KYBERNETIKA — VOLUME *44* (2008), NUMBER 4, PAGES 455 – 468

# ANALYSIS OF AN ON–OFF INTERMITTENCY SYSTEM WITH ADJUSTABLE STATE LEVELS

Shi-Jian Cang, Zeng-Qiang Chen and Zhu-Zhi Yuan

We consider a chaotic system with a double-scroll attractor proposed by Elwakil, composing with a second-order system, which has low-dimensional multiple invariant subspaces and multi-level on-off intermittency. This type of composite system always includes a skew-product structure and some invariant subspaces, which are associated with different levels of laminar phase. In order for the level of laminar phase be adjustable, we adopt a nonlinear function with saturation characteristic to tune the range of a certain state variable so that the number and position of the laminar phase level can be arbitrarily controlled. We find that there exist many interesting statistical characteristics in this complex system, such as the probability distribution of the laminar lengths with $-3/2$ exponent in the power law and random jumping of the system trajectories.

*Keywords:* on-off intermittency, multi-state, invariant subspace, control analysis, statistical analysis

*AMS Subject Classification:* 37C70, 62J09, 93C10

## 1. INTRODUCTION

Because of the chaotic dynamical behavior, interpreted by theory and observed in practice, a simple system may exhibit many exceptional phenomena such as aperiodic switching between laminar states and chaotic bursts of oscillation. Sustained alternation between these two distinct states is called intermittency. The theory of intermittency, when being applied to practice, can help us to better understand some abnormal phenomena, such as the variability of solar and stellar in astronomy [17], the motion of excitable brain cells and of neuronal firing in neuroscience [5, 13], the argon measure in a gas discharge plasma system [6, 7], the driven electro-hydrodynamic convection in nematics [9], and so on.

On-off intermittency, which has been studied recently [1, 14, 15, 16], is one kind of intermittency. This intermittency is related to the blowout bifurcation and the transverse instability of chaotic attractors confined to a manifold whose dimension is smaller than that of the full phase space [2, 10]. This structure of the phase space is typical for dynamical systems with symmetry. As a parameter varies across a threshold, a blowout bifurcation takes place, and the attractor becomes transversely unstable. Just after losing the transverse stability of the invariant subspace, the system orbit can still wander for a long time near the invariant subspace. If there are no other attractors offside the invariant subspace, the orbit being pushed away from the invariant subspace will eventually return to its neighborhood. If there are some other attractors come over, the orbit will jump out of their neighborhood randomly and will move back to its neighborhood again for a while. The former is single-level intermittency and the latter is multiple-level intermittency. Single and multiple are associated with riddled basins [12, 14] and intermingled basins [11], respectively. On-off intermittency with a single level has been explored [1, 12, 14, 15, 16], but on-off intermittency with multiple levels has received relatively little attention [8, 11].

Our work lies in designing a new system to generate on-off intermittency with adjustable multiple laminar state levels in this paper. We adopt a chaotic system with double-scroll attractors as the drive system and construct a simple second-order nonlinear system as the response system. We thus obtain a typical on-off intermittency system by composing the drive system and the response system together. Moreover, we design an efficient strategy based on the model to control the range of a state variable. By doing so, single or multiple levels intermittent chaos can be generated. The different levels and positions of the laminar state can be governed by the threshold of a nonlinear saturation function. Therefore, the number of invariant subspaces and the emergent positions of the laminar state can be controlled arbitrarily. Numerical simulations are given to confirm the single or multiple level intermittent phenomena. Finally, the statistical analysis on the multi-level on-off intermittency, including the analysis of the power law and the strength of random jumping will be and discussed.

## 2. MODEL OF A MULTI–LEVEL ON–OFF INTERMITTENCY SYSTEM

### 2.1. Model construction

In this section, we introduce the construction of the system model and the conditions for generating on-off intermittency phenomenon from the model. The model construction includes two parts. One is the drive system and the other is the response system. We adopt a chaotic system proposed by Elwakil [4], with a double-scroll chaotic attractor, as the drive system, which is described by

$$\begin{cases} \dot{x} = y \\ \dot{y} = z \\ \dot{z} = -a(x + y + z - f(x)) \end{cases} \tag{1}$$

where

$$f(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0. \end{cases} \tag{2}$$

It has been proved [4] that the system produces a double-scroll chaotic attractor for $a = 0.8$. Here, we construct the following second-order system as the response system:

$$\begin{cases} \dot{u} = 2v \\ \dot{v} = -0.05v - 2g(u)(x - b) \end{cases} \tag{3}$$

where parameter $b \in \mathbb{R}$ and $g(u)$ is a modulating function and satisfying

$$\begin{cases} g(u) = -g(-u) \\ g(u) = g(u+T). \end{cases} \tag{4}$$

Drive signal $x$ is from the simple chaotic system (1). Composing system (1) and system (3) together, we obtain a system with five first-order differential equations as follows.

$$\begin{cases} \dot{x} = y \\ \dot{y} = z \\ \dot{z} = -a(x+y+z-f(x)) \\ \dot{u} = 2v \\ \dot{v} = -0.05v - 2g(u)(x-b). \end{cases} \tag{5}$$

### 2.2. Skew-product structure

On-off intermittency system always contains a skew-product structure [3] and a low-dimension invariant subspace. Consider the following system:

$$\begin{cases} \dot{s} = p(s) \\ \dot{r} = q(s,r). \end{cases} \tag{6}$$

Here, $s, r \in \mathbb{R}^n$ are state variables. Obviously, $s$ is independent of $r$. Such type of systems is said to have a skew-product structure. Comparing system (6) to system (5), the first and second expressions of system (6) are related to system (1) and system (3), respectively.

The skew-product structure can help us to find invariant subspaces. If we let $r = 0$, then one or more invariant subspaces are obtained. Clearly, system (5) has a skew-product structure. Moreover, there exist invariant subspaces in its full phase space.

### 2.3. Invariant subspaces and transverse Lyapunov exponents

Consider system (5) with five state variables, and let $\dot{u} = 0$, $\dot{v} = 0$. Then a class of invariant subspaces are obtained because of the existence of the periodic odd function $g(u)$. They are

$$M = \{(x,y,z,u,v) \,|\, g(u) = 0,\ v = 0\} \tag{7}$$

where function $g(u)$ determines the number of invariant subspaces, which provides us a strategy to control the laminar state levels of the on-off intermittency system in later discussions.

Since each subspace is invariant, initial conditions result in trajectories which can not run away from the subspace ultimately. If the largest Lyapunov exponent is positive for some given parameters, the attractors in all subspaces are chaotic attractors and they are also attractors of the whole system. When the largest transversely Lyapunov exponent(TLE) is negative, all invariant subspaces attract trajectories transversely in the phase space and all the chaotic attractors are the global attractors in the whole phase space. When the largest TLE is positive, trajectories in the vicinity of any subspaces are repelled away from the subspace. Consequently, the attractors in all subspaces are unstable and they are hence not attractors of the whole system. In this case, attractors for trajectories are restricted to subspaces. In practice, we are interested in the changes of parameters where the largest TLE is changing from negative to positive, which can help us to analyze the mechanism of generating on-off intermittency.

Knowing the relation between invariant subspaces and TLE with special parameters, we may calculate the TLE in light of different invariant subspaces. Considering system (5) and its invariant subspaces (7), we obtain

$$\begin{cases} \delta\dot{u} = 2\delta v \\ \delta\dot{v} = -0.05\delta v - 2(x-b)\dot{g}(u)\delta u \end{cases} \tag{8}$$

where variable $x$ in the invariant subspace (7) is produced by system (1) and acts as the drive signal of system (3). The TLE $h_\perp$ is computed via the following formula [11], based on system (1) and expression (8):

$$h_\perp = \lim_{t\to\infty} \frac{1}{t} \ln \frac{\delta(t)}{\delta(0)} \tag{9}$$

where $\delta(t) \equiv \sqrt{(\delta u(t))^2 + (\delta v(t))^2}$.

Because we do not know the explicit form of the continuous periodic odd function $g(u)$, we use the sine function, which is a typical continuous periodic odd function, as the modulating function. Of course, there are many functions satisfying with condition (4), but we only choose a simple function to describe my approach. In order to analyze the characteristic transition of attractors in invariant subspaces along with the change of the parameter $b$, we construct $g(u)$ as follows.

$$g(u) = \sin(\gamma u) \tag{10}$$

where $\gamma$ is a periodic control parameter, and the period $T$ should satisfy the following expression:

$$T = \frac{2\pi}{\gamma}. \tag{11}$$

If we choose $\gamma = 1$, then period $T = 2\pi$ will be obtained. Therefore, we can deduce two classes of invariant subspaces, $M_1$ and $M_2$, from function (10). The two classes of invariant subspaces are

$$M_1 = \{(x,y,z,u,v)|u = 2k\pi, v = 0; k = 0, \pm1, \pm2, \ldots\} \tag{12}$$

$$M_2 = \{(x,y,z,u,v)|u = (2k+1)\pi, v = 0; k = 0, \pm1, \pm2, \ldots\}. \tag{13}$$

For invariant subspaces $M_1$, we obtain two first-order differential equations which can be used to calculate the TLE $h_\perp^1$, as follows:

$$\begin{cases} \delta \dot{u} = 2\delta v \\ \delta \dot{v} = -0.05\delta v - 2(x-b)\delta u. \end{cases} \tag{14}$$

For $M_2$, similarly, we can obtain two first-order differential equations which can be used to calculate the TLE $h_\perp^2$, as follows:

$$\begin{cases} \delta \dot{u} = 2\delta v \\ \delta \dot{v} = -0.05\delta v + 2(x-b)\delta u. \end{cases} \tag{15}$$

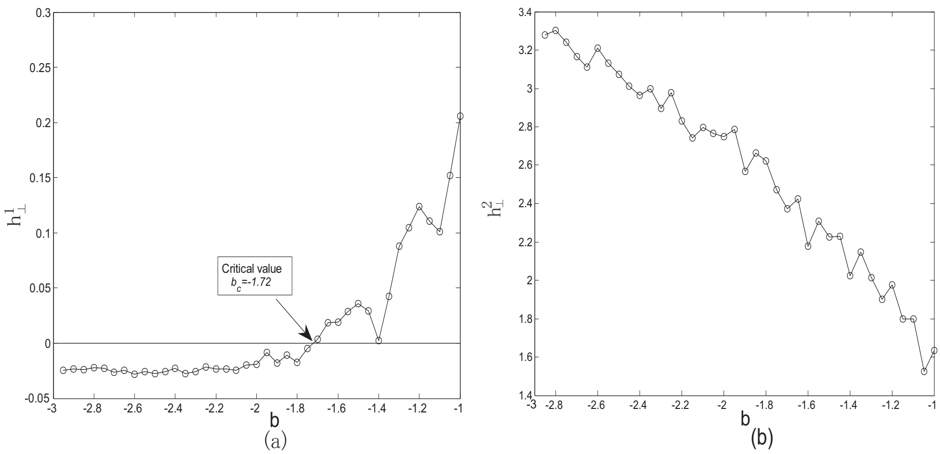


**Fig. 1.** TLEs of $M_1$ and $M_2$ versus b.

Figure 1(a) and Figure 1(b) show that the two TLEs $h_\perp^1$ and $h_\perp^2$, which belong to different subspaces $M_1$ and $M_2$, vary along with parameter $b$. It can be easily found that $h_\perp^1$ varies from negative to positive at the critical point $b_c = -1.72$ and $h_\perp^2$ keeps being positive as $b$ increases in the interval $[-3, -1]$.

The TLE obtained on invariant subspaces is a scale which can help us to find on-off intermittency. When the TLE $h_\perp^1$ is near to zero, intermittency can be found. However, the invariant subspaces discussed above are related to parameter $\gamma$. When parameter $\gamma$ is changed, the TLEs will be changed. The critical parameter $b_c$ at blowout bifurcation point is also fluctuant when the TLE is traversing to zero. In order to discuss the problem of on-off intermittency expediently, we choose $\gamma = 1$.

Since the drive system (1) is odd and symmetric, parameter $b$ can be substituted by $-b$ as we substitute $x$ by $-x$ , $y$ by $-y$ and $z$ by $-z$. Therefore, the TLE of the invariant space $M_1$ with $b$ is equal to that of $M_2$ with $-b$. Similarly, the critical point parameter $b_c$ can be substituted by $-b_c$ in $M_2$.

## 3. CONTROL ANALYSIS OF THE LAMINAR PHASE LEVELS

Because $h_{\perp}^{2}$ keeps being positive as $b$ increases in the interval $[-3,-1]$, trajectories in the vicinity of the $M_2$ subspaces are repelled away from the subspace. Therefore, in the following set (16) there will not be attractors:

$$M_2 = \{(u,v)|u = (2k+1)\pi,\ v = 0;\ k = 0, \pm 1, \pm 2, \ldots\}. \tag{16}$$

However, $h_{\perp}^{1}$ varies from negative to positive at the critical point $b_c$ belonging to the interval $[-3,-1]$, trajectories near the invariant subspace $M_1$ result in transverse instability of attractors in the following set:

$$M_1 = \{(u,v)|u = 2k\pi, v = 0; k = 0, \pm 1, \pm 2, \ldots\}. \tag{17}$$

When the TLE $h_{\perp}^{1}$ changes to be slightly positive, on-off intermittency will emerge, so the trajectories get arbitrarily close to an attractor in the invariant subspace (laminar phase), intermittently yielding large deviations (burst phase). In order to obtain better simulation results, we choose $b = -1.7$ (namely, $h_{\perp}^{1}$ becomes slightly positive) in the following analysis.

If we can control the range of the state variable $u$, the number of attractors or the levels of the laminar phase will be confirmed. In view of the fact that laminar states can be controlled by state variable $u(t)$, how can we realize the control in the mathematical model? By means of theoretical analysis and numerical simulations, we choose a saturation function to realize the variable control of $u(t)$.

During the course of numerical simulations, a simple odd saturation function is found to satisfy the requirement of control. The saturation function is

$$G(u(t)) = \begin{cases} Th^+ & u \ge Th^+ \\ u(t) & Th^- < u < Th^+ \\ Th^- & u \le Th^- \end{cases} \tag{18}$$

where the state variable $u(t)$ of system (5) is the input signal and $Th^-$, $Th^+$ are the lower threshold and upper threshold, respectively. Figure 2 shows the transfer characteristic of the saturation function.

The input signal of $G(u)$ is $u(t)$, and the output of $G(u)$ may be regarded as the state variable of function $g(\cdot)$ in the original system (5). Namely, the function $g(u)$ of system (5) is $\sin(G(u))$ in our numerical simulations, which still satisfies the equation (4). Therefore, the following system can be used to replace system (5):

$$\begin{cases} \dot{x} = y \\ \dot{y} = z \\ \dot{z} = -a(x + y + z - f(x)) \\ \dot{u} = 2v \\ \dot{v} = -0.05v - 2g(G(u))(x - b). \end{cases} \tag{19}$$

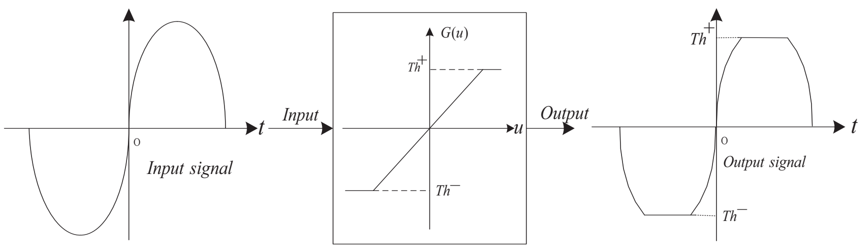


**Fig. 2.** The transfer characteristic of the saturation function.

Adopting this method, we can overcome the shortcoming of the fact that $u(t)$ jumps randomly among all invariant subspaces and makes the trajectories only move near the orbits of the given invariant subspaces when $u(t)$ comes into the saturation state.

Can the mathematical model really control the state variable $u(t)$ and adjust the levels of the laminar phase?

Considering the saturation function (18), the input $u(t)$ can be divided into three intervals by $Th^-$ and $Th^+$:

$$I_- = (-\infty, Th^-], \quad I = (Th^-, Th^+), \quad I_+ = [Th^+, +\infty). \tag{20}$$

If the state variable $u \in I$, then $g(G(u)) = \sin(u)$. Obviously, there exist different levels of the laminar phase at $u = 2k\pi (k = \pm 1, \pm 2, \ldots)$ due to the occurrence of blowout bifurcation. If the input $u(t)$ of system (19) jumps out of the interval $I$ irregularly, the trajectories of $u(t)$ can return to the interval $I$ again.

Let us consider the case when $u(t_0) \in I_+$. In this case, $g(G(u)) = \sin(Th^+)$, and system (19) becomes

$$\begin{cases} \dot{x} = y \\ \dot{y} = z \\ \dot{z} = -a(x + y + z - f(x)) \\ \dot{u} = 2v \\ \dot{v} = -0.05v - 2\sin(Th^+)(x - b). \end{cases} \tag{21}$$

The last two equations of system (21) can be rewritten as

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 0 & -0.05 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} w(t) \tag{22}$$

where

$$w(t) = -2\sin(Th^+)(x - b). \tag{23}$$

Solving (22) gives

$$u(t) = S_1 + S_2 \tag{24}$$

where

$$\begin{cases} S_1 = u(t_0) + 40(1 - \mathrm{e}^{-0.05(t-t_0)})v(t_0) - \displaystyle\int_{t_0}^{t} 40\mathrm{e}^{-0.05(t-\tau)}w(\tau)\,\mathrm{d}\tau \\ S_2 = \displaystyle\int_{t_0}^{t} 40w(\tau)\,\mathrm{d}\tau \end{cases} \tag{25}$$

in which $u(t_0), v(t_0)$ represent initial conditions. Since the attractor of system (1) is bounded when parameter $a = 0.8$, $|w(t)|$ is bounded. If there exists a constant $M$ which is large enough and $|w(t)| \leq M$, we obtain that

$$\begin{aligned} S_1 &\leq |u(t_0)| + |40(1 - \mathrm{e}^{-0.05(t-t_0)})v(t_0)| + \left| \int_{t_0}^{t} 40\mathrm{e}^{-0.05(t-\tau)}w(\tau)\,\mathrm{d}\tau \right| \\ &\leq |u(t_0)| + |40v(t_0)| + 800M \ (\text{as } t \to \infty). \end{aligned} \tag{26}$$

Therefore, the term $S_1$ is bounded. Consequently, considering the bounded attractor of system (1), we conclude that

$$S_2 = \int_{t_0}^{t} 40w(\tau)\,\mathrm{d}\tau = -80 \int_{t_0}^{t} \sin(Th^+)(x-b)\,\mathrm{d}\tau \approx 80\sin(Th^+)\,bt\ (\text{as } t \to \infty). \tag{27}$$

If we choose an appropriate value $Th^+$, and make sure $\sin(Th^+) > 0$, we will have $S_2 \to -\infty$ as $t \to +\infty$ when $b = -1.7$. Furthermore, from expression (24) to (27), this implies that $u(t)$ cannot stay in the interval $I_+$ forever and must go back to the interval $I$ in a finite time.

Similarly, when $u(t_0) \in I_-$, we choose an appropriate value $Th^-$ and make sure $\sin(Th^-) < 0$, we will have $S_2 \to +\infty$ as $t \to +\infty$ when $b = -1.7$. In other words, the state variable $u(t)$ will go back to the interval $I$ again in a finite time. Therefore, we can draw a conclusion that the levels of the laminar phase only exist in the interval $I$ even if the trajectories of $u(t)$ jumps out of the interval $I$.

According to the theoretical analysis mentioned above, we can arbitrarily control the levels of the laminar phase. For example, when $u(t)$ varies in interval $[-9.24, 9.24]$, there are only three invariant subspaces in the whole space according to the limit of sets (17); that is, $k = 0$ or $k = \pm 1$. We set the thresholds ($Th^+ = -TH^- = 9.24$) for the saturation function $G(u)$, and then calculate that

$$\sin(Th^+) = 0.184 > 0, \quad \sin(Th^-) = -0.184 < 0 \tag{28}$$

which are used to ensure $u(t)$ to return to the given laminar levels when $u(t)$ jumps out of the interval $I$ irregularly.

In view of this control method, the attractors of the three invariant subspaces projected on the $u - v$ plane are as shown in the following. Figure 3 shows a three-level on-off intermittency phase portrait and time series. Because we confine the range of the state variable $u(t)$, system (5) has on-off intermittency with three states in the case of $u \in [-9.24, 9.24]$. The time series $u(t)$ has three laminar levels when $u(t)$ jumps unpredictably among all the invariant subspaces.

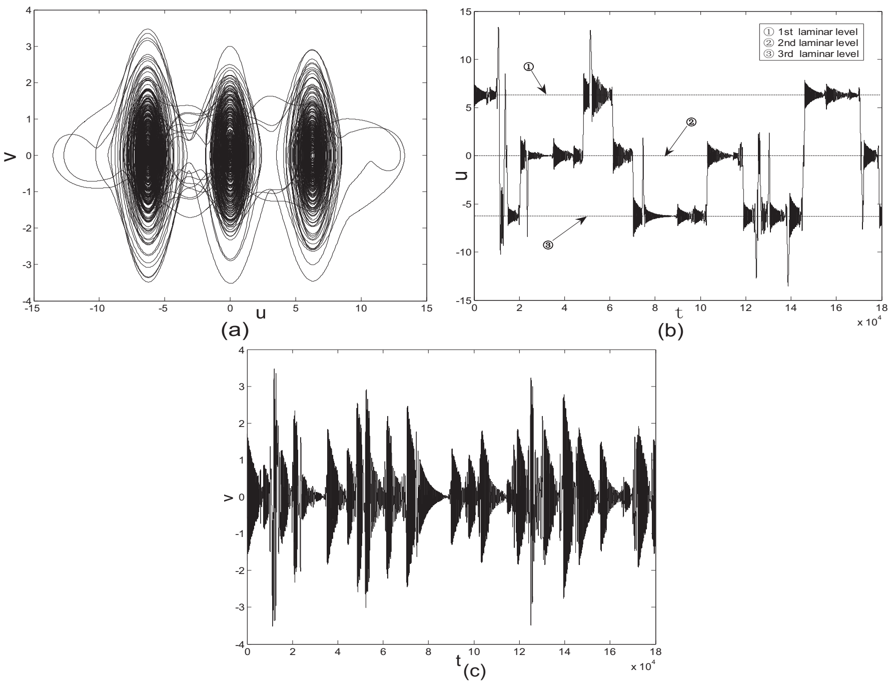


**Fig. 3.** Phase portrait and time series of on-off intermittency with three laminar levels when $b = -1.7$.

Figure 3(a) shows the phase portrait of $u(t)$ vs. $v(t)$. We can find that the trajectory projected on the $u - v$ plane will form three-stroll attractors. Figure 3(b) demonstrates that $u(t)$ has a step-like behavior. The time series $u(t)$ only jumps among three levels of laminar states like a typical random-walk motion because of the invariant subspaces $M_1$ limited by parameter $k = 0$ or $k = \pm 1$. Thus, it can be seen that multi-stroll attractors imply multiple state of on-off intermittency. However, $v(t)$ shown in Figure 3(c) has only one single state of on-off intermittency, which reflects the changes of the extent of the variable $u(t)$. The trajectories beyond the orbits of the three given invariant subspaces will be forced to enter the nearby orbit again due to the effect of the saturation function, as shown in Figure 3(a). The trajectories shown in Figure 3(a) can explain it clearly.

## 4. NUMERICAL SIMULATIONS

In order to further explain the control method, we obtain the results of different numbers of levels of laminar state with a single or multiple stroll attractors by numerical simulations. According to set (17), the laminar state level only exists in the interval $I$ even if the trajectories of $u(t)$ jumps out of the interval $I$. Figure 4, Figure 5 and Figure 6 illustrate the phenomenon vividly.

(1) When $Th^+ = -TH^- = 3.11$, which implies that $\sin(Th^+) > 0$ and also

$\sin(Th^-) < 0$, there exists only one single level of laminar state or one single attractor of on-off intermittency, as shown in Figure 4.

(2) When $Th^+ = 15.61$, $TH^- = -21.61$, there are six levels of laminar state or six-stroll attractors of on-off intermittency, as shown in Figure 5.

(3) When $Th^+ = -TH^- = 27.72$, there are nine levels of laminar state and nine-stroll attractors of on-off intermittency, as shown in Figure 6.

From the numerical simulation results, we can see multi-scroll attractors located in different invariant spaces of the multi-state on-off intermittency system. Each attractor in an invariant subspace is transversely unstable, therefore they intertwine with each other to form a stable global attractor.

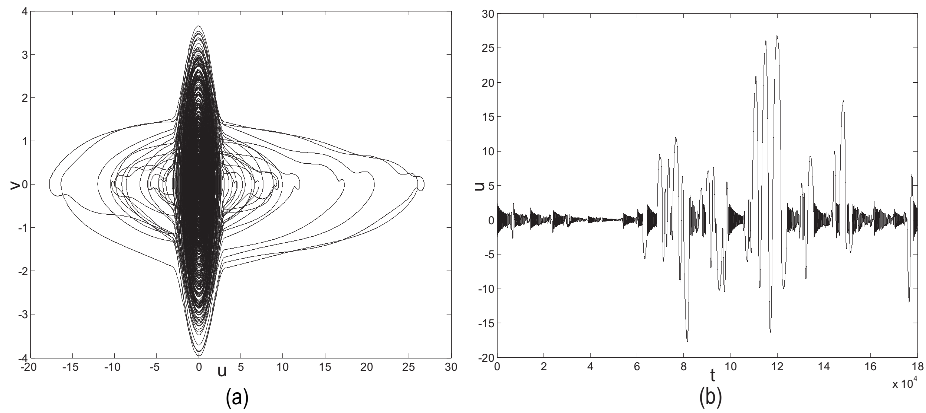

**Fig. 4.** Phase portrait and time series of on-off intermittency with single level.

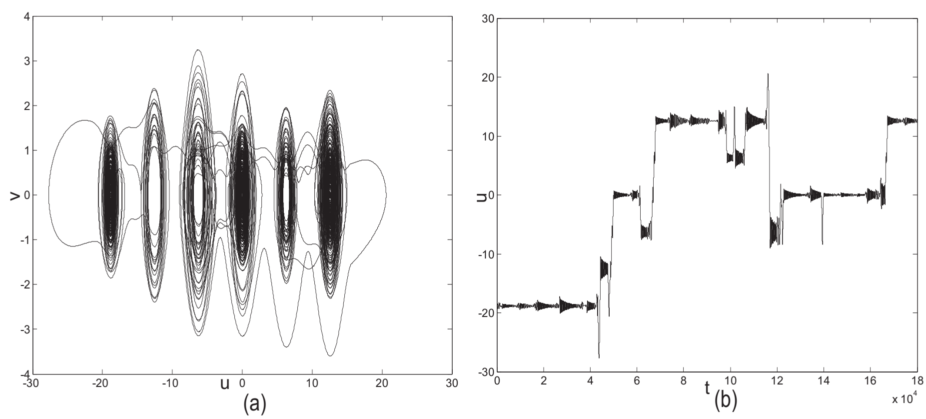

**Fig. 5.** Phase portrait and time series of on-off intermittency with six levels.

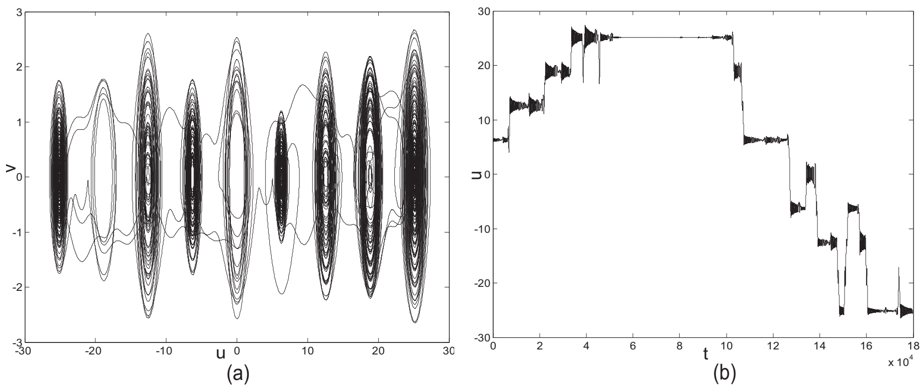


**Fig. 6.** Phase portrait and time series of on-off intermittency with nine levels.

## 5. STATISTICAL ANALYSIS OF MULTI–LEVEL ON–OFF INTERMITTENCY

### 5.1. Probability distribution of the laminar phase length

In this section, we discuss the "regular" probability distribution of "irregular" on-off intermittency.

A commonly used measure for analyzing the characteristic of on-off intermittency from time series is the laminar phase length distribution. For a single level on-off intermittency system, the laminar length probability distribution has been studied extensively. It follows a power law with exponent $-3/2$ see [3, 8]. For multi-level on-off intermittency, the power law is also in existence.

Now, we take three levels of the laminar phase as an example. The first step to demonstrate the presence of a kind of intermittency is to choose a vertical threshold of the time series $u(t)$, which is small enough and has a sufficient number of digits for good statistics. Let $\varepsilon$ denote the threshold value. The phase state is divided as "on" state for $u(t) \geq \varepsilon$ and "off" state for $u(t) < \varepsilon$. For the situation with three-state intermittency, "on" state and "off" state should be confined to three different laminar levels. Next step is to find the time intervals $\delta t$ between successive crossings of the threshold in the upward direction. These time intervals $\delta t$ are laminar phases. If we choose $\varepsilon = 0.001$ and collect 20000 iterations for each level, three-state laminar phase normalized probability distribution will be obtained. Here, $\delta t = N \times t\,\text{step}$, $t\,\text{step}$ is the step length of time $t$ in numerical simulations, $N$ is the number of laminar length sampled by $t\,\text{step}$. We adopt $P(N)$ and $N$ as coordinates. Figure 7 shows a double log plot of the normalized probability distribution $P(N)$ having three levels of laminar phases versus $N$, which is based on the results of three-state on-off intermittency shown in Figure 3.

From Figure 7, we can easily find that this plot is essentially linear, which implies that the fluctuations of the time series $u(t)$ is governed by a power law with exponent $-3/2$. The $-3/2$ exponent in power law is a typical characteristic of on-off intermittency.

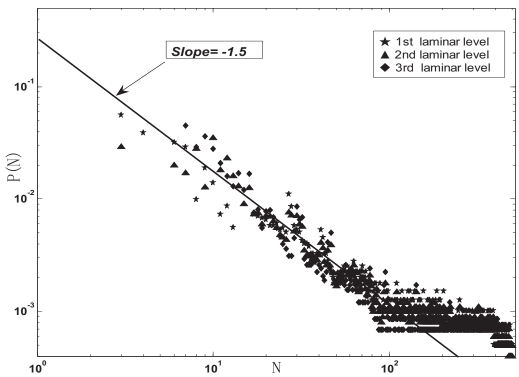


**Fig. 7.** The log-log plot for the laminar phase probability distribution $P(N)$ of three-level on-off intermittency: $P(N)$ vs. $N$. The diamonds, stars and triangles represent different levels of the laminar phase probability distribution with heavy-tails, respectively. The solid line represents a power law with exponent $-3/2$.

### 5.2. Analysis of random jumping

Figure 3, Figure 4, Figure 5, and Figure 6 show the time series $u(t)$, which jumps randomly between laminar levels restricted to the threshold of the saturation function $G(u)$. If we let the lower threshold and the upper threshold of $G(u)$ be large enough, the time series $u(t)$ will jump to potential laminar levels. We define the leap strength by $D = |k_1 - k_2|,\ k_1 \neq k_2$. Here, parameter $k_1$ denotes the sequence number of the laminar phase level at a time. After a short burst phase, the time series $u(t)$ jumps to another laminar phase level, whose sequence number can be denoted by parameter $k_2$. In short, there may be an intermittent leap interval between two different laminar phase levels. The leap strength $D$ (integer) obeys the following probability distribution:

$$P(D) \propto \mathrm{e}^{-1.176D}. \tag{29}$$

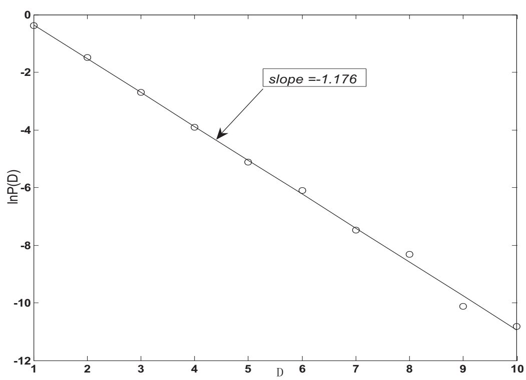


**Fig. 8.** The probability distribution of jumping intervals: $P(D)$ vs. $D$.

Figure 8 shows the leap interval probability distribution, which is calculated from 40000 iterations between different levels of laminar state. Obviously, the probability distribution $P(D)$ manifests exponential decreasing with the increase of the leap strength. The distribution shows that the variable $u(t)$ in some laminar phase levels tend to jump to another adjacent laminar level rather than a distant laminar phase level.

## 6. CONCLUSION

In this paper, we use a simple chaotic system with double-stroll attractors to drive a second-order system with a modulating function, and obtain a dynamic system which can generate multiple levels of on-off intermittency. We have analyzed the mechanism of emergence of this type of intermittency and have also designed a strategy to adjust the levels and the state positions of the laminar phase. By numerical simulations and theoretical analysis, it is shown that the control method is effective. In addition, statistical analysis has shown that the laminar length probability distribution follows a power law with exponent $-3/2$ and the leap strength of jumping between different laminar levels obeys an exponent distribution.

## ACKNOWLEDGEMENT

This work was supported in part by the Natural Science Foundation of China (Nos. 60774088 and 60574036), the Program for New Century Excellent Talents in University of China (NCET), the Specialized Research Fund for the Doctoral Program of Higher Education of China (No. 20050055013), and the Science and Technology Research Key Project of Education Ministry of China (No.107024).

(Received September 30, 2007.)

## REFERENCES

[1] P. Ashwin, J. Buescu, and I. Stewart: From attractor to chaotic saddle: A tale of transverse instability. Nonlinearity *9* (1996), 703–737.

[2] M. Bojowald and H. A. Kastrup: Symmetry reduction for quantized diffeomorphism invariant theories of connections. Class. Quantum. Grav. *17* (2000), 3009–3043.

[3] E. Covas, R. Tavakol, P. Ashwin, A. Tworkowski, and J. M. Brooke: In-out intermittency in PDE and ODE models of axisymmetric mean field dynamos. Chaos *11* (2001), 404–409.

[4] A. S. Elwakil, K. N. Salama, and M. P. Kennedy: A system for chaos generation and its implementation in monolithic form. Proc. IEEE Internat. Symposium on Circuits and Systems *5* (2000), 217–220.

[5] P. Faure and H. Korn: Is there chaos in the brain? I. Concepts of nonlinear dynamics and methods of investigation. Life Sciences *324* (2001), 773–793.

[6] D. L. Feng, C. X. Yu, J. L. Xie, and W. X. Ding: On-off intermittencies in gas discharge plasma. Phys. Rev. E *58* (1998), 3678–3685.

[7] D. L. Feng, J. Zheng, W. Huang, C. X. Yu, and W. X. Ding: Type-I-like intermittent chaos in multicomponent plasmas with negative ions. Phys. Rev. E *54* (1996), 2839–2843.

[8] J. F. Heagy, N. Platt, and S. M. Hammel: Characterization of on-off intermittency. Phys. Rev. E *49* (1994), 1140–1150.
[9] T. John, R. Stannarius, and U. Behn: On-off intermittency in stochastically driven electro-hydrodynamic convection in nematics. Phys. Rev. Lett. *83* (1999), 749–752.
[10] T. Kapitaniak: Blowout bifurcation and on-off intermittency in pulse neural networks with multiple modules. International Journal of Bifurcation and Chaos *16* (2006), 3309–3321.
[11] T. Kapitaniak, Y. C. Lai, and C. Grebogi: Blowout bifurcation of chaotic saddles. Discrete Dynamics in Nature and Society *3* (1999), 9–13.
[12] T. Kapitaniak, Y. Maistrenko: Riddling bifurcations in coupled piecewise linear maps. Physica D *126* (1999), 18–26.
[13] H. Korn and P. Faure: Is there chaos in brain? II. Experimental evidence and related models. Biologies *326* (2003), 787–840.
[14] E. Ott and J. C. Sommerer: Blowout bifurcations: The occurrence of riddled basins and on-off intermittency. Phys. Lett. A *188* (1994), 39–47.
[15] N. Platt, S. M. Hammel, and J. F. Heagy: Effects of additive noise on on-off intermittency. Phys. Rev. Lett. *72* (1994), 3498–3501.
[16] N. Platt, E. A. Spiegel and C. Tresser: On-off intermittency: a mechanism for bursting. Phys. Rev. Lett. *70* (1993), 279–282.
[17] R. Tavakol and E. Covas: Intermittency as a possible underlying mechanism for solar and stellar variability. arXiv:astro-ph/9904394v1

*Shi-Jian Cang, Department of Industry Design, Tianjin University of Science and Technology, Tianjin 300222 and Department of Automation, Nankai University, Tianjin 300071. China.*
*e-mail: csj98231@tust.edu.cn*

*Zeng-Qiang Chen and Zhu-Zhi Yuan, Department of Automation, Nankai University, Tianjin 300071. China*
*e-mails: chenzq@nankai.edu.cn, yuanzhzh@nankai.edu.cn*