(t),a) &= \Psi_i(a)\eta(t) & (7) \end{aligned}$$

where $Q_i \in \mathbb{R}^{1\times\bar{m}}$, $\Phi_i \in \mathbb{R}^{(n_i-1)\times\bar{m}}$ and $\Psi_i \in \mathbb{R}^{1\times\bar{m}}$ are constant matrices. Furthermore, there exist two constant matrices $\Pi_i \in \mathbb{R}^{(n_i-1)\times\bar{m}}$ and $\bar{L}_i(a) \in \mathbb{R}^{1\times\bar{m}}$, such that if

$$\pi_i(t,a) := \Pi_i(a)\eta(t) \tag{8}$$

$$l_i(t,a) := \bar{L}_i(a)\eta(t) \tag{9}$$

with $\pi_i(t,a) \in \mathbb{R}^{n_i-1}$ and $l_i(t,a) \in \mathbb{R}$, then

$$\frac{\mathrm{d}}{\mathrm{d}t}\pi_i(t,a) = B_i\pi_i(t,a) + \phi_i^{[1]}(w(t),a) \tag{10}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}q_i(w(t),a) = H(a)_i\pi_i(t,a) + \psi_i^{[1]}(w(t),a) + h_i(a)l_i(t,a). \tag{11}$$

Proof. Since $q_i(w,a)$, $\phi_i^{[1]}$ and $\psi_i^{[1]}$, $i = 1,\dots,N$, are polynomials of w, there are finite frequency components in each of the terms, and those frequency components can be combinations of frequency components contained in w. Let Ω be the set containing all the different individual frequencies that appear in $q_i(w,a)$, $\phi_i^{[1]}$ and $\psi_i^{[1]}$, $i = 1,\dots,N$, ie, $\Omega = \{0,\omega_1,\dots,\omega_{n_\omega}\}$. We can construct $\bar{S}$ as

$$\bar{S} = \mathrm{diag}\{0, s_1, \dots, s_{n_\omega}\}$$

where $s_i = \begin{bmatrix} 0 & \omega_i \\ -\omega_i & 0 \end{bmatrix}$, and $\bar{m} = 2n_\omega + 1$. If the initial value of η is set as $\eta^T(0) = [1,0,1,\dots,0,1]$, it can be shown that

$$\eta(t) = \begin{bmatrix} 1 \\ \sin\omega_1 t \\ \cos\omega_1 t \\ \vdots \\ \sin\omega_{n_\omega} t \\ \cos\omega_{n_\omega} t \end{bmatrix}.$$

Therefore, there exist Q_i, Φ_i and Ψ_i to satisfy equations (5), (6) and (7), depending on the amplitudes and phases of each frequency components in each of the elements in $q_i(w,a)$, $\phi_i^{[1]}$ and $\psi_i^{[1]}$.

Let $\Pi_i(a)$ be the unique solution of the following equation

$$\Pi_i(a)\bar{S} = B(a)_i\Pi_i(a) + \Phi_i(a)$$

which is guaranteed by the fact that $\bar{S}$ and B_i have mutually exclusive eigenvalues as each of the subsystem is of minimum phase. It is then easy to verify that $\pi_i(t,a) = \Pi_i(a)\eta(t)$ satisfies (10). It is ready to verify (11) if $\bar{L}_i(a)$ is defined as

$$\bar{L}_i(a) = h(a)^{-1}[Q_i(a)\bar{S} - H_i(a)\Pi_i(a) - \Psi_i(a)].$$

This completes the proof. □

Remark 4. In the proof of Lemma 3, the first element of $\bar{S}$ is set 0. This is for the generation of a constant bias in the desired feedforward control. If the constant bias is zero, we have $\bar{S} = \mathrm{diag}\{s_1, \ldots, s_{n_\omega}\}$. In the example included in this paper, the constant bias is zero.

Note that $\pi_i(t,a) = \Pi_i(a)\eta(t)$ is often referred to as the invariant manifold for z_i and $l_i(t,a)$ is referred to as the desired feedforward control input for output regulation for the ith subsystem.

Introduce the following transformation based on the invariant manifold with

$$\begin{cases} \tilde{z}_i(t) = z_i(t) - \pi_i(t,a), \\ e_i(t) = y_i(t) - q_i(w(t),a), \qquad i = 1, \ldots, N. \end{cases}$$

From the results shown in Lemma 2, we have the model for the control design in the next section

$$\begin{cases} \dot{\tilde{z}}_i(t) = B_i(a)\tilde{z}_i(t) + \tilde{\phi}_i(e_1(t), \ldots, e_N(t), e_1(t-d_1), \ldots, e_N(t-d_N), w(t), a), \\ \dot{e}_i(t) = H_i(a)\tilde{z}_i(t) + \tilde{\psi}_i(e_1(t), \ldots, e_N(t), e_1(t-d_1), \ldots, e_N(t-d_N), w(t), a) \\ \qquad\qquad + h_i(a)(u_i(t) - l_i(t,a)), \qquad i = 1, \ldots, N, \end{cases} \tag{12}$$

where

$$\begin{cases} \quad \tilde{\phi}_i(e_1(t), \ldots, e_N(t), e_1(t-d_1), \ldots, e_N(t-d_N), w(t), a) \\ = \phi_i(y_1(t), \ldots, y_N(t), y_1(t-d_1), \ldots, y_N(t-d_N), w(t), a) - \phi_i^{[1]}(w(t), a), \\ \quad \tilde{\psi}_i(e_1(t), \ldots, e_N(t), e_1(t-d_1), \ldots, e_N(t-d_N), w(t), a) \\ = \psi_i(y_1(t), \ldots, y_N(t), y_1(t-d_1), \ldots, y_N(t-d_N), w(t), a) - \psi_i^{[1]}(w(t), a). \end{cases}$$

For the functions $\tilde{\phi}_i$ and $\tilde{\psi}_i$, we have the following properties which are useful for control design and stability analysis.

Lemma 3. There exist positive real constants $r_{i,j}^{[1]}$ and $r_{i,j}^{[2]}$, for $i, j = 1, \ldots, N$, and a positive integer p such that

$$\|\tilde{\phi}_i\|^2 \leq \sum_{j=1}^{N} r_{i,j}^{[1]}[(e_j^2(t) + e_j^{2p}(t)) + (e_j^2(t-d_j) + e_j^{2p}(t-d_j))] \tag{13}$$

$$|\tilde{\psi}_i|^2 \leq \sum_{j=1}^{N} r_{i,j}^{[2]}[(e_j^2(t) + e_j^{2p}(t)) + (e_j^2(t-d_j) + e_j^{2p}(t-d_j))]. \tag{14}$$

P r o o f. From the definition of $\tilde{\phi}_i$, we have

$$\tilde{\phi}_i(0, \ldots, 0, 0, \ldots, 0, w(t), a) = 0.$$

As a consequence, each element of $\tilde{\phi}_i$ is a polynomial of $e_j(t)$ and $e_j(t-d_j)$ for $j = 1, \ldots, N$ without constant bias. Notice that $w(t)$ and a or their functions may appear as coefficients of the elements in $\tilde{\phi}_i$. For any given initial state $w(0)$, $w(t)$ remains bounded. Hence, each element of $\tilde{\phi}_i$ is a polynomial of $e_j(t)$ and $e_j(t-d_j)$ for $j = 1, \ldots, N$ with bounded coefficients. Notice that for any cross terms involving more than one error terms can be spitted using the property $|xy| < \frac{1}{2}(x^2+y^2)$ for any error terms x and y. Therefore, there exist $r_{i,j}^{[1]}$ depending on the actual polynomials and $w(t)$ and a and an integer p, depending on the actual polynomials, such that (13) holds. Similarly, we can establish (14).

Internal models are designed to estimate the desired feedforward control input $l_i(t,a)$. Based on the result shown in Lemma 2, we can further introduce a state transformation for the augmented exosystem for each subsystem as

$$\xi_i = M_i \eta, , \qquad i = 1, \ldots, N,$$

where $M_i \in \mathbb{R}^{\bar{m} \times \bar{m}}$ satisfies

$$M_i(a)\bar{S} - FM_i(a) = G\bar{L}_i(a)$$

with $F \in \mathbb{R}^{\bar{m} \times \bar{m}}$ and $G \in \mathbb{R}^{\bar{m}}$, and F being Hurwitz and $\{F, G\}$ being controllable. Note the unique solution of $M_i(a)$ satisfying (15) is guaranteed by the controllability of $\{F, G\}$ and the observability of $\{\bar{S}, L_i(a)\}$. Under the coordinate ξ_i, the desired feedforward control is given by

$$\begin{cases} \dot{\xi}_i(t) = F\xi_i(t) + Gl_i(t,a), \\ l_i(t,a) = L_i(a)\xi_i, \qquad i = 1, \ldots, N \end{cases}$$

where $L_i(a) = \bar{L}_i(a)M_i^{-1}(a)$. We define the internal models as

$$\dot{\hat{\xi}}_i(t) = F\hat{\xi}_i(t) + Gu_i(t), \qquad i = 1, \ldots, N. \tag{15}$$

Define an auxiliary error

$$\tilde{\xi}_i = \xi_i - \hat{\xi}_i + h_i^{-1}(a)Ge_i. \tag{16}$$

It can be shown that

$$\dot{\tilde{\xi}}_i \quad = \quad F\tilde{\xi}_i - h_i^{-1}(a)FGe_i + h_i^{-1}(a)GH_i(a)\tilde{z}_i + h_i^{-1}(a)G\tilde{\psi}_i.$$

Since F is Hurwitz, there exists a positive definite matrix $P \in \mathbb{R}^{\bar{m} \times \bar{m}}$ such that

$$F^T P + PF = -5I$$

where I is a generic notation for an identity matrix with a proper dimension. Let

$$V_{\xi,i} = \tilde{\xi}_i^T P \tilde{\xi}_i.$$

It then can be obtained that

$$\begin{aligned}\dot{V}_{\xi,i}(t) &= -5\tilde{\xi}_i^T\tilde{\xi}_i - 2\tilde{\xi}_i^T Ph_i^{-1}(a)FGe_i + 2\tilde{\xi}_i^T Ph_i^{-1}(a)GH_i(a)\tilde{z}_i + 2\tilde{\xi}_i^T Ph_i^{-1}(a)G\tilde{\psi}_i \\ &\leq -2\|\tilde{\xi}_i(t)\|^2 + r_{i,0}^{[1]}e_i^2(t) + r_{z,i}\|\tilde{z}_i(t)\|^2 \\ &\quad + \sum_{j=1}^{N} r_{i,j}^{[3]}[(e_j^2(t) + e_j^{2p}(t)) + (e_j^2(t-d_j) + e_j^{2p}(t-d_j))]\end{aligned}$$

where

$$\begin{aligned} r_{i,0}^{[1]} &\geq \|h_i^{-1}(a)PFG\|^2, \\ r_{z,i} &\geq \|h_i^{-1}(a)PGH_i(a)\|^2, \\ r_{i,j}^{[3]} &\geq \|h_i^{-1}(a)PG\|^2 r_{i,j}^{[2]}. \end{aligned}$$

□

Remark 5. When introducing desired control input for the ith subsystem for the internal model design, we assume the observability of $\{\bar{S}, L_i(a)\}$. The condition for this assumption to hold is that the desired feedforward control input $l_i(t,a)$ contains all the frequency modes specified in $\bar{S}$, ie, for all the frequency components of ω_i, $i = 1, \ldots, n_w$. The observability of $\{\bar{S}, L_i(a)\}$ and the controllability of $\{F, G\}$ ensure the unique and nonsingular solution of $M_i(a)$. If the pair $\{\bar{S}, L_i(a)\}$ is not observable, it implies that $l_i(t,a)$ does not contain some frequency components. In this case, all the frequency components that are contained in $l_i(t,a)$, are a subset of ω_i, $i = 1, \ldots, n_w$, and they can be used to form an $\bar{S}'$ with a lower dimension, say, $\bar{m}' < \bar{m}$. Then there exists an $L_i(a)' \in \mathbb{R}^{1\times\bar{m}'}$ such that $\{\bar{S}', L_i(a)'\}$ is observable, and the internal model design can still be carried out in the same way with any controllable pair $\{F', G'\}$, $F' \in \mathbb{R}^{\bar{m}'\times\bar{m}'}$ and $G' \in \mathbb{R}^{\bar{m}'}$.

4. CONTROL DESIGN

With the definition of $\tilde{\xi}_i$ in (16), the dynamics of e_i in (12) can be rewritten as

$$\begin{aligned}\dot{e}_i(t) &= H_i(a)\tilde{z}_i(t) + \tilde{\psi}_i + h_i(a)[u_i(t) - L_i(a)(\tilde{\xi}_i + \hat{\xi}_i - h_i^{-1}(a)Ge_i)] \\ &= H_i(a)\tilde{z}_i(t) + \tilde{\psi}_i - h_i(a)L_i(a)\tilde{\xi}_i + L_i(a)Ge_i + h_i(a)(u_i(t) - L_i(a)\hat{\xi}_i).\end{aligned}$$

The decentralized control for the ith sub-system is designed as

$$\begin{cases} u_i(t) = -c_i(t)(e_i(t) + e_i^{2p-1}(t)) + \hat{L}_i(t)\hat{\xi}_i(t) \\ \dot{c}_i(t) = \gamma_i(e_i^2(t) + e_i^{2p}(t)), \qquad c_i(0) \geq 0, \\ \dot{\hat{L}}_i(t) = -e_i(t)\hat{\xi}_i^T(t)\Gamma_i \end{cases} \tag{17}$$

where $\gamma_i \in \mathbb{R}$ is a positive real number and $\Gamma_i \in \mathbb{R}^{\bar{m}\times\bar{m}}$ is a positive definite matrix. Under this control law, we have

$$\begin{aligned}\dot{e}_i(t) &= H_i(a)\tilde{z}_i(t) + \tilde{\psi}_i(t) - h_i(a)L_i(a)\tilde{\xi}_i + L_i(a)Ge_i(t) \\ &\quad -h_i(a)c_i(t)(e_i(t) + e_i^{2p-1}(t)) - h_i(a)\tilde{L}_i\hat{\xi}_i(t)\end{aligned}$$

where $\tilde{L}_i = L_i - \hat{L}_i$. Let

$$V_{e,i}(t) = \frac{1}{2}e_i^2(t).$$

It can be obtained that

$$\begin{aligned}\dot{V}_{e,i}(t) &= -h_i(a)c_i(t)(e_i^2(t) + e_i^{2p}) + e_i(t)H_i(a)\tilde{z}_i(t) + e_i(t)\tilde{\psi}_i \\ &\quad -e_i(t)h_i(a)L_i(a)\tilde{\xi}_i + L_i(a)Ge_i^2 - h_i(a)\tilde{L}_ie_i(t)\hat{\xi}_i \\ &\leq -h_i(a)c_i(t)(e_i^2(t) + e_i^{2p}) + r_{i,0}^{[2]}e_i^2(t) + \|\tilde{z}_i\|^2 + \|\tilde{\xi}_i\|^2 - h_i(a)\tilde{L}_ie_i(t)\hat{\xi}_i \\ &\quad + \sum_{j=1}^{N} r_{i,j}^{[2]}[(e_j^2(t) + e_j^{2p}(t)) + (e_j^2(t-d_j) + e_j^{2p}(t-d_j))]\end{aligned}$$

where $r_{i,0}^{[2]} \geq \frac{1}{4}\|H_i(a)\|^2 + \frac{1}{4}\|h_i(a)L_i(a)\|^2 + \|L_i(a)G\| + \frac{1}{4}$.

From Assumption 2, we know that $B_i(a)$ is Hurwitz, and therefore there exists a positive definite matrix $P_i \in \mathbb{R}^{(n_i-1)\times(n_i-1)}$ satisfying

$$B_i^T(a)P_i + P_iB_i(a) = -(3 + r_{z,i})I.$$

Let

$$V_{z,i}(t) = \tilde{z}_i^T(t)P_i\tilde{z}_i(t).$$

It can be obtained from (12) and Lemma 3 that

$$\begin{aligned}\dot{V}_{z,i}(t) &= -(3 + r_{z,i})\tilde{z}_i^T(t)\tilde{z}_i(t) + 2\tilde{z}_i^T(t)P_i\tilde{\phi}_i \\ &\leq -(2 + r_{z,i})\|\tilde{z}_i(t)\|^2 + \sum_{j=1}^{N} r_{i,j}^{[4]}[(e_j^2(t) + e_j^{2p}(t)) + (e_j^2(t-d_j) + e_j^{2p}(t-d_j))]\end{aligned}$$

where $r_{i,j}^{[4]} = \|P_i\|^2 r_{i,j}^{[1]}$.

To analyze the stability for the ith subsystem, we introduce a few notations,

$$\begin{aligned}r_{i,0} &= r_{i,0}^{[1]} + r_{i,0}^{[2]}, \\ r_{i,j} &= r_{i,j}^{[2]} + r_{i,j}^{[3]} + r_{i,j}^{[4]}, \\ \bar{c}_i &= h_i^{-1}(a)\left[1 + r_{i,0} + 2\sum_{j=1}^{N} r_{j,i}\right],\end{aligned}$$

and we let

$$V_i(t) = V_{\xi,i}(t) + V_{z,i}(t) + \frac{1}{2}[e_i^2(t) + h_i(a)\gamma_i^{-1}\tilde{c}_i^2 + h_i(a)\tilde{L}_i\Gamma^{-1}\tilde{L}_i^T]$$

where $\tilde{c}_i = \bar{c}_i - c_i$, and $\tilde{L} = L_i - \hat{L}_i$. It can be obtained that

$$\begin{aligned}
\dot{V}_i(t) \quad &\leq \quad -\|\tilde{\xi}_i(t)\|^2 + r_{i,0}^{[1]} e_i^2(t) \\
&\quad + \sum_{j=1}^{N} r_{i,j}^{[3]} [(e_j^2(t) + e_j^{2p}(t)) + (e_j^2(t-d_j) + e_j^{2p}(t-d_j))] \\
&\quad - \|\tilde{z}_i(t)\|^2 + \sum_{j=1}^{N} r_{i,j}^{[4]} [(e_j^2(t) + e_j^{2p}(t)) + (e_j^2(t-d_j) + e_j^{2p}(t-d_j))] \\
&\quad - h_i(a)\bar{c}_i(e_i^2(t) + e_i^{2p}(t)) + r_{i,0}^{[2]} e_i^2(t) \\
&\quad + \sum_{j=1}^{N} r_{i,j}^{[2]} [(e_j^2(t) + e_j^{2p}(t)) + (e_j^2(t-d_j) + e_j^{2p}(t-d_j))] \\
&= \quad -\|\tilde{\xi}_i(t)\|^2 - \|\tilde{z}_i(t)\|^2 - |e_i(t)|^2 - 2\sum_{j=1}^{N} r_{j,i} [(e_i^2(t) + e_i^{2p}(t))] \\
&\quad + \sum_{j=1}^{N} r_{i,j} [(e_j^2(t) + e_j^{2p}(t)) + (e_j^2(t-d_j) + e_j^{2p}(t-d_j))].
\end{aligned}$$

To deal with delays in the system, we introduce a Lyapunov–Krasovskii functional

$$V(t) = \sum_{i=1}^{N} V_i(t) + \sum_{i=1}^{N}\sum_{j=1}^{N} r_{j,i} \int_{t-d_i}^{t} [e_i^2(\tau) + e_i^{2p}(\tau)]\, \mathrm{d}\tau.$$

From the earlier results, we have

$$\begin{aligned}
\dot{V}(t) \quad &\leq \quad -\sum_{i=1}^{N} [\|\tilde{\xi}_i(t)\|^2 + \|z_i(t)\|^2 + |e_i(t)|^2] - 2\sum_{i=1}^{N}\sum_{j=1}^{N} r_{j,i} [(e_i^2(t) + e_i^{2p}(t))] \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N} r_{i,j} [(e_j^2(t) + e_j^{2p}(t)) + (e_j^2(t-d_j) + e_j^{2p}(t-d_j))] \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N} r_{j,i} [(e_i^2(t) + e_i^{2p}(t)) - (e_i^2(t-d_i) + e_i^{2p}(t-d_i))] \\
&= \quad -\sum_{i=1}^{N} [\|\tilde{\xi}_i(t)\|^2 + \|z_i(t)\|^2 + |e_i(t)|^2].
\end{aligned}$$

This implies the boundedness of all the variables in the closed loop control system, including $\hat{\xi}_i$, $\tilde{z}_i$, e_i, $\hat{L}_i$ and c_i, and $\tilde{\xi}_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, $\tilde{z}_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $e_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, and the boundedness of $\dot{\tilde{z}}_i$, $\dot{\tilde{\xi}}_i$, $\dot{e}_i$, for $i = 1, \ldots, N$. Therefore, from Babalat's Lemma, we conclude that $\lim_{t\to\infty} \tilde{\xi}_i(t) = 0$, $\lim_{t\to\infty} \tilde{z}_i(t) = 0$ and $\lim_{t\to\infty} e_i(t) = 0$, for $i = 1, \ldots, N$.

We summarize the stability result in the following theorem.

Theorem 4. For a large scale nonlinear system (1) satisfying Assumptions 1 and 2, the decentralized control laws (17) together with the internal models (15) solve the decentralized output regulation problem with the regulated errors asymptotically converging to zero.

5. EXAMPLE

Consider a nonlinear system

$$\begin{aligned}
\dot{x}_{1,1}(t) &= x_{1,2}(t) + a_1 y_1^2(t-d_1)w_1(t) + a_2 y_2^2(t-d_2)w_1(t) + b_{1,1}(a_5)u_1(t),\\
\dot{x}_{1,2}(t) &= b_{1,2}(a_6)u_1(t),\\
y_1(t) &= x_{1,1}(t),\\
e_1(t) &= x_{1,1}(t) - w_1(t),\\
\dot{x}_{2,1}(t) &= x_{2,2}(t) + a_3 y_2^2(t-d_2)w_1(t) + a_4 y_1^2(t-d_1)w_1(t) + b_{2,1}(a_7)u_2(t),\\
\dot{x}_{2,2}(t) &= b_{2,2}(a_8)u_2(t),\\
y_2(t) &= x_{2,1}(t),\\
e_2(t) &= x_{2,1}(t) - w_1(t),\\
x_{1,1}(\theta_1) &= \delta_1(\theta_1), \theta_1 \in [-d_1, 0],\\
x_{2,1}(\theta_2) &= \delta_2(\theta_2), \theta_2 \in [-d_2, 0]
\end{aligned}$$

with the exosystem

$$\begin{aligned}
\dot{w}_1(t) &= \omega w_2(t),\\
\dot{w}_2(t) &= -\omega w_1(t)
\end{aligned}$$

where a_1, a_2, a_3, and a_4 are complete unknown constants, $b_{1,1}(a_5) > 0$, $b_{1,2}(a_6) > 0$, $b_{2,1}(a_7) > 0$, $b_{2,2}(a_8) > 0$ positive real constants with unknown values, d_1 and d_2 are unknown positive real constants for delays in y_1 are y_2, and ω in an unknown positive real constant for the frequency of the exosystem. It is easy to see that the system for the example considered above is in the form of (1) with $N = 2$ and satisfies Assumptions 1 and 2. Based on the result shown in Lemma 2, it can be established that the desired feedforward input for both subsystems can be generated an augmented exosystem with

$$\bar{S} = \begin{bmatrix} 0 & \omega & 0 & 0 \\ -\omega & 0 & 0 & 0 \\ 0 & 0 & 0 & 3\omega \\ 0 & 0 & -3\omega & 0 \end{bmatrix}.$$

It can also be shown that the results in Lemma 3 hold with $p = 2$. The internal models are designed in the form of (15) with $N = 2$ and the control designed as in (17) with $N = 2$ and $p = 2$.

In the simulation study, the parameters are set as $a_1 = a_2 = a_3 = 1.0$, $a_4 = 0.5$, $b_{1,1} = b_{2,1} = 1.0$, $b_{1,2} = 5$, $b_{2,2} = 6$, $d_1 = 1$ second, $d_2 = 1.5$ seconds, $\omega = 1$ rad/s, $\delta_1 = \delta_2 = 0$, $x_1(0) = x_2(0) = (1, 0)$, $w(0) = (1, 0)$, $c_1(0) = c_2(0) = 20$,

$L_1(0) = L_2(0) = 0$, $\gamma_1 = \gamma_2 = 1$, $\Gamma_1 = \Gamma_2 = 10000I$ and

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -16 & -32 & -24 & -8 \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \end{bmatrix},$$

with the eigenvalues of F at $\{-2, -2, -2, -2\}$. Figures 1 and 3 shows the regulated errors and the control inputs of the two subsystems, while Figures 2 and 4 shows the adaptive parameters $\hat{L}_1$ and $\hat{L}_2$, which converge to their ideal values $[0.7, 3.2, 1.4, 0.8]$ respectively. Note that the convergent $\hat{L}_1$ and $\hat{L}_2$ can be used to estimate the unknown frequency of the exosystem.

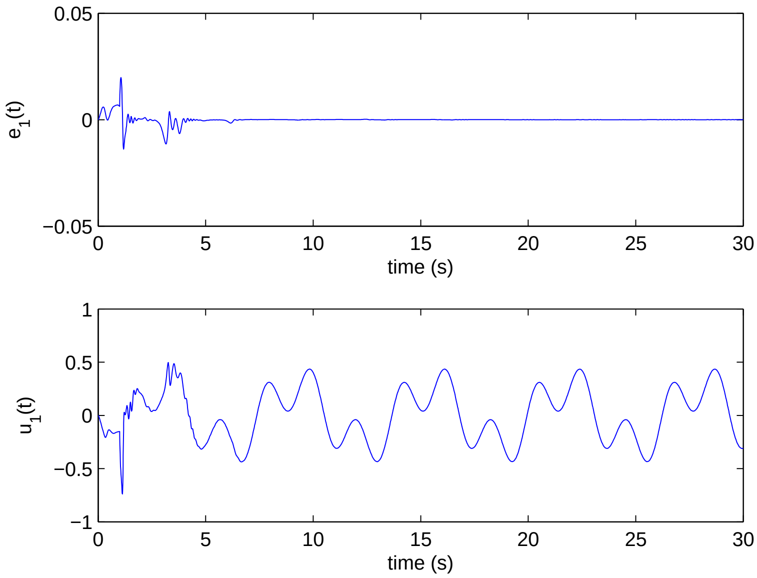

Fig. 1. The output and control input of subsystem 1.

6. CONCLUSIONS

In this paper, we have proposed a decentralized control strategy for output regulation of a class of large scale nonlinear systems with unknown time delays. The proposed control strategy makes use of various design techniques in decentralized control, output regulation and control of time delay systems. In particular, we have established the existence of augmented exosystems for generating the invariant manifolds and the desired feedforward inputs in the presence of unknown time delays. Internal models are designed based on the resultant augmented exosystems. With the proper use of adaptive control coefficients, the proposed decentralized control

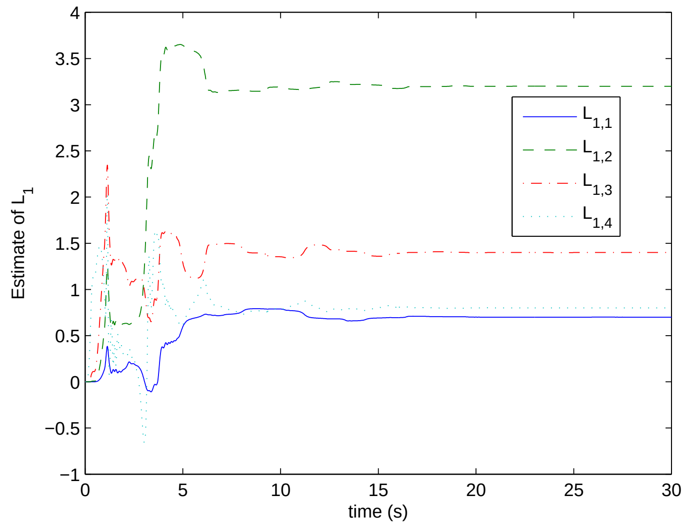

Fig. 2. Adaptive parameters for subsystem 1.

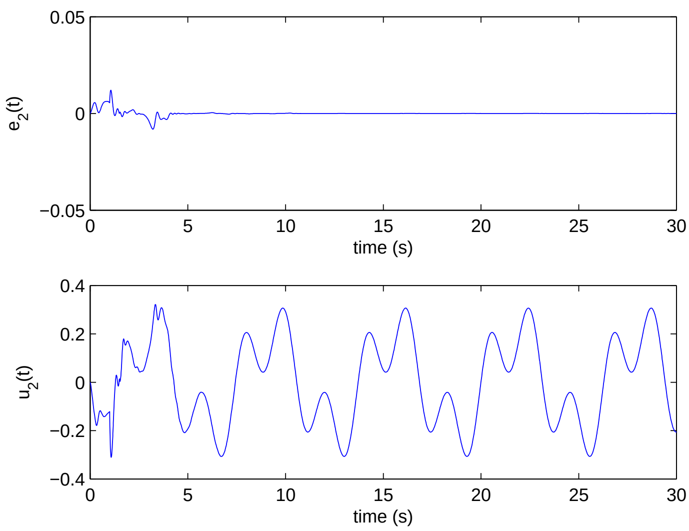

Fig. 3. The output and control input of subsystem 2.

is capable of tackling the uncertainty in the nonlinear system and the exosystem, and ensure the global stability of the closed loop nonlinear system. The regulation errors are guaranteed to converge to zero. In the simulation results for the included example, the estimates also converge to their ideal values respectively, providing the

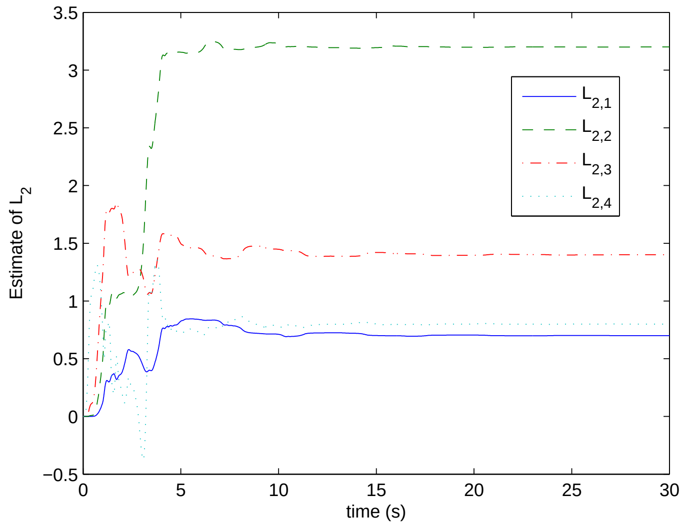

Fig. 4. Adaptive parameters for subsystem 2.

possibility to estimate unknown frequencies of the unknown disturbances.

(Received April 17, 2008.)

REFERENCES

[1] C. Chen and Z. Ding: Global output regulation of nonlinear time-delay systems with unknown exosystems. In: Proc. 17th IFAC World Congress, Seoul 2008, pp. 12141–14146.

[2] Z. Ding: Global output regulation of uncertain nonlinear systems with exogenous signals. Automatica *37* (2001), 113–119.

[3] Z. Ding: Output regulation of uncertain nonlinear systems with nonlinear exosystems. IEEE Trans. Automatic Control *51* (2006), 498–503.

[4] E. Fridman: Output regulation of nonlinear systems with delay. Systems Control Lett. *50* (2003), 81–93.

[5] D. S. Gilliam, V. I. Shubov, C. I. Byrnes, and E. D. Vugrin: Output regulation for delay systems: tracking and disturbance rejection for an oscillator with delayed damping. In: Proc. 2002 IEEE Internat. Conference on Control Application, Glasgow 2002, pp. 554–558.

[6] J. Huang and W. J. Rugh: On a nonlinear multivariable servomechanism problem. Automatica *26* (1990), 963–972.

[7] A. Isidori and C. I. Byrnes: Output regulation of nonlinear systems. IEEE Trans. Automat. Control *35* (1990), 131–140.

[8] S. Jain and F. Khorrami: Decentralized adaptive output feedback design for large-scale nonlinear systems. IEEE Trans. Automat. Control *42* (1986), 729–736.

[9] Z. P. Jiang: Decentralized and adaptive nonlinear tracking of large-scale systems via output feedback. IEEE Trans. Automat. Control *45* (2000), 2122–2128.

[10] Z. Jiang, D. W. Repperger, and D. J. Hill: Decentralized nonlinear output-feedback stabilization with disturbance attenuation. IEEE Trans. Automat. Control *46* (2001) 1623–1629.

[11] F. D. Priscoli: Output regulation with nonlinear internal models. Systems Control Lett. *53* (2004), 177–185.

[12] A. Serrani and A. Isidori: Global robust output regulation for a class of nonlinear systems. Systems Control Lett. *39* (2000), 133–139.

[13] Z. Xi and Z. Ding: Global decentralised output regulation for a class of large-scale nonlinear systems with nonlinear exosystems. IET Control Theory Appl. *1* (2007), 1504–1511.

[14] Z. Xi and Z. Ding: Global adaptive output regulation of a class of nonlinear systems with nonlinear exosystems. Automatica *43* (2007), 143–149.

[15] X. Ye and J. Huang: Decentralized adaptive output regulation for large-scale nonlinear systems. IEEE Trans. Automat. Control *48* (2003), 276–281.

Zhengtao Ding, Control Systems Centre, School of Electrical and Electronic Engineering, University of Manchester, P. O. Box 88, Manchester M60 1QD. U. K.
e-mail: zhengtao.ding@manchester.ac.uk