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ON ROBUST STABILITY OF NEUTRAL SYSTEMS

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This paper focuses on the problem of uniform asymptotic stability of a class of linear neutral systems including some constant delays and time-varying cone-bounded nonlinearities. *Sufficient* stability conditions are derived by taking into account the weighting factors describing the nonlinearities. The proposed results are applied to the stability analysis of a class of lossless transmission line models.

1. INTRODUCTION

It is relatively well known that the existence of a delay in a physical system may induce instability or bad performance [12, 14] in open or closed-loop schemes. In certain control problems, one encounters linear hyperbolic differential equations with mixed initial and derivative boundary conditions, see, e. g. processes including steam or water pipes, lossless transmission lines. In some cases, the connection through the partial differential equations can be rewritten by using some appropriate delay (inter)connections. Thus, using a technique proposed in Hale and Verduyn Lunel [10] (see also the works of Brayton [4], Abolinia and Myshkis [1], Cooke and Krumme [6] or Răsvan [9, 18]), a nonlinear lossless transmission line [3] can be easily described by a functional differential equation of *neutral* type. Further examples and discussions can be found in [19]. The particularity of neutral systems is that the delay argument occurs also in the derivative of the state variables.

A different example is proposed in [16], where the effect of force measurements delays on the stability of manipulators in contact with a rigid environment is considered. The closed-loop system is represented by a linear time-invariant neutral equation. In this case, the time-delay may be a *cause* of *possible bouncing* of the robot's tip on the environment.

There are several methods to analyze the stability of such systems. Without being exhaustive, one can mention in the frequency-domain class, a frequency-dependent matrix pencil technique [5], or the singular value test [24]; in the time-domain class, the Lyapunov's second method (with the Razumikhin and Krasovskii methodologies) [10, 20], or the comparison methods [13]. Thus, using an appropriate Lyapunov–Krasovskii functional, [24] proves that the stability of such system can be reduced to the existence of a positive-definite solution of a *continuous* Riccati equation coupled with a *discrete* Lyapunov equation. A guided tour of the general corresponding methods for the stability and robust stability of linear systems with delayed states can be found in [17].

In this paper, we consider a particular class of uncertain time-delay systems described by linear neutral differential equations, including cone-bounded and time-varying nonlinear uncertainty. We are interested in analyzing stability conditions for such systems. The approach adopted here is based on Lyapunov's second method and makes use of an appropriate Lyapunov–Krasovskii functional [10, 12, 24]. *Sufficient* delay-independent stability conditions are given in terms of positive solution to some linear matrix inequalities (LMIs). Note that the proposed conditions extend the results of [24] to handle nonlinear uncertainty, and/or multiple delays. Furthermore, an appropriate optimization problem related to the nonlinearity description can be also considered. As an application, a simplified neutral model of a nonlinear lossless transmission line is proposed.

The paper is organized as follows: in Section 2, we give the problem formulation. The main results are presented in Section 3. The case of a lossless transmission line model described as a neutral system is considered in Section 4. Some concluding remarks end the paper.

Notation. The following notations will be used throughout the paper. $\mathbf{R}$ denotes the set of real numbers, $\mathbf{R}^n$ denotes the n dimensional Euclidean space, and $\mathbf{R}^{n\times m}$ denotes the set of all $n \times m$ real matrices. $\mathcal{C}_{n,\tau} = \mathcal{C}([-\tau, 0], \mathbf{R}^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-\tau, 0]$ into $\mathbf{R}^n$ with the topology of uniform convergence. The following norms will be used: $\|\cdot\|$ refers to the Euclidean vector norm; $\|\phi\|_c = \sup_{-\tau\leq t\leq 0} \|\phi(t)\|$ stands for the norm of a function $\phi \in \mathcal{C}_{n,\tau}$. Moreover, we denote by $\mathcal{C}^v_{n,\tau}$ the set defined by $\mathcal{C}^v_{n,\tau} = \{\phi \in \mathcal{C}_{n,\tau} \ : \ \|\phi\|_c < v\}$, where v is a positive real number.

2. PROBLEM STATEMENT

Consider the following class of linear neutral systems:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\mathcal{D}x_t &= Ax(t) + Bx(t-\tau_2) + \Delta A(x_t(0), t) + \Delta B(x_t(-\tau_2), t) \\ &\quad + \Delta D(\mathcal{D}x_t, t), \end{aligned} \tag{1}$$

with the initial condition

$$x_{t_0}(\theta) = \phi(\theta),\ \forall\, \theta \in [-\bar{\tau}, 0]; \quad (t_0, \phi) \in \mathbf{R}^+ \times \mathcal{C}^v_{n,\tau}, \tag{2}$$

where $\bar{\tau} = \max\{\tau_1, \tau_2\}$, and the operators $x_t, \mathcal{D} : \mathcal{C}_{n,\tau} \mapsto \mathbf{R}^n$ are defined as follows:

$$\begin{aligned} x_t(\theta) &= x(t+\theta), \\ \mathcal{D}x_t &= x(t) - Dx(t-\tau_1). \end{aligned}$$

The delays of the system are $\tau_{1,2} > 0$ and are assumed constant, and D, A and B are constant matrices of appropriate dimension.

The mappings $\Delta A, \Delta B, \Delta D : \mathcal{C}_{n,\tau} \times \mathbf{R} \mapsto \mathbf{R}^n$ denote unknown nonlinear functions satisfying the following assumption:

Assumption 1. The uncertain functions ΔA, ΔB and ΔD are gain bounded smooth functions described by

$$\begin{cases} \Delta A(x_t(0),t) = E_a\delta_a(x_t(0),t), & \delta_a(y,t)^T\delta_a(y,t) \leq y^T W_a^T W_a y, \\ \Delta B(x_t(-\tau),t) = E_b\delta_b(x_t(-\tau),t), & \delta_b(y,t)^T\delta_b(y,t) \leq y^T W_b^T W_b y, \\ \Delta D(\mathcal{D}x_t,t) = E_d\delta_d(\mathcal{D}x_t,t), & \delta_d(y,t)^T\delta_d(y,t) \leq y^T W_d^T W_d y, \end{cases} \tag{3}$$

for all $y \in \mathbf{R}^n$ and all $t \in \mathbf{R}$, with known matrices E_a, E_b and E_d. The matrices W_a, W_b and W_d are given weighting matrices. The unknown mappings δ_a, δ_b, δ_d satisfy the conditions

$$\delta_a(0,t) = 0, \ \ \delta_b(0,t) = 0, \ \ \delta_d(0,t) = 0.$$

This assumption implies that the origin $x = 0$ is an equilibrium point of the system (1) with δ_a, δ_b and δ_d uniformly bounded by $x(t)$, $x(t-\tau)$ and $\mathcal{D}x_t$, respectively.

If $C \equiv 0$, $\delta_d \equiv 0$ the proposed model represents a 'classical' description of linear uncertain 'approximations' of nonlinear delay systems [15]. The advantage of the representation (3) lies in the ability to analyze stability properties via an appropriate (simple) quadratic Lyapunov–Krasovskii candidate. A unifying formalism for large classes of uncertain systems can be found in [7]. Note that the nonlinearity described by the $\delta_d(\cdot,t)$-term is specific to the application presented in Section 4.

Throughout the paper, we shall say that the system (1) – (2) is *robustly delay-independent stable* if it is uniformly asymptotically stable for each uncertainty ΔA, ΔB and ΔD satisfying Assumption 1 (see also [15]).

With these notation, definitions and assumptions, the stability problem can be formulated as follows: *find conditions to ensure the stability of the system (1) – (2) for all the class of nonlinearities (3). If there exists a solution, one may introduce a "measure" to describe how robust is the stability property with respect to the considered nonlinearity.*

3. MAIN RESULTS

With the notations given in the previous section, we have the following stability result:

Theorem 1. The neutral system (1) – (2) satisfying Assumption 1 is delay-independent robustly stable if

(i) A is a Hurwitz stable matrix;

(ii) D is a Schur–Cohn stable matrix;

(iii) there exist three symmetric and positive definite matrices $P, S_1, S_2 > 0$ such that the following LMI holds:

$$\begin{bmatrix} \begin{array}{c} A^TP + PA + S_1 + S_2 \\ +W_a^TW_a + W_d^TW_d \end{array} & PE & \begin{array}{c} (PA + S_1 + S_2 + \\ +W_a^TW_a)D \end{array} & PB \\ E^TP & -I & 0 & 0 \\ \begin{array}{c} D^T(A^TP + S_1 + S_2 + \\ +W_a^TW_a) \end{array} & 0 & \begin{array}{c} D^T(S_1 + S_2 + \\ +W_a^TW_a)D - S_1 \end{array} & 0 \\ B^TP & 0 & 0 & W_b^TW_b - S_2 \end{bmatrix} < 0, \quad (4)$$

where $E = [E_a\ E_b\ E_d]$.

The proof is included in Appendix B, and makes use of the following Lyapunov–Krasovskii functional:

$$\begin{aligned} V(x_t) &= (x(t) - Dx(t-\tau_1))^T P(x(t) - Dx(t-\tau_1)) \\ &\quad + \sum_{i=1}^{2} \int_{-\tau_i}^{0} x(t+\theta)^T S_i x(t+\theta)\,\mathrm{d}\theta. \end{aligned}$$

Note that if $\tau_1 = \tau_2$, the result above becomes:

Corollary 1. $[\tau_1 = \tau_2 = \tau]$ The neutral system (1)–(2) satisfying Assumption 1 is delay-independent robustly stable if

(i) A is a Hurwitz stable matrix;

(ii) D is a Schur–Cohn stable matrix;

(iii) there exist two symmetric and positive definite matrices $P > 0$ and $S > 0$ such that the following LMI holds:

$$\begin{bmatrix} \begin{array}{c} A^TP + PA + S + \\ +W_a^TW_a + W_d^TW_d \end{array} & PE & \begin{array}{c} P(AD + B) + SD + \\ +W_a^TW_aD \end{array} \\ E^TP & -I & 0 \\ \begin{array}{c} (B^T + D^TA^T)P + \\ +D^TS + D^TW_a^TW_a \end{array} & 0 & \begin{array}{c} D^TSD - S + \\ +W_b^TW_b + D^TW_a^TW_aD \end{array} \end{bmatrix} < 0, \quad (5)$$

where $E = [E_a\ E_b\ E_d]$.

Remark 1. The proof of this corollary follows the same steps as Theorem 1 and makes use of the following Lyapunov–Krasovskii functional candidate:

$$V(x_t) = (x(t) - Dx(t-\tau))^T P(x(t) - Dx(t-\tau)) + \int_{-\tau}^{0} x(t+\theta)^T Sx(t+\theta)\,\mathrm{d}\theta.$$

Remark 2. The Schur–Cohn stability of the matrix D ensures the *stability* of the "discrete" operator $\mathcal{D} : \mathcal{C}_{n,\tau} \to \mathbf{R}^n$:

$$\mathcal{D}(\phi) = \phi(0) - D\phi(-\tau_1),$$

which is a *necessary* condition to have the stability of the neutral differential equation (1) – (2) [10].

Note also that the Hurwitz stability of the matrix A is a *necessary* condition for the existence of a symmetric positive definite solution to the LMI (5) ($A^TP + PA$ should be negative definite), but is not a *sufficient* one.

Remark 3. Corollary 1 recovers the results given in [24] in the case when we have no uncertainties, i. e. $\Delta A \equiv 0$, $\Delta B \equiv 0$ and $\Delta D \equiv 0$. In this case, the corresponding LMI is:

$$\begin{bmatrix} A^TP + PA + S & P(AD + B) + SD \\ D^TS + (B^T + D^TA^T)P & D^TSD - S \end{bmatrix} < 0.$$

Furthermore, if we suppose that the system is of retarded type, i. e. $C \equiv 0$, the proposed result recovers the sufficient conditions for *delay-independent* stability given in [2].

A different Lyapunov–Krasovskii functional to study the robust stability of systems of the form (1) – (2) is:

$$\begin{aligned} V(x(t), x_t, \dot{x}_t) &= x(t)^T P_1 x(t) + \int_{-\tau_2}^{0} x(t+\theta)^T P_2 x(t+\theta)\,\mathrm{d}\theta \\ &\quad + \int_{-\tau_1}^{0} \dot{x}(t+\theta)^T P_3 \dot{x}(t+\theta)\,\mathrm{d}\theta, \end{aligned} \tag{6}$$

where P_i ($i = 1, 2, 3$) are symmetric and positive definite matrices satisfying some appropriate Riccati inequalities (see [22]).

The form of the Lyapunov functional (6) includes "information" on the state derivatives $\dot{x}_t$. In this case, a proper norm for this stability analysis study is given by:

$$\|x_t\|_{c1} = \sup_{-\tau \le \theta \le 0} \{\|x(t+\theta)\|, \|\dot{x}(t+\theta)\|\}.$$

Some connections between the stability results obtained using the *norms* $\|\cdot\|_c$ and $\|\cdot\|_{c1}$ can be found in Els'golts' and Norkin [8]. For the sake of simplicity, we do not consider this approach here.

Due to the particular form of the LMI (5), (which is affine in the weighting factors $W_a^TW_a$, $W_b^TW_b$ and $W_d^TW_d$, respectively), one may consider the following natural way to analyze the *robustness* of the system in terms of weighting factors, i. e. the following standard *LMI optimization problem* [2]:

$$\begin{aligned} &\text{maximize} \quad \mathbf{Tr}(W_a^TW_a + W_b^TW_b + W_d^TW_d) \text{ s.t.} \\ &\qquad (4) \text{ or } (5) \text{ holds.} \end{aligned}$$

Roughly speaking, this maximization problem is related to the stability radius of the matrix A [15], that is, the uncertainty terms are not allowed to "exceed" (in norm, or in trace) some bounds specified by this quantity. Other robustness formulations and comments for robustness issues for delay systems can be found in [15].

Consider now a special case: the existence only of the uncertainty term ΔD. Thus, one has:

Corollary 2. $[\tau_1 = \tau_2]$ The neutral system (1)–(2) (with $\Delta A \equiv 0$ and $\Delta B \equiv 0$) such that ΔD satisfies Assumption 1 is delay-independent robustly stable if

(i) A is a Hurwitz stable matrix;

(ii) D is a Schur–Cohn stable matrix;

(iii) there exist two symmetric and positive definite matrices $P > 0$ and $S > 0$ such that the following LMI holds:

$$\begin{bmatrix} A^TP + PA + S + W_d^TW_d & PE_d & P(AD+B) + SD \\ E_d^TP & -I & 0 \\ (B^T + D^TA^T)P + D^TS & 0 & D^TSD - S \end{bmatrix} < 0. \quad (7)$$

4. APPLICATION TO A NONLINEAR TRANSMISSION LINE

Let us consider the following nonlinear transmission line system described by the following set of partial differential equations [3, 10]:

$$\begin{cases} L\dfrac{\partial i}{\partial t} = -\dfrac{\partial v}{\partial x}, \qquad C\dfrac{\partial v}{\partial t} = -\dfrac{\partial i}{\partial x} \\ 0 < x < 1, \quad t > 0, \end{cases}$$

with the boundary conditions:

$$\begin{cases} E - v(0,t) - Ri(0,t) = 0 \\ C_1\dfrac{\mathrm{d}}{\mathrm{d}t}v(1,t) = i(1,t) - g(v(1,t)), \end{cases}$$

where $g(\cdot)$ is an appropriate nonlinear function [10].

Using one of the transformation techniques proposed in Hale and Verduyn Lunel [10] (the "transformation" is not unique), this system can be rewritten in the following form,

$$\begin{aligned} C_1 \cdot \frac{\mathrm{d}}{\mathrm{d}t}\left[u(t) - qu(t - 2\sqrt{LC})\right] &= -\alpha\left(u(t) + qu(t - 2\sqrt{LC})\right) \\ &\quad -g\left(u(t) - qu(t - 2\sqrt{LC})\right) + k, \end{aligned} \quad (8)$$

with an appropriate k and where

$$\begin{aligned} q &= \frac{\sqrt{L}-R\sqrt{C}}{\sqrt{L}+R\sqrt{C}}, \quad |q|<1 \\ \alpha &= \sqrt{\frac{C}{L}}. \end{aligned}$$

Without loss of generality, consider now the equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{D}x_t = -\alpha x(t) - q\alpha x(t-\tau) - g(\mathcal{D}x_t), \tag{9}$$

where $\mathcal{D}x_t = x(t) - qx(t-\tau)$.

Applying Corollary 2, we will have to find $\varepsilon, \gamma > 0$ such that the following matrix:

$$M(\varepsilon,\gamma,w_d) = \begin{bmatrix} -\alpha+\varepsilon+\gamma & w_d & (\gamma-\alpha)q \\ w_d & -\varepsilon & 0 \\ (\gamma-\alpha)q & 0 & -\gamma(1-q^2) \end{bmatrix}$$

is negative definite. Note that ε is a tuning parameter for reducing the conservativeness of the robust stability condition.

If one chooses $\varepsilon = \frac{|w_d|}{2}$ and $\gamma = \alpha|q|$, we shall have:

Proposition 1. Supposing that the nonlinearity g satisfies the Assumption 1, then the neutral system (9) is delay-independent robustly stable if $|q| < 1$ and:

$$\frac{g(x)^2}{x^2} < \alpha^2\left(\frac{1-|q|}{1+|q|}\right)^2. \tag{10}$$

5. CONCLUDING REMARKS

In this paper, we have considered the problem of robust stability of a class of neutral systems including time-varying cone-bounded uncertainty. We have derived sufficient delay-independent conditions expressed in terms of the existence of symmetric and positive-definite solutions for some appropriate linear matrix inequalities. The proposed results are applied for the stability study of a neutral model associated to a nonlinear lossless transmission line. The results proposed here extend similar ones in the literature [24].

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APPENDIX A: STABILITY THEORY

Consider the following functional differential equation of neutral type:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\mathcal{D}x_t\right] = f(x_t), \tag{11}$$

with an appropriate initial condition:

$$x_{t_0}(\theta) = \phi(\theta), \quad \forall\, \theta \in [-\tau, 0]; \quad (t_0, \phi) \in \mathbf{R}^+ \times \mathcal{C}^v_{n,\tau}, \tag{12}$$

where $\mathcal{D} : \mathcal{C}_{n,\tau} \to \mathbf{R}^n$, $\mathcal{D}\phi = \phi(0) - D\phi(-\tau)$ and $x(t) \in \mathbf{R}^n$. We say that the operator $\mathcal{D}$ is *stable* if the zero solution of the corresponding homogeneous difference equation is uniformly asymptotically stable. For our choice, this condition is replaced by the *Schur–Cohn* stability of the matrix D. For a general framework, see e. g. Hale and Verduyn Lunel [10].

If $V : \mathbf{R} \times \mathcal{C}_{n,\tau} \to \mathbf{R}^n$ is continuous and $x(t_0, \phi)$ is the solution of the neutral differential equation (11) through (t_0, ϕ) defined by (12), we define:

$$\dot{V}(t_0, \phi) \;=\; \limsup_{h \to 0+} \frac{1}{h}\left[V(t+h, x_{t+h}(t_0, \phi) - V(t_0, \phi)\right].$$

We have the following result:

Theorem A.1. [10] Suppose $\mathcal{D}$ is stable, $f : \mathbf{R} \times \mathcal{C}_{n,\tau} \to \mathbf{R}^n$ takes bounded sets of $\mathcal{C}_{n,\tau}$ into bounded sets of $\mathbf{R}^n$ and suppose $u(s)$, $v(s)$ and $w(s)$ are continuous, nonnegative and nondecreasing functions with $u(s)$, $v(s) > 0$ for $s \neq 0$ and $u(0) = v(0) = 0$.

If there is a continuous function $V : \mathbf{R} \times \mathcal{C}_{n,\tau} \to \mathbf{R}^n$ such that

(i) $u(\|\mathcal{D}\phi\|) \leq V(t, \phi) \leq v(\|\phi\|_c)$,

(ii) $\dot{V}(t, \phi) \leq -w(\|\mathcal{D}\phi(0)\|)$ then the solution $x = 0$ of the neutral equation (11)–(12) is uniformly stable.

If $u(s) \to \infty$ as $s \to \infty$ the solutions are uniformly bounded.

If $w(s) > 0$ for $s > 0$, then the solution $x = 0$ is uniformly asymptotically stable. The same conclusions hold if the upper bound on $\dot{V}(t, \phi)$ is given by $-w(\|\phi(0)\|)$.

APPENDIX B: PROOF OF THEOREM 1

Let us consider the following Lyapunov–Krasovskii functional candidate:

$$\begin{aligned} V(x_t) \;&=\; (x(t) - Dx(t-\tau_1))^T P (x(t) - Dx(t-\tau_1)) \\ &\quad + \sum_{i=1}^{2} \int_{-\tau_i}^{0} x(t+\theta) S_i x(t+\theta)\, \mathrm{d}\theta, \end{aligned} \tag{13}$$

where P and S_i are solutions of the linear matrix inequality (4).

It is easy to see that the functional V satisfies the condition:

$$u(|\mathcal{D}\phi|) \quad \leq \quad V(\phi) \leq v(\|\phi\|_c), \tag{14}$$

where $u(s) = \lambda_{\min}(P)s^2$ and $v(s) = [\lambda_{\max}(P) + \sum \tau_i \lambda_{\max}(S_i)]]s^2$. The derivative of $V(\cdot)$ along the trajectory of the neutral system (1) is given by:

$$\begin{aligned}\dot{V}(x_t) \;=\; & (Ax(t)+Bx(t-\tau_2)+\Delta A(x_t(0),t)+\Delta B(x_t(-\tau_2))+\Delta D(\mathcal{D}x_t,t))^T P\mathcal{D}x_t \\ & +(x(t)-Dx(t-\tau_1))^T P(Ax(t)+Bx(t-\tau_2)+\Delta A(x_t(0),t)+\Delta B(x_t(-\tau_2)) \\ & +g(\mathcal{D}x_t,t)) + \sum_{i=1}^{2}\left[x(t)^T S_i x(t) - x(t-\tau_i)]^T S_i x(t-\tau_i)\right] .\end{aligned} \tag{15}$$

Using the following inequality [15]:

$$2\mathcal{D}x_t^T Fh(y,t) \quad \leq \quad \mathcal{D}x_t^T FF^T\mathcal{D}x_t + h(y,t)^T h(y,t), \tag{16}$$

for any matrix F and function h (of appropriate dimensions), we have (via Assumption 1):

$$\begin{aligned} 2\mathcal{D}x_t PE_a\delta_a(x_t(0),t) &\leq \mathcal{D}x_t^T PE_aE_a^T P\mathcal{D}x_t + x(t)^T W_a^T W_a x(t), \\ 2\mathcal{D}x_t PE_b\delta_b(x_t(-\tau_2),t) &\leq \mathcal{D}x_t^T PE_bE_b^T P\mathcal{D}x_t + x(t-\tau_2)^T W_b^T W_b x(t-\tau_2), \\ 2\mathcal{D}x_t PE_d\delta_d(\mathcal{D}x_t,t) &\leq \mathcal{D}x_t^T PE_dE_d^T P\mathcal{D}x_t + \mathcal{D}x_t^T W_d^T W_d\mathcal{D}x_t. \end{aligned}$$

Using additions and subtractions of appropriate terms, we can rewrite each expression containing the quantity "$x(t)$" as an expression containing "$\mathcal{D}x_t$" and "$x(t-\tau_1)$." For example,

$$\begin{aligned} x(t)^T Sx(t) \;=\; & \mathcal{D}x_t^T S\mathcal{D}x_t + x(t-\tau_1)^T D^T SDx(t-\tau_1) \\ & +(\mathcal{D}x_t)^T SDx(t-\tau_1) + x(t-\tau_1)^T D^T S\mathcal{D}x_t. \end{aligned}$$

With all these inequalities and transformations, simple computations allow to obtain the following form from (15) and (16):

$$\begin{aligned}\dot{V}(x_t) \;=\; & \begin{bmatrix} \mathcal{D}x_t^T & x(t-\tau_1)^T & x(t-\tau_2)^T \end{bmatrix} \cdot \\ & \cdot \begin{bmatrix} \begin{matrix} A^TP+PA+S_1+S_2+ \\ W_a^TW_a+W_d^TW_d+ \\ +PEE^TP \end{matrix} & \begin{matrix}(PA+S_1+S_2+ \\ +W_a^TW_a)D\end{matrix} & PB \\ \begin{matrix} D^T(A^TP+S_1+S_2+ \\ +W_a^TW_a) \end{matrix} & \begin{matrix} D^T(S_1+S_2)D-S_1+ \\ +D^TW_a^TW_aD \end{matrix} & 0 \\ B^TP & 0 & W_b^TW_b-S_2 \end{bmatrix} \cdot \\ & \cdot \begin{bmatrix} \mathcal{D}x_t \\ x(t-\tau_1) \\ x(t-\tau_2) \end{bmatrix} . \end{aligned} \tag{17}$$

Thus, if the matrix inequality (4) holds, it follows (via an appropriate Schur transformation) that:

$$\begin{bmatrix} A^TP+PA+S_1+S_2+ \\ W_a^TW_a+W_d^TW_d+ \\ +PEE^TP & (PA+S_1+S_2+ \\ & +W_a^TW_a)D & PB \\ D^T(A^TP+S_1+S_2+ & D^T(S_1+S_2)D-S_1+ & 0 \\ +W_a^TW_a) & +D^TW_a^TW_aD & \\ B^TP & 0 & W_b^TW_b-S_2 \end{bmatrix} < 0.$$

In conclusion, there exists some $\beta > 0$ such that:

$$\dot{V}(x_t) \leq -\beta\|\mathcal{D}x_t\|^2. \tag{18}$$

The inequalities (14) and (18) allow us to conclude the uniform asymptotic stability of the trivial solution of the neutral differential equation 1 (see Appendix A above or [10], Theorem 8.1, pp. 292–293).

Furthermore, the negativity of the Lyapunov functional candidate does not use any information about the delay size and in conclusion, we have the *delay-independent* robust stability property. □

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