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DECENTRALIZED CONTROL AND SYNCHRONIZATION OF TIME–VARYING COMPLEX DYNAMICAL NETWORK

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A new class of controlled time-varying complex dynamical networks with similarity is investigated and a decentralized holographic-structure controller is designed to stabilize the network asymptotically at its equilibrium states. The control design is based on the similarity assumption for isolated node dynamics and the topological structure of the overall network. Network synchronization problems, both locally and globally, are considered on the ground of decentralized control approach. Each sub-controller makes use of the information on the corresponding node's dynamics and the resulting overall controller is composed of those sub-controllers. The overall controller can be obtained by means of a combination of typical control designs and appropriate parametric tuning for each isolated node. Several numerical simulation examples are given to illustrate the feasibility and the efficiency of the proposed control design.

Keywords: decentralized control, complex dynamical network, similarity, stabilization, synchronization

AMS Subject Classification: 34D20, 65L99, 70G60, 70K99

1. INTRODUCTION

There are many inherent complexity issues that lead to tremendous difficulties in understanding various aspects of complex networks including structural complexity, network evolution dynamics, connection diversity, node diversity, meta-complication, etc. One of the crucial findings is that the network topology affects essentially their functioning. In order to investigate further and to understand better dynamical behaviors of various complex networks, both the dynamics of each individual node and the topological connectivity of a network should be considered.

Traditionally, a network of complex topology is described by a completely random graph, the so-called E-R model, due to famous discoveries of Paul Erdös and Alfred Rényi [5]. More recently, Watts and Strogatz introduced the concept of so-called small-world network [12], which demonstrates the transition from a regular network to a random network, since many real-world complex networks are neither completely regular nor completely random. Another significant discovery in the field of complex networks is the observation that a number of complex networks are scale-free [1].

Small-world phenomenon and scale-free feature have been shown to play critical roles in complexity. More recently, a general scale-free dynamical network model was discussed in [14] first bringing in the significant result: the synchronizability of a scale-free dynamical network is robust against random removal of nodes yet it is fragile to a specific removal of the most highly connected nodes. Synchronization and dynamical behaviors in complex networks were studied further on the grounds of this model [3, 4, 6, 7, 8, 10, 15, 16, 19]. In these investigations, an essential requirement is that the structure of the network and the coupling functions are known in advance. Yet the precise state equations and other exact prior knowledge, such as coupling structures and different weighting, are usually unavailable as a result of the inevitable uncertainties and limits to phenomena measurability.

The adaptive synchronization of uncertain complex dynamical networks were investigated in [9]. Paper [20] designed several robust adaptive controllers for complex dynamical networks with unknown but bounded nonlinear couplings. Based on the stability analysis of impulsive systems for the proposed uncertain complex dynamical networks several adaptive-impulsive network synchronization criteria in local and global senses are established in [11]. The uniform and non-uniform pinning control strategies are used to stabilize scale-free networks [13, 18]. Stabilization of complex network with hybrid impulsive and switching control is discussed in [21]. Stability analysis and decentralized control problems are addressed for linear and sector-nonlinear complex dynamical networks in [22].

Motivated by these recent works, this paper proposes a new controlled uncertain time-varying complex dynamical network model and investigates its asymptotic stabilization and synchronization by designing decentralized state feedback and output feedback controllers with holographic-structure, first proposed in [17]. By exploring the combined application of Lyapunov stability theory and nonlinear robust decentralized control method [2], the present paper shows some robust decentralized controllers can indeed be designed for the same tasks as described in [9, 11, 20] and [22]. Moreover, when the node number is rather large the proposed controllers appear fairly easy to implement. For practically the controllers with the same structure may be designed first, and thereafter by tuning some transformation parameters a family of controllers can be created. This way an appropriate decentralized controller with holographic-structure in a given domain can be obtained.

This article is organized as follows. An uncertain time-varying complex dynamical network model is presented and some preliminaries are introduced in Section 2. In Section 3, the decentralized state feedback and output feedback controllers with holographic-structure are designed to stabilize the proposed complex network. At the same time, the locally and globally decentralized synchronization approaches of the network are studied and several network synchronization criteria are given. Section 4 presents application to several illustrative examples along with the respective numerical and simulation results in order to verify the theoretical results and demonstrate the effectiveness of the proposed controls. Conclusions and references follow thereafter.

2. MODEL DESCRIPTION AND PRELIMINARIES

2.1. Model description

Consider an uncertain dynamical networks consisting of N coupled nodes, with each node being an $n-$dimensional dynamical system.

$$
\begin{aligned}
\dot{x}_i(t) &= f_i(x_i(t), t) + h_i(x_1(t), x_2(t), \ldots, x_N(t), t) + G_i(x_i(t), t)u_i, \\
y_i(t) &= y_i(x_i(t)), \quad i = 1, 2, \ldots, N,
\end{aligned} \tag{1}
$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in \mathbb{R}^n$ represents the state variable of the ith node, nonlinear vector field $f_i : \Omega_i \times \mathbb{R}^+ \to \mathbb{R}^n$ is continuously differentiable, $h_i : \Omega_1 \times \ldots \times \Omega_N \times \mathbb{R}^+ \to \mathbb{R}^n$ are unknown nonlinear smooth coupling functions, $G_i(x_i(t), t) = (g_{i1}(x_i(t), t), g_{i2}(x_i(t), t), \ldots, g_{in}(x_i(t), t)), \ g_{ij} : \Omega_i \times \mathbb{R}^+ \to \mathbb{R}^n, 1 \le j \le n$ are continuous vectors.

2.2. Mathematical preliminaries

Definition 2.1.   The network (1) is said to possess similarity if its isolated nodes are similar. That is, there exist coordinates transformations $T_i : x_i \to z_i$ and feedback laws $u_i = \alpha_i(x_i) + \beta_i(x_i)\, v_i$, such that the closed-loop isolated node dynamics in the new coordinates $z = (z_1^T, z_2^T, \ldots, z_n^T)^T$ have the following structure

$$
\dot{z}_i = s(z_i, t) + \Gamma(z_i, t)\, v_i, \quad 1 \le i \le N, \tag{2}
$$

where $\alpha_i, \beta_i : \Omega_i \to \mathbb{R}^+$ are smooth vectors. Then (T_i, α_i, β_i) is called the transformation parameter of the ith isolated node of the network (1).

Definition 2.2.   The network (1) is said to be stabilized by decentralized output feedback holographic-structure controllers $u_i = u(y_i, t, T_i(x_i), \alpha(x_i), \beta(x_i)), \ x_i \in \Omega_i$ if in the closed loop both the network and the isolated nodes are asymptotically stable in the domain $\bar{\Omega} = \Omega_1 \times \ldots \times \Omega_N$. Especially, when $y_i(x_i) = x_i$, the controllers $u_i = u(x_i, t, T_i(x_i), \alpha(x_i), \beta(x_i)), \ x_i \in \Omega_i$ are said to be decentralized state feedback holographic-structure controllers.

Lemma 2.1.   Suppose that the dynamical network (1) possesses a similarity structure. Then the solution $x = 0$ of its isolated nodes can be asymptotically stabilized if and only if the solution $z = 0$ of the system (2) can be asymptotically stabilized.

Suppose that $f_i(x_i, t) = f(x_i, t)$ and the coupling-control terms satisfy $h_i(s, s, \ldots, s, t) + G_i(s, t)u_i = 0$, where s will be given as in Definition 2.3. Then we have the following rigorous definition of synchronization.

Definition 2.3.   Let $x_i = (t; t_0, X_0)$ be a solution of the dynamical network (1), where $X_0 = ((x_1^0)^T, (x_2^0)^T, \ldots, (x_N^0)^T)^T$, $f : \Omega \times \mathbb{R}^+ \to \mathbb{R}^+$, and $h_i : \Omega \times \Omega \times \ldots \times$

$\Omega \times \mathbb{R}^+ \to \mathbb{R}^n$ are continuously differentiable, $\Omega \in \mathbb{R}^n$. If there is a nonempty subset $\Lambda \subseteq \Omega$, with $x_i^0 \in \Lambda$, such that $x_i = (t; t_0, X_0) \in \Omega$ for all $t \geq t_0, 1 \leq i \leq N$, and

$$\lim_{t \to \infty} \|x_i(t; t_0, X_0) - s(t; t_0, x_0)\|_2 = 0, \tag{3}$$

where $s(t; t_0, x_0)$ is a solution of the system $\dot{x} = f(x, t)$ with $x_0 \in \Omega$, then the dynamical network (1) is said to realize synchronization and $\Lambda \times \ldots \times \Lambda$ is called the region of synchronicity for the dynamical network (1).

3. MAIN RESULTS

3.1. Decentralized state feedback

Consider a class of time-varying dynamical network model with similarity structure of system (1) consisting of N linearly and diffusively coupled non-identical nodes

$$\dot{x}_i(t) = f_i(x_i(t), t) + G_i(x_i(t), t)u_i + \sum_{j=1}^{N} c_{ij}(t)A(t)x_j(t), \quad i = 1, 2, \ldots, N, \tag{4}$$

where $A(t) \in \mathbb{R}^{N \times N}$ is the inner-coupling matrix of the network at time t, $C(t) = (c_{ij}(t))_{N \times N}$ is the coupling configuration matrix representing the coupling strength and the topological structure of the network at time t, $c_{ii}(t) = -\sum_{j=1, j \neq i}^{N} c_{ij}(t)$ and $C(t)$ is irreducible.

Remark 3.1. In practice, the node dynamics of complex networks are usually similar and most of the proposed complex network models are special cases of similarity complex dynamical networks.

Assumption 3.1. (A1) There exists some isolated dynamical node of network (4) that can be stabilized and the corresponding Lyapunov function is given by $V(T_i(x_i, t))$ with $\|\frac{\partial V(T_i(x_i, t))}{\partial T_i(x_i)}\| \leq k_2 \|x_i\|$, where k_2 is a specific positive constant.

Assumption 3.2. (A2) There exists a matrix function $W^T(t) + W(t) > 0$ in $\Omega \backslash \{0\}$ for all $t > 0$, where $W(t) = w_{ij}(t)_{N \times N}$ is defined by

$$w_{ij}(t) = \begin{cases} k_1 - k_2 |c_{ij}(t)| \|A(t)\|, & i = j, \\ -k_2 |c_{ij}(t)| \|A(t)\|, & i \neq j, \end{cases}$$

and k_1 is another positive constant.

Theorem 3.1. Suppose that A1 and A2 are satisfied. Then network (4) can be stabilized by decentralized state feedback controllers with holographic structure.

P r o o f . According to Definition 2.1, the similarity structure of the network (4) implies that there exist coordinates transformation $T_i : x_i \to z_i$ and state feedback

laws $u_i = \alpha_i(x_i) + \beta_i(x_i)\, v_i$ such that in the new coordinates $z = (z_1^T, z_2^T, \ldots, z_n^T)^T$ the closed-loop network have the following structure

$$\dot{z}_i = s(z_i, t) + \Gamma(z_i, t)\, v_i + H(z, t), \qquad (5)$$

where

$$s(z_i, t) = \left(\frac{\partial T(x_i)}{\partial x_i}\right)^T_{x_i = T^{-1}(z_i)}(f_i(T^{-1}(z_i, t)) + G_i(T^{-1}(z_i), t)\alpha_i(T^{-1}(z_i), t))$$

$$\Gamma(z_i\,, t) = \left(\frac{\partial T(x_i)}{\partial x_i}\right)^T_{x_i = T^{-1}(z_i)}(G_i(T^{-1}(z_i), t)\beta_i(T^{-1}(z_i), t))$$

$$H(z, t) = \left(\frac{\partial T(x_i)}{\partial x_i}\right)^T_{x_i = T^{-1}(z_i)}\left(f_i(T^{-1}(z_i, t))\sum_{j=1}^{N} c_{ij}(t)A(t)(T^{-1}(z_j))\right).$$

According to A1, and without loss of generality, we suppose that the first isolated node can be locally asymptotically stabilized. By Lemma 2.1, there exists a state feedback $v_1 = v(z_1, t)$ such that the zero solution of the closed-loop system

$$\dot{z}_1 = s(z_1, t) + \Gamma(z_1, t)\, v_1,$$

is asymptotically stable. Using similarity structure, the zero solutions of the closed-loop system constructed by substituting feedback $v_i = v(z_i, t), i = 1, 2, \ldots, N$ into the isolated node of network (5) are asymptotically stable. Namely, the solution $z = 0$ of the closed-loop system

$$\dot{z}_i = s(z_i, t) + \Gamma(z_i, t)\, v, \qquad (6)$$

is asymptotically stable. That is to say, there exist a positive function $V(z_i, t)$ defined in $\Omega_i \times \mathbb{R}^+$ and a positive constant k_1 such that

$$\left.\frac{\mathrm{d}V(z_i, t)}{\mathrm{d}t}\right|_{(6)} \le -k_1\|z_i\|^2. \qquad (7)$$

Upon substituting the feedback controllers $u_i = \alpha_i(x_i, t) + \beta_i(x_i, t)\, v(T_i(x_i), t)$ into (4) the following closed-loop network is obtained

$$\dot{x}_i = f_i(x_i, t) + G_i(x_i, t)\alpha_i(x_i, t) + G_i(x_i, t)\beta_i(x_i, t)\, v(T_i(x_i, t)) + \sum_{j=1}^{N} c_{ij}(t)A(t)x_j(t), \qquad (8)$$

In the next step, select the following Lyapunov function candidate

$$\bar{V}(x, t) = \sum_{i=1}^{N} V(T_i(x_i), t), \qquad (9)$$

and then the time derivative of $\bar{V}(x, t)$ along the solution of the closed-loop network (8) is found as follows:

$$\left.\frac{\mathrm{d}\bar{V}}{\mathrm{d}t}\right|_{(8)} = \sum_{i=1}^{N}\left(\frac{\partial V}{\partial t} + \frac{\partial V(T_i(x_i, t))}{\partial z_i}\right)^T (f_i(z_i, t) + G(z_i, t)\, v(z_i, t)))$$

$$+ \sum_{i=1}^{N}\left(\frac{\partial V(T_i(x_i, t))}{\partial z_i}\right)^T \sum_{j=1}^{N} c_{ij}(t)A(t)x_j$$

$$\leq \sum_{i=1}^{N} k_1 \|x_i\|^2 - \sum_{i=1}^{N}\sum_{j=1}^{N} k_2 |c_{ij}(t)| \|A(t)\| \|x_i\| \|x_j\|$$
$$= -\frac{1}{2}(\|x_1\|, \|x_2\|, \ldots, \|x_N\|)(W^T(t) + W(t))(\|x_1\|, \|x_2\|, \ldots, \|x_N\|)^T.$$

From the positive definitiveness of function matrix $W^T(t) + W(t)$, it follows that $\frac{\mathrm{d}\overline{V}}{\mathrm{d}t}\big|_{(8)} < 0$ in $\Omega \setminus \{0\}$. In turn, the asymptotic stability of the network (4) follows as well. And this completes the proof. $\qquad\square$

Now we will discuss the locally and globally decentralized state feedback synchronization of the network (1) consisting of N identical nodes with diffusively coupling, which is described by

$$\dot{x}_i(t) = f(x_i(t), t) + g_i(x_1(t), x_2(t), \ldots, x_N(t), t) + u_i, \quad 1 \leq i \leq N, \qquad (10)$$

where $g_i \in \mathbb{R}^n$ are unknown nonlinear smooth diffusive coupling functions, $u_i \in \mathbb{R}^n$ are the control inputs, and the coupling-control terms satisfy $g_i(s(t), s(t), \ldots, s(t), t) + u_i = 0$ for all $t \geq 0$ with $s(t)$ being a synchronous solution of the isolated node system $\dot{x}(t) = f(x(t), t)$. Here, $s(t)$ can be an equilibrium point, a periodic or a non-periodic orbit, or a chaotic orbit in the phase space. Then $S(t) = (s^T(t), s^T(t), \ldots, s^T(t))^T$ is a synchronous solution of the uncertain dynamical network (10).

The objective is to synchronize the uncertain complex network (10) by designing controllers u_i. That is, the trajectories of the closed-loop systems should satisfy:

$$\lim_{t\to\infty} \|x_i(t) - s(t)\|_2 = 0, \quad 1 \leq i \leq N. \qquad (11)$$

For this purpose define first the error vector by $e_i(t) = x_i(t) - s(t)$. Then the error dynamical system can be given as follows:

$$\dot{e}_i(t) = \bar{f}(x_i(t), s(t), t) + \bar{g}_i(x_1(t), x_2(t), \ldots, x_N(t), s(t), t) + u_i, \quad 1 \leq i \leq N, \quad (12)$$

where $\bar{f}(x_i(t), s(t), t) = f(x_i(t), t) - f(s(t), t)$, $\bar{g}_i(x_1(t), x_2(t), \ldots, x_N(t), s(t), t) = g_i(x_1(t), x_2(t), \ldots, x_N(t), t) - g(s(t), s(t), \ldots, s(t), t)$.

Linearization of the error system (12) around the zero state gives

$$\dot{e}_i(t) = A(t)\, e_i(t) + \bar{g}_i(x_1(t), x_2(t), \ldots, x_N(t), s(t), t) + u_i, \quad 1 \leq i \leq N, \qquad (13)$$

where $A(t) = Df(s(t), t)$ is the Jacobian of f evaluated at $s(t)$.

Obviously, it follows from there that if the pair $(A(t), I)$ is controllable, then there exist matrices $K(t), P(t) > 0, Q(t) > 0$ such that

$$\dot{P}(t) = -(A(t) + K(t))^T P(t) - P(t)(A(t) + K(t)) - Q(t). \qquad (14)$$

Assumption 3.3. (A3) There exist known first-order continuously differentiable positive definite functions $\varphi_i(\cdot)$ with $\varphi_i(0) = 0$ and nonnegative functions $r_{ij}(t)$ such that

$$\|\bar{g}_i(x_1(t), x_2(t), \ldots, x_N(t), s(t), t))\| \leq \sum_{j=1}^{N} r_{ij}(t)\varphi_i(\|e_j(t)\|), \quad 1 \leq i \leq N, \quad (15)$$

for $x(t) \in \Omega, t \in \mathbb{R}^+$.

Assumption 3.4. (A4) There exists a neighborhood about the origin of $\bar{\Omega} \subseteq \Omega$ such that the matrix function $W^T(e(t)) + W(e(t)) > 0$ in $\bar{\Omega}\backslash\{0\}$, where $W(e(t)) = (w_{ij}(e(t)))_{N \times N}$ is defined by

$$w_{ij}(e(t)) = \begin{cases} \lambda_m(Q(t)) - 2\lambda_M(P(t)) \, r_{ij}(t)\phi_i(\|e_j(t)\|), & i = j, \\ -2\lambda_M(P(t)) \, r_{ij}(t)\phi_i(\|e_j(t)\|), & i \neq j, \end{cases}$$

and where $\lambda_m(\cdot)$ and $\lambda_M(\cdot)$ stand for the smallest and largest eigenvalues respectively, $\phi_i(r) = \int_0^1 \frac{\partial \varphi_i(r\zeta)}{\partial \zeta} d\zeta$ with $r \in \mathbb{R}^+$.

Theorem 3.2. Suppose that A3 and A4 are satisfied. Then the synchronization solution $S(t)$ of the uncertain dynamical network (10) is locally asymptotically stable under the decentralized controllers

$$u_i = K(t) \, e_i(t), \quad 1 \leq i \leq N. \quad (16)$$

P r o o f . Substituting (16) into (13) gives the following closed-loop error system

$$\dot{e}_i(t) = (A(t) + K(t)) \, e_i(t) + \bar{g}_i(x_1(t), x_2(t), \ldots, x_N(t), s(t), t), \quad 1 \leq i \leq N. \quad (17)$$

Thus select the following Lyapunov function candidate

$$V(e(t)) = \sum_{i=1}^{N} e_i^T(t)P(t) \, e_i(t), \quad (18)$$

where $e(t) = (e_1^T(t), e_2^T(t), \ldots, e_N^T(t))^T$ and $P(t)$ is defined by (14). Then the time derivative of $V(e(t))$ along the solution of the closed-loop error system (17) is

$$\dot{V}(e(t)) = \sum_{i=1}^{N} \dot{e}_i^T(t)P(t) \, e_i(t) + e_i^T(t)\dot{P}(t) \, e_i(t) + e_i^T(t)P(t)\dot{e}_i(t)$$

$$= \sum_{i=1}^{N} e_i^T(t)((A(t) + K(t))^T P(t) + P(t)(A(t) + K(t)) + \dot{P}(t)) \, e_i(t) \quad (19)$$

$$+ 2 \sum_{i=1}^{N} e_i^T(t)P(t)\bar{g}_i(x_1(t), x_2(t), \ldots, x_{N(t)}, s(t), t).$$

By virtue of A3 and A4, we obtain

$$\sum_{i=1}^{N} e_i^T(t)((A(t)+K(t))^T P(t) + P(t)(A(t)+K(t)) + \dot{P}(t)) \, e_i(t)$$

$$\leq - \sum_{i=1}^{N} e_i^T(t) Q(t) \, e_i(t). \tag{20}$$

$$2 \sum_{i=1}^{N} e_i^T(t) P(t) \bar{g}_i(x_1(t), x_2(t), \ldots, x_{N(t)}, s(t), t)$$

$$\leq 2 \sum_{i=1}^{N} \sum_{j=1}^{N} \lambda_M(P(t)) \, r_{ij}(t) \phi_i(\|e_j(t)\|) \|e_i(t)\| \|e_j(t)\|. \tag{21}$$

Now, substitution of (20) and (21) into (19) yields

$$\dot{V}(e(t)) \leq - \sum_{i=1}^{N} (\lambda_m(Q(t)) \|e_i(t)\|^2 - 2 \sum_{j=1}^{N} \lambda_M(P(t)) \, r_{ij}(t) \phi_i(\|e_j(t)\|) \|e_i(t)\| \|e_j(t)\|)$$

$$= -\frac{1}{2} E(t)(W^T(e(t)) + W(e(t))) E^T(t),$$

where $E(t) = (\|e_1(t)\|, \|e_2(t)\|, \ldots, \|e_N(t)\|)$. From the positive definitiveness of function matrix $W^T(e(t)) + W(e(t))$ in $\Omega \setminus \{0\}$, it follows that $\dot{V}(e(t))$ is a negative definite function in domain Ω. Therefore, the error dynamic system (13) is locally asymptotically stabilized by the controllers (16), i.e., $\lim_{t \to \infty} \|x_i(t) - s(t)\|_2 = 0$, $1 \leq i \leq N$. Consequently, the synchronous solution $S(t)$ of uncertain dynamical network (10) is locally asymptotically stable under the decentralized controllers (16). The proof is thus completed. $\qquad \square$

Remark 3.2. The above result generalizes the average linear coupling to nonlinear coupling, and yet the proposed controllers are fairly simple in form as well as the synchronizability of the network is being reinforced.

Assume that the coupling terms of network (10) are bounded by some linear functions, namely, the inequalities (15) satisfy $\|\bar{g}_i(x_1(t), x_2(t), \ldots, x_N(t), s(t), t)\| \leq \sum_{j=1}^{N} r_{ij}(t) \|e_j(t)\|$, $1 \leq i \leq N$. Then one can obtain the following corollary.

Corollary 3.1. Suppose there exists a neighborhood about the origin of $\bar{\Omega} \subseteq \Omega$ such that the matrix function $W^T(e(t)) + W(e(t)) > 0$ in $\bar{\Omega} \setminus \{0\}$. Then the synchronization solution $S(t)$ of the network (10) with linear coupling is locally asymptotically stable under the decentralized set of controllers (16), where

$$w_{ij}(e(t)) = \begin{cases} \lambda_m(Q(t)) - 2\lambda_M(P(t)) \, r_{ij}(t) \|e_j(t)\|, & i = j, \\ -2\lambda_M(P(t)) \, r_{ij}(t) \|e_j(t)\|, & i \neq j. \end{cases}$$

In the sequel, we investigate the global decentralized synchronization of the complex network (10) in a similar way to that in [9]. We rewrite node dynamics $\dot{x}_i(t) = f(x_i(t), t)$ as $\dot{x}_i = A(t)x_i(t) + h(x_i(t), t)$, where $A(t) \in \mathbb{R}^{n \times n}$ and $h : \Omega \times \mathbb{R}^+ \to \mathbb{R}^n$ is a smooth nonlinear function. Thus network (10) is represented by the following model:

$$\dot{x}_i(t) = A(t)x_i(t) + h(x_i(t), t) + g_i(x_1(t), x_2(t), \dots, x_N(t), t) + u_i, \quad 1 \le i \le N. \tag{22}$$

As before, one can obtain the system model for the error dynamics as

$$\dot{e}_i(t) = A(t)\, e_i(t) + \bar{f}(x_i(t), s(t), t) + \bar{g}_i(x_1(t), x_2(t), \dots, x_N(t), s(t), t) + u_i, \quad (23)$$

where $\bar{f}(x_i(t), s(t), t) = h(x_i(t), t) - h(s(t), t)$.

Assumption 3.5. (A5) There exist known first-order continuously differentiable positive functions $\gamma_i(\cdot)$ with $\gamma_i(0) = 0$ such that $\|\bar{f}(x_i(t), s(t), t)\| \le \gamma_i(\|e_i(t)\|)$.

Assumption 3.6. (A6) There exists a neighborhood about origin of $\bar{\Omega} \subseteq \Omega$ such that the matrix function $W^T(e(t)) + W(e(t)) > 0$ in $\bar{\Omega} \backslash \{0\}$, where $W(e(t)) = (w_{ij}(e(t)))_{N \times N}$ is defined by

$$w_{ij}(e(t)) = \begin{cases} \lambda_m(Q(t)) - 2\lambda_M(P(t))\kappa_i(\|e_i(t)\|) - \lambda_M(P(t))\, r_{ij}(t)\phi_i(\|e_j(t)\|), & i = j, \\ -\lambda_M(P(t))\, r_{ij}(t)\phi_i(\|e_j(t)\|), & i \ne j, \end{cases}$$

for $i, j = 1, 2, \dots, N$, $\phi_i(r) = \int_0^1 \frac{\partial \varphi_i(r\zeta)}{\partial \zeta}\, \mathrm{d}\zeta$ and $\kappa_i(r) = \int_0^1 \frac{\partial \gamma_i(r\zeta)}{\partial \zeta}\, \mathrm{d}\zeta$ with $r \in \mathbb{R}^+$.

Theorem 3.3. Suppose that A1, A5 and A6 are satisfied. Then the synchronization solution $S(t)$ of the uncertain dynamical network (10) is globally asymptotically stable under the decentralized controllers (16).

P r o o f .   The proof is rather similar to that of Theorem 3.2 and thus omitted in here. $\qquad\square$

Assumption 3.7. (A7) Assume that $\bar{f}(x_i(t), s(t), t) = 0$ for $e_i(t) \in \Xi$ with $\Xi = \{(e_i(t), t) | P(t)\, e_i(t) = 0, t \in \mathbb{R}^+\}$ for $1 \le i \le N$.

Corollary 3.2. Suppose that A4, A5 and A7 are satisfied. Then the synchronization solution $S(t)$ of the uncertain dynamical network (10) is globally asymptotically stable under the decentralized controllers

$$u_i = K(t)\, e_i(t) + \rho(e_i(t)), \quad 1 \le i \le N, \tag{24}$$

where $\rho(\cdot)$ is given by

$$\rho(e_i(t)) = \begin{cases} -\frac{P(t)\, e_i(t)}{\|P(t)\, e_i(t)\|^2} \lambda_M(P(t))\gamma_i(\|e_i(t)\|)\|e_i(t)\|, & P(t)\, e_i(t) \ne 0, \\ 0, & P(t)\, e_i(t) = 0. \end{cases} \tag{25}$$

P r o o f . This proof is also rather similar to that of the Theorem 3.2 and thus omitted.   □

3.2. Decentralized output feedback

For this purpose we assume that all nodes of network (4) are identical and that an output vector is available. Then this new complex dynamical network is described by means of the following model:

$$
\begin{aligned}
\dot{x}_i(t) &= f(x_i(t), t) + G(x_i(t), t)u_i + \sum_{j=1}^{N} c_{ij}(t)A(t)x_j(t), \\
y_i(t) &= y_i(x_i(t)), \quad i = 1, 2, \ldots, N.
\end{aligned}
\tag{26}
$$

Assumption 3.8. (A8) There exist a first-order continuously differentiable positive definite function $V_i(x_i, t)$, a continuous control law $u_i = \psi(y_i, t)$ and functions $r_{i1}(\cdot), r_{i2}(\cdot)$ of class κ, such that for all $x_i \in \Omega$ and $t \in \mathbb{R}^+$

(i) $r_{i1}(\|x_i\|) \leq V_i(x_i, t) \leq r_{i2}(\|x_i\|)$,

(ii) $\dfrac{\partial V_i(x_i, t)}{\partial t} + \left(\dfrac{\partial V_i(x_i, t)}{\partial x_i} \right)^T (f(x_i, t) + G(x_i, t)\psi(y_i, t)) \leq -k_i \|x_i\|^2$,

(iii) $\left\| \dfrac{\partial V_i(x_i, t)}{\partial t} \right\| \leq \bar{k}\|x_i\|$.

Assumption 3.9. (A9) Assume the matrix function $W^T(t) + W(t) > 0$ in $\Omega \setminus \{0\}$ for all $t > 0$, where $W(t) = w_{ij}(t)_{N \times N}$ is defined by

$$
w_{ij}(t) = \begin{cases} k_i - \bar{k}|c_{ij}(t)|\|A(t)\|, & i = j, \\ -\bar{k}|c_{ij}(t)|\|A(t)\|, & i \neq j, \end{cases}
$$

and k_i are positive constants.

Theorem 3.4. Suppose A8 and A9 are satisfied. Then network (26) can be stabilized by decentralized output feedback controllers with holographic structure.

P r o o f . By substituting the output feedback controllers $u_i = \psi(y_i, t)$ into network (26), we can get the representation model of network (26) in closed-loop as follows:

$$
\begin{aligned}
\dot{x}_i &= f(x_i, t) + G(x_i, t)\psi(y_i, t) + \sum_{j=1}^{N} c_{ij}(t)A(t)x_j, \\
y_i &= y_i(x_i).
\end{aligned}
\tag{27}
$$

Next, we choose the same Lyapunov function candidate as defined in (9). Then the derivative $\bar{V}(x,t)$ along the trajectories of (27) is given by

$$
\begin{aligned}
\dot{V}(x_i,t)\big|_{(27)} &= \sum_{i=1}^{N}\left(\frac{\partial V_i(x_i,t)}{\partial t}+\frac{\partial V_i(x_i,t)}{\partial x_i}\right)^{T}(f(x_i,t)+G(x_i,t)\psi(y_i,t))) \\
&\quad + \sum_{i=1}^{N}\left(\frac{\partial V_i(x_i,t)}{\partial x_i}\right)^{T}\sum_{j=1}^{N}c_{ij}(t)A(t)x_j \\
&\leq -\left(\sum_{i=1}^{N}k_i\|x_i\|^2-\sum_{i=1}^{N}\sum_{j=1}^{N}\bar{k}|c_{ij}(t)|\|A(t)\|\|x_i\|\|x_j\|\right) \\
&= -\frac{1}{2}(\|x_1\|,\|x_2\|,\ldots,\|x_N\|)(W^T(t)+W(t))(\|x_1\|,\|x_2\|,\ldots,\|x_N\|)^{T}.
\end{aligned}
$$

From the positive definitiveness of function matrix $W^T(t)+W(t)$, it follows that $\frac{\mathrm{d}\bar{V}}{\mathrm{d}t}\big|_{(27)}<0$ in $\Omega\setminus\{0\}$. Thus the asymptotic stability of the network (26) follows in turn. This completes the proof. $\qquad\square$

4. EXAMPLES

Example 1. Consider the following controlled time-varying complex dynamical network that is consisted of 2 non-identical, third-order nodes.

$$
\begin{aligned}
\begin{pmatrix}\dot{x}_{11}\\ \dot{x}_{12}\\ \dot{x}_{13}\end{pmatrix} &= \begin{pmatrix}0\\ x_{11}+x_{12}^2\\ x_{11}-x_{12}\end{pmatrix} - 0.025\cos(t)A(t)\begin{pmatrix}x_{11}\\ x_{12}\\ x_{13}\end{pmatrix} \\
&\quad + 0.025\cos(t)A(t)\begin{pmatrix}x_{21}\\ x_{22}\\ x_{23}\end{pmatrix} + \begin{pmatrix}e^{x_{12}}\\ e^{x_{12}}\\ 0\end{pmatrix}u_1,
\end{aligned}
$$

$$
\begin{aligned}
\begin{pmatrix}\dot{x}_{21}\\ \dot{x}_{22}\\ \dot{x}_{23}\end{pmatrix} &= \begin{pmatrix}x_{21}-x_{23}\\ -3x_{23}^5+3x_{23}^2+x_{21}\\ x_{21}-x_{23}^3\end{pmatrix} + 0.03\sin(t)A(t)\begin{pmatrix}x_{11}\\ x_{12}\\ x_{13}\end{pmatrix} \\
&\quad - 0.03\sin(t)A(t)\begin{pmatrix}x_{21}\\ x_{22}\\ x_{23}\end{pmatrix} + \begin{pmatrix}0\\ x_{22}^2+1\\ 0\end{pmatrix}u_2,
\end{aligned}
$$

where $A(t)=\begin{pmatrix}1+e^{-t} & 0 & 0\\ 0 & 1+e^{-2t} & 0\\ 0 & 0 & 1+e^{-3t}\end{pmatrix}$.

A straightforward calculation yields: $\alpha_1=-2x_{12}(x_{11}+x_{12}^2)/(1+2x_{12})\,e^{x_{12}}$, $\beta_1=-((1+2x_{12})\,e^{x_{12}})^{-1}$, $T_1:z_{11}=x_{13},z_{12}=x_{11}-x_{12},z_{13}=-x_{11}-x_{12}^2$ and $\alpha_2=(x_{23}^3-x_{23}-x_{22})/(x_{22}^2+1)$, $\beta_2=-(x_{22}^2+1)^{-1}$, $T_2:z_{21}=x_{21},z_{22}=x_{21}-x_{23},z_{23}=x_{23}^3-x_{23}-x_{22}+x_{21}$.
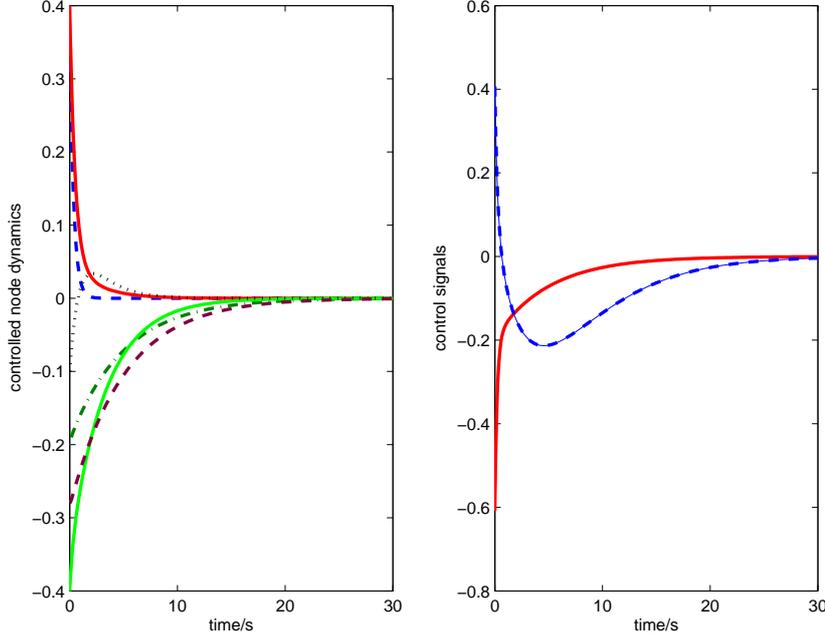
Fig. 1. Evolution of state variables and control signals.

The first dynamical node can be stabilized by a feedback $v_1 = -z_{11} - 2z_{12} - 3z_{13}$, and the corresponding Lyapunov function is $V(z) = z^T P z$ with

$$P = \begin{pmatrix} 2.3 & 2.1 & 0.5 \\ 2.1 & 4.6 & 1.3 \\ 0.5 & 1.3 & 0.6 \end{pmatrix},$$

$$W(t) = \begin{pmatrix} 1 - 0.62|\sin(t)|(1 + e^{-t}) & 0.62|\sin(t)|(1 + e^{-t}) \\ 0.37|\cos(t)|(1 + e^{-t}) & 1 - 0.37|\cos(t)|(1 + e^{-t}) \end{pmatrix}$$

is positive definite for any $t > 0$, $k_1 = 1, k_2 = 12.3654$. The conditions of the Theorem 3.1 are satisfied. Therefore, the above network can be stabilized by the following decentralized state feedback controller with holographic structure

$$u_1 = \frac{-2x_{11}x_{12} - 2x_{12}^3 + x_{13} - x_{11} - 3x_{12}^2}{(1 + 2x_{12})\, e^{x_{12}}}, \quad u_2 = \frac{-4x_{23}^4 + 6x_{23} + 4x_{22} - 6x_{21}}{-(x_{22}^2 + 1)}.$$

We choose the initial state as $x^0 = (0.35, 0.4, -0.2, -0.3, -0.1, -0.28)$ and obtain the simulation results that are depicted in Figure 1.

Example 2. Consider the following controlled time-varying complex dynamical network that is consisted of 5 identical second-order nodes.

$$
\begin{pmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \end{pmatrix} = \begin{pmatrix} -x_{i2} \\ -\sin(-x_{i1}+2x_{i2}) \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_1
$$

$$
+ \sum_{j=1}^{5} c_{ij}(t) \begin{pmatrix} \frac{2t+\sin(t)}{t} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_{j1} \\ x_{j2} \end{pmatrix},
$$

$$
y_i = x_{i1} - 2x_{i2}, \quad i = 1,2,\ldots,5,
$$

where we define $C(t) = (c_{ij}(t))_{5\times 5}$,

$$
C(t) = 0.01 \times \begin{pmatrix}
-4\sin(t) & \sin(t) & \sin(t) & \sin(t) & \sin(t) \\
\cos(t) & -4\cos(t) & \cos(t) & \cos(t) & \cos(t) \\
3\sin(t) & 3\sin(t) & -12\sin(t) & 3\sin(t) & 3\sin(t) \\
3\cos(t) & 3\cos(t) & 3\cos(t) & -12\cos(t) & 3\cos(t) \\
4\sin(t) & 4\sin(t) & 4\sin(t) & 4\sin(t) & -16\sin(t)
\end{pmatrix}
$$

Let $u_i = \sin(-x_{i1}+2x_{i2}) + x_{i1} - 2x_{i2} = \sin(y_i) - y_i$, $i = 1,2,\ldots,5$, $V(z_1,z_2) = 3z_1^2 - 2z_1 z_2 + z_2^2$. A direct computation gives: $r_{i1}(\tau) = r_{i2}(\tau) = (2 - \sqrt{2})\tau^2$, $i = 1,2,\ldots,5$, $k_i = 3$, $i = 1,2,\ldots,5$, $\bar{k} = 7$,

$$
W(t) = \begin{pmatrix}
3 & 0 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 & 0 \\
0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 3
\end{pmatrix}
$$

$$
+ \begin{pmatrix}
-0.28|\sin(t)| & 0.07|\sin(t)| & 0.07|\sin(t)| & 0.07|\sin(t)| & 0.07|\sin(t)| \\
0.07|\cos(t)| & -0.28|\cos(t)| & 0.07|\cos(t)| & 0.07|\cos(t)| & 0.07|\cos(t)| \\
0.21|\sin(t)| & 0.21|\sin(t)| & -0.84|\sin(t)| & 0.21|\sin(t)| & 0.21|\sin(t)| \\
0.21|\cos(t)| & 0.21|\cos(t)| & 0.21|\cos(t)| & -0.84|\cos(t)| & 0.21|\cos(t)| \\
0.28|\sin(t)| & 0.28|\sin(t)| & 0.28|\sin(t)| & 0.28|\sin(t)| & -1.12|\sin(t)|
\end{pmatrix}
$$

is positive definite for any $t > 0$. The conditions of the Theorem 3.4 are satisfied. Therefore, the above network can be stabilized by the decentralized output feedback controller.

We choose the initial state as $x^0 = (-0.5, 0.5, 0.6, -0.4, 0.4, 0.8, 0.3, -0.3, 0.25, 0.7)$ and the simulation results are depicted in Figure 2.

Example 3. Consider now a dynamical network that is consisted of 5 identical second-order nodes, which are described by

$$
\begin{pmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} x_{i1} \\ x_{i2} \end{pmatrix} + \begin{pmatrix} \sin^2(x_{i1})x_{i2} \\ \cos(x_{i2})x_{i1} \end{pmatrix} + u_i
$$

$$
+ \sum_{j=1}^{5} c_{ij}(t) \begin{pmatrix} \cos^2(x_{i2}) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_{j1} \\ x_{j2} \end{pmatrix}, \quad i = 1,2,\ldots,5,
$$
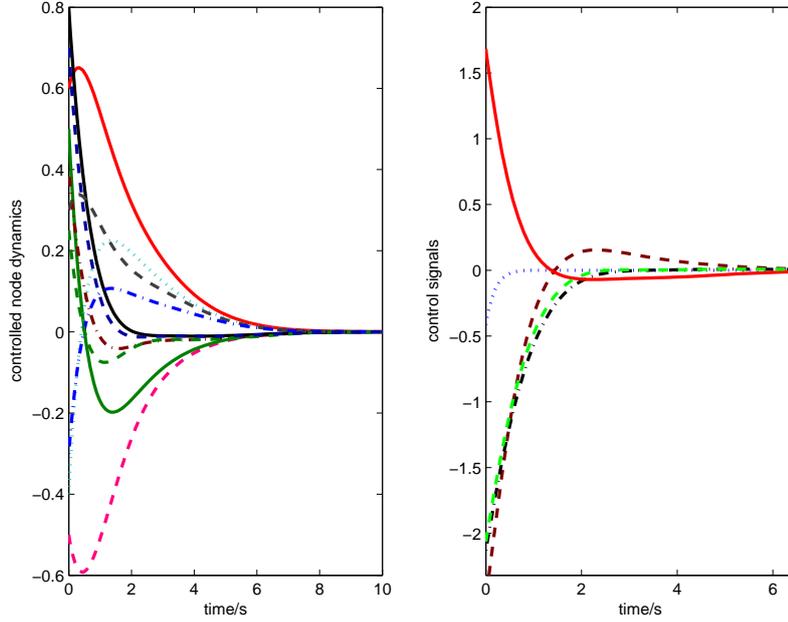
Fig. 2. Evolution of state variables and control signals.

where we define $C(t) = (c_{ij}(t))_{5 \times 5}$,

$$C(t) = 0.5 \times \begin{pmatrix} 5\sin(\pi/t) & \sin(t)\,e^{-t} & \cos(t)\,e^{-2t} & \sin(t) & \cos(t) \\ \cos(t) & 5\sin(\pi/t) & \sin(t)\,e^{-t} & \cos(t)\,e^{-2t} & \sin(t) \\ \sin(t) & \cos(t) & 5\sin(\pi/t) & \sin(t)\,e^{-t} & \cos(t)\,e^{-2t} \\ \cos(t)\,e^{-2t} & \sin(t) & \cos(t) & 5\sin(\pi/t) & \sin(t)\,e^{-t} \\ \sin(t)\,e^{-t} & \cos(t)\,e^{-2t} & \sin(t) & \cos(t) & 5\sin(\pi/t) \end{pmatrix}$$

For the control design and computer simulation, the controller gain matrix and other parameter matrix are chosen as follows:

$$K(t) = \begin{pmatrix} -3 & 1 \\ 2 & -4 \end{pmatrix}, \quad P(t) = \begin{pmatrix} 0.6406 & 0.2031 \\ 0.2031 & 0.5156 \end{pmatrix}.$$

By direct calculation we have

$$Q(t) \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix},$$

$$W(t) = -0.4 \times \begin{pmatrix} w_{11}(t) & |\sin(t)|e^{-t} & |\cos(t)|e^{-2t} & |\sin(t)| & |\cos(t)| \\ |\cos(t)| & w_{22}(t) & |\sin(t)|e^{-t} & |\cos(t)|e^{-2t} & |\sin(t)| \\ |\sin(t)| & |\cos(t)| & w_{33}(t) & |\sin(t)|e^{-t} & |\cos(t)|e^{-2t} \\ |\cos(t)|e^{-2t} & |\sin(t)| & |\cos(t)| & w_{44}(t) & |\sin(t)|e^{-t} \\ |\sin(t)|e^{-t} & |\cos(t)|e^{-2t} & |\sin(t)| & |\cos(t)| & w_{55}(t) \end{pmatrix}$$
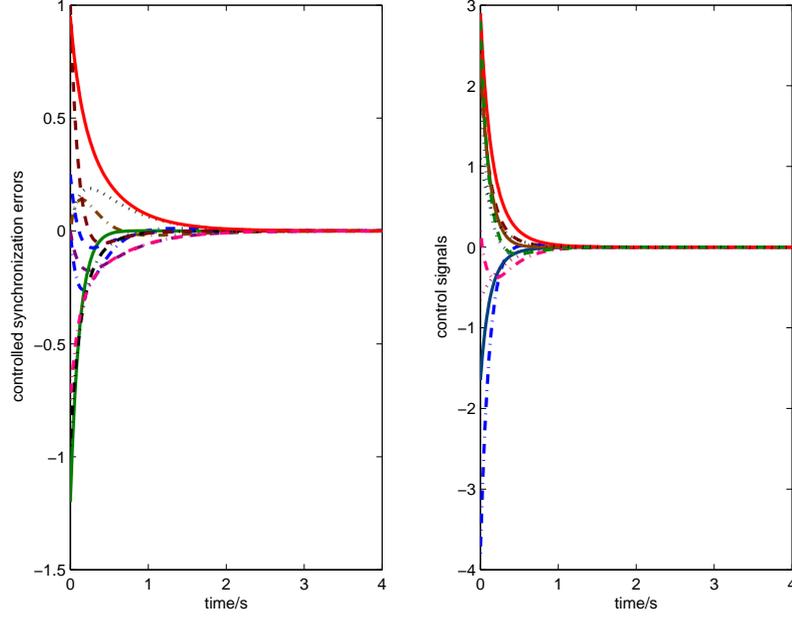
Fig. 3. Synchronization errors and control signals.

$\|\bar{h}_1(x,s,t)\|_2 \leq 0.25|\sin(\pi/t)|\|e_1\|_2 + 0.5|\sin(t)|e^{-t}\|e_2\|_2 + 0.5|\cos(t)|e^{-2t}\|e_3\|_2 + 0.5|\sin(t)|\|e_4\|_2 + 0.5|\cos(t)|\|e_5\|_2$, $\|\bar{h}_2(x,s,t)\|_2 \leq 0.5|\cos(t)|\|e_1\|_2 + 0.25|\sin(\pi/t)| \|e_2\|_2 + 0.5|\sin(t)| \ e^{-t}\|e_3\|_2 + 0.5|\cos(t)|e^{-2t}\|e_4\|_2 + 0.5|\sin(t)|\|e_5\|_2$, $\|\bar{h}_3(x,s,t)\|_2 \leq 0.5|\sin(t)|\|e_1\|_2 + 0.5|\cos(t)|\|e_2\|_2 + 0.25|\sin(\pi/t)| \ \|e_3\|_2 + 0.5|\sin(t)| \ e^{-t}\|e_4\|_2 + 0.5|\cos(t)|e^{-2t}\|e_5\|_2$, $\|\bar{h}_4(x,s,t)\|_2 \leq 0.5|\cos(t)|e^{-2t}\|e_1\|_2 + 0.5|\sin(t)|\|e_2\|_2 + 0.5 |\cos(t)|\|e_3\|_2 + 0.25|\sin(\pi/t)| \ \|e_4\|_2 + 0.5|\sin(t)| \ e^{-t}\|e_5\|_2$, $\|\bar{h}_5(x,s,t)\|_2 \leq 0.5|\sin(t)| \ e^{-t}\|e_1\|_2 + 0.5|\cos(t)|e^{-2t}\|e_2\|_2 + 0.5|\sin(t)|\|e_3\|_2 + 0.5|\cos(t)|\|e_4\|_2 + 0.25|\sin(\pi/t)| \ \|e_5\|_2$, $\|\bar{g}_i(x_i,s,t)\|_2 \leq \|e_i\|_2$, $\lambda_M(P(t)) = 0.7906$, $\lambda_m(Q(t)) = 6$.
Thus $W(t)$ is positive definite for all $t > 0$, where $w_{ii}(t) = 11.05 - 5|\sin(\pi/t)|, i = 1, 2, \ldots, 5$.

It is readily seen above that the conditions of the Theorem 3.3 are satisfied. We choose the initial state as $x^0 = (-0.5, 0.7, 0.45, -0.3, -0.45, 0.5, 0.45, 0.3, 0.2, -0.35)$ and obtain the synchronization errors e_i shown in Figure 3.

5. CONCLUSIONS

A representation model for a class of controlled time-varying complex dynamical networks with similarity structure is proposed in this article. For this class of dynamical networks, the both decentralized state feedback and output feedback controllers with holographic-structure that asymptotically stabilize the network in closed loop are designed. Two stabilization criteria have been proved by using Lyapunov stability theory. Furthermore, the related synchronization problems were also investigated

hence several criteria for local and global network synchronization on the grounds of decentralized state feedback control were proposed too. In comparison with the relevant literature, the assumptions adopted to derive the presented main results are more general.

The proposed control designs are composed of a number of such sub-controllers, which not only possess decentralization but also the holographic property too. Each sub-controller possesses all the structural information of the others. More precisely, all sub-controllers share the same structure except for the different transformation parameters. Therefore once a sub-controller is designed, also all the others are obtained too. Practically, controllers with the same structure may be designed and thereafter adjusted by tuning some transformation parameters to create a family of controllers. An appropriate decentralized controller with holographic-structure in a given domain can be obtained this way. The control infrastructure is thus fairly easy to implement.

Three typical examples were explored and the respective computer simulations given. These demonstrate the effectiveness of the proposed control designs and the achievable performance. It should be noted, the effectiveness of the proposed method is even better exploited in cases with large node numbers.

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