

GUEST EDITORIAL: SPECIAL ISSUE ON MAX-PLUS ALGEBRAS

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1. FOREWORD

The twelve papers of this special issue were selected from communications presented at the Workshop on Max-Plus Algebras, which was held in Prague, August 25–27, 2001, within the First IFAC Symposium on System Structure and Control. The latter followed the IFAC Workshops and Conferences organized in Prague, 1989 and 1992, and in Nantes, 1995 and 1998. This series of meetings will now on be organized every three years by IFAC, which reflects the permanence of the activity on the field of System Structure and Control.

A relatively new domain of research, in turn, is based on the study of the so-called Max-Plus semiring $(\mathbb{R} \cup \{-\infty\}, \max, +)$, and of other similar mathematical structures that are called exotic or extremal algebras, which appear in several fields, including graph theory, optimization, optimal control, game theory, Hamilton-Jacobi partial differential equations, asymptotic analysis, large deviations, complexity and language theory, and in systems theory, where they provide basic tools for modeling discrete-event systems. The Max-Plus workshop aimed at bringing together the researchers from these different areas and to do a state-of-the-art of the current developments in the field. The issue presents a representative set of contributions to various fields related to Max-Plus algebras.

2. SCANNING THE ISSUE

The three first contributions deal with general algebraic properties of the Max-Plus semiring.

— R. A. Cuninghame-Green considers functions formed by composition of the operators Max, Min, and Plus, as rational functions in the Max-Plus Algebra.

— P. Butkovič studies the problem of computing the characteristic polynomial and the characteristic equation of a matrix in the Max-Plus algebra.

— K. Cechlárová and R. A. Cuninghame-Green show how to compute a Chebyshev best soluble approximation of an insoluble system of Max-Plus linear equations.

The three following contributions concern the modeling, analysis and control of Max-Plus linear systems, which arise in discrete-event systems and dynamic programming.

— S. Lahaye, L. Hardouin and J.-L. Boimond analyse the interconnection of Max-Plus linear systems, which was used in a software for the simulation and analysis of a real production process.

— L. Truffet exploits the similarity between Markov chains, i.e. usual linear systems, and Bellman chains, i.e. Max-Plus linear systems, and gives tools for the comparison of Bellman chains.

— B. Cottenceau, L. Hardouin and J.-L. Boimond propose a synthesis method for the stabilization of a Max-Plus linear system by output feedback.

One paper focuses in language theory and series representations.

— I. Klimann compares different types of representations for series with coefficients in complete idempotent semirings.

In the Max-Plus algebra, all numbers are greater than the ‘zero element’, that is $-\infty$, so that max-plus numbers may be thought of as elements of a ‘positive cone’. The three following papers deals with maps, and systems, defined on cones.

— C. Walsh and J. Gunawardena study self maps of symmetric cones, that are non-expansive in Hilbert’s and Thompson’s metric, and study the asymptotic behaviour of the iterates of such maps.

— A. D. Burbanks, C. T. Sparrow and R. D. Nussbaum give conditions under which a homogeneous order-preserving map on a cone extends continuously to the boundary of the cone, and use this continuous extension to study the dynamic behavior in the interior of the cone.

— L. Benvenuti and L. Farina survey the minimality problem for positive realizations of linear systems.

Finally the two last papers deal with analysis and optimization.

— O. V. Gulinsky studies large deviation type asymptotic problems using idempotent measure theory.

— M. Gondran and R. Hoblos-Saade introduce a complex calculus of variations.

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