KYBERNETIKA — VOLUME 35 (1999), NUMBER 1, PAGES 83-91

AN APPLICATION OF THE EXPECTATION-MAXIMIZATION ALGORITHM TO INTERFERENCE REJECTION FOR DIRECT-SEQUENCE SPREAD-SPECTRUM SIGNALS

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For a direct-sequence spread-spectrum (DS-SS) system we pose and solve the problem of maximum-likelihood (ML) sequence estimation in the presence of narrowband interference, using the expectation-maximization (EM) algorithm. It is seen that the iterative EM algorithm obtains at each iteration an estimate of the interference which is then subtracted from the data before a new sequence estimate is produced. Both uncoded and trellis coded systems are studied, and the EM-based algorithm is seen to perform well, outperforming a receiver that uses an optimized notch filter to remove the intereference, especially for large interference levels.

1. INTRODUCTION

With the proliferation of wireless communication products and the crowding of the radio frequency spectrum, the problem of combatting interference has become more pronounced. For example, in the unlincensed industrial, scientific and medical (ISM) bands, in which the so-called FCC Part 15 devices (cordless phones, wireless ethernet cards, etc.) operate, users must be able to sustain interference. Many of the systems in these bands use spread spectrum technology, which is known to be robust to narrowband interference and multipath. Spread spectrum alone, however, is not enough to alleviate the interference problem, and further steps are needed to combat it, especially in severe interference environments.

There are in general two ways to further reduce interference: 1) by preventing it from entering the receiver front-end through appropriate antenna design (i.e. "smart antennas"), and/or 2) by suitably processing the received signal in order to negate the effects cf interference. The work we present next belongs to the second category of interference rejection techniques. In contrast to most algorithms, however, which focus on estimating the interference (using one technique or another) and then subtracting it from the received signal, in this paper we pose the problem as one of maximum-likelihood (ML) sequence estimation (i.e. we use a minimum error probability criterion).

To make the problem of obtaining ML estimates tractable, we use the expectation-maximization (EM) algorithm [2, 10], and apply it first to the simple problem of single-tone interference, where the interfering frequency is known, but either the phase or the amplitude are unknown. This problem is admittedly not realistic, but it does serve to illustrate the use of the EM algorithm and to assess its potential performance compared to other techniques. For the single-tone interference problem, an obvious (but suboptimal) technique for combatting the interference is to use a notch filter, which however, besides suppressing the interference, also suppresses part of the signal. We will see next, that the EM-based algorithm significantly outperforms the notch filter approach, particularly at large interference levels.

For an excellent tutorial on interference rejection techniques the interested reader is referred to [8]. Other applications of the EM algorithm to communication scenarios include [1] – [6].

Section 2 introduces the EM-based algorithms, Section 3 looks at performance and makes comparisons, and Section 4 concludes.

2. THE EM-BASED ALGORITHMS FOR INTERFERENCE REJECTION

The EM algorithm is based on the notion of *complete* and *incomplete* data. The incomplete data consist of the data actually oberved, from which a ML estimates must be obtained. The complete data is a set of desirable data, whose availability makes the estimation problem easy in some sense.

The EM algorithm proceeds as follows. Suppose $\boldsymbol{x}$ and $\boldsymbol{y}$ are the complete and incomplete data respectively, and $\boldsymbol{b}$ is a parameter vector to be estimated. The two-step iterative algorithm at the ith iteration is:

1. E–step: Compute $Q(\boldsymbol{b}|\boldsymbol{b}^i) = E[\log p(\boldsymbol{x}|\boldsymbol{b})|\boldsymbol{y}, \boldsymbol{b}^i]$,
2. M–step: Compute $\boldsymbol{b}^{i+1} = \arg\max_{\boldsymbol{b}} Q(\boldsymbol{b}|\boldsymbol{b}^i)$,

where $\boldsymbol{b}^i$ is the estimated parameter at the ith step, and $p(\boldsymbol{x}|\boldsymbol{b})$ is the conditional density of $\boldsymbol{x}$, given $\boldsymbol{b}$.

We apply the algorithm to the case of single-tone interference next.

A. Single-tone interference with random phase

Let the single-tone interference be

$$J(t) = B\cos(\omega t + \theta), \tag{1}$$

where θ is a uniformly distributed random phase, and B and ω are known amplitude and frequency respectively. The received signal in an additive white Gaussian noise $n(t)$ of spectral density $N_0/2$ is then

$$r(t) = S(t; \boldsymbol{a}) + J(t) + n(t), \tag{2}$$

where

$$S(t; \boldsymbol{a}) = A\sum_i \sum_k a_i c_k p(t - kT_c - iT) \tag{3}$$

is the baseband spread spectrum signal, $\{c_k\}$ is the spreading sequence, $p(t)$ is the baseband pulse, T_c and T are the chip and bit intervals, respectively, A is the signal amplitude, and $\{a_i\}$ is the data sequence with data taking values in $\{-1,+1\}$. The problem is to estimate **a** from $r(t)$, using the EM algorithm. Towards this end, we choose the complete data as $[r(t),\theta]$. Then the E-Step of the EM iteration is:

$$Q(\boldsymbol{a}|\boldsymbol{a}^k) = E\big[\log p[r(t),\theta|\boldsymbol{a}]|r(t),\boldsymbol{a}^k\big], \tag{4}$$

where $\boldsymbol{a}^k$ is the sequence estimate at the kth iteration, and $\log p[r(t),\theta|\boldsymbol{a}]$ is the log-likelihood function for the complete data. After some simplifications and manipulations, we obtain

$$Q(\boldsymbol{a}|\boldsymbol{a}^k) = \int \Big[r(t) - \hat{J}(t,\boldsymbol{a}^k)\Big] S(t,\boldsymbol{a})\,\mathrm{d}t, \tag{5}$$

where

$$\hat{J}(t,\boldsymbol{a}^k) = B\frac{I_1[C(\boldsymbol{a}^k)]}{I_0[C(\boldsymbol{a}^k)]}\cos\big[\omega t - \hat{\theta}(\boldsymbol{a}^k)\big], \tag{6}$$

$$C_1(\boldsymbol{a}^k) = \tfrac{2B}{N_0}\int_{-\infty}^{+\infty} \big[r(t) - S(t,\boldsymbol{a}^k)\big]\cos(\omega t)\,\mathrm{d}t, \tag{7}$$

$$C_2(\boldsymbol{a}^k) = \tfrac{2B}{N_0}\int_{-\infty}^{+\infty} \big[r(t) - S(t,\boldsymbol{a}^k)\big]\sin(\omega t)\,\mathrm{d}t, \tag{8}$$

$$C(\boldsymbol{a}^k) = \sqrt{C_1^2(\boldsymbol{a}^k) + C_2^2(\boldsymbol{a}^k)}, \tag{9}$$

$$\hat{\theta}(\boldsymbol{a}^k) = \arctan\left[\tfrac{C_2(\boldsymbol{a}^k)}{C_1(\boldsymbol{a}^k)}\right]. \tag{10}$$

Here the $I_0[\cdot]$ and $I_1[\cdot]$ are the zeroth and first order modified Bessel functions respectively.

The data sequence can be obtained by maximizing $Q(\boldsymbol{a}|\boldsymbol{a}^k)$ over all data sequences $\boldsymbol{a}$. This can be done efficiently through symbol-by-symbol detection when no coding is used, or by using the Viterbi algorithm if trellis coding is used. In initializing the algorithm, we assume (at the start of the iteration process) that $\theta = 0$.

The general structure of the EM-based algorithm is shown in Figure 1.

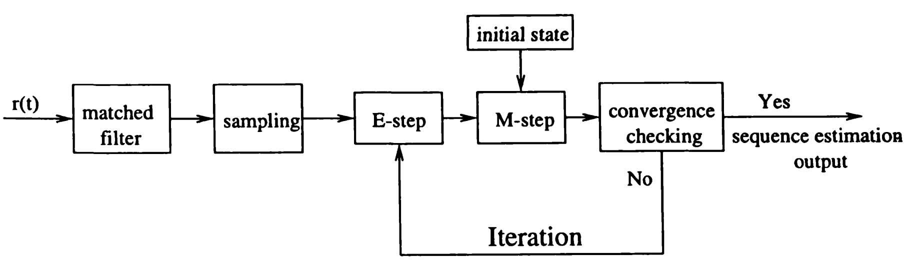

Fig. 1. Structure of the EM-based receiver.

Random amplitude interference

As another application, we assume here that only the amplitude B of the tone interferer is random. We consider two example cases, but others can be solved as easily: 1) when B is uniformly distributed over a known interval; and 2) when B is Rayleigh distributed. In other words,

$$p_B(B) = \frac{1}{\eta}, \qquad 0 \le B \le \eta \tag{11}$$

for a uniform distribution, and

$$p_B(B) = Be^{-\frac{B^2}{2}} \tag{12}$$

for a Rayleigh distribution.

It is easily seen that equation (5) still holds (in fact it holds in general for any interference $J(t)$), where

$$\hat{J}(t, \boldsymbol{a}^k) = \hat{B}\cos(\omega t + \theta). \tag{13}$$

Skipping the derivations, we have:

— Uniform case:

$$\hat{B} = \frac{\int_0^\eta Be^{-K_2(B-\frac{K_1}{2K_2})^2}\mathrm{d}B}{\int_0^\eta e^{-K_2(B-\frac{K_1}{2K_2})^2}\mathrm{d}B}. \tag{14}$$

— Rayleigh case:

$$\hat{B} = \frac{K_6}{K_5}, \tag{15}$$

where,

$$K_1(\boldsymbol{a}^k) = \frac{1}{N_0}\int \left[r(t) - S(t, \boldsymbol{a}^k)\right]\cos(\omega t + \theta)\,\mathrm{d}t, \tag{16}$$

$$K_2 = \frac{1}{2N_0}\int \cos^2(\omega t + \theta)\,\mathrm{d}t, \tag{17}$$

$$K_5 = \int_0^{+\infty} Be^{K_1 B - (K_2 + \frac{1}{2})B^2}\mathrm{d}B, \tag{18}$$

$$K_6 = \int_0^{+\infty} B^2 e^{K_1 B - (K_2 + \frac{1}{2})B^2}\mathrm{d}B. \tag{19}$$

All the time-integrals above are over the data sequence length.

We look at performance next.

3. SIMULATION RESULTS

In this section, we investigate the error-probability performance of the EM-based algorithms for both coded and uncoded systems and compare it to that obtained using a notch filter. Simulations are run for various parameters, such as the observed data sequence length, processing gain, and interference strength. A sampling rate of 10 samples per chip (more than adequate) was used in the simulations. The frequency offset of the tone interferer from the carrier was fixed to about 1/6 of the chip rate. Other offsets were also tried, but it was seen that there was no observable difference in the performance of the EM-based algorithms as a function of frequency offset.

In the figures, J/S is the interference to signal ratio in dB, defined as the ratio of the interference power to the signal power, and L is the observed sequence length in bits. Figure 2 shows the performance of the EM algorithm for interference levels of 10 dB and 20 dB. The comparison is to a conventional detector that ignores the interference, and to the performance of a ML detector in the absence of intereference.

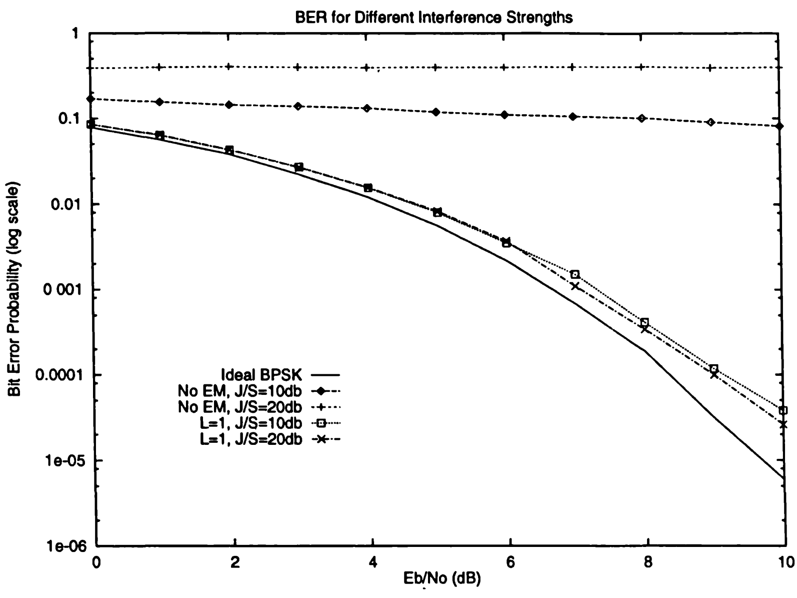

Fig. 2. Performance with and without the EM-algorithm.

It can be seen from Figure 2 that the EM estimator is effective for interference rejection for a large range of interference levels, even when $L = 1$.

Figure 3, which plots performance as a function of interference for an SNR of 8 dB, illustrates this further.

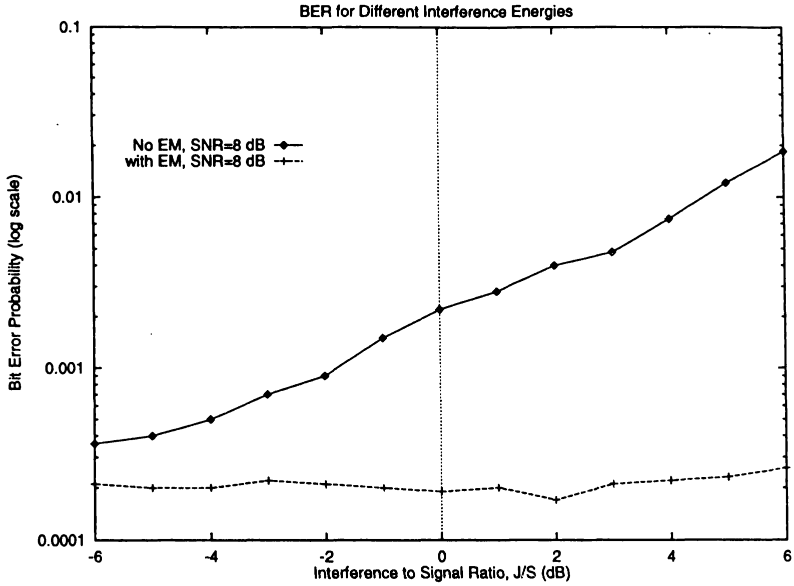

Fig. 3. Performance as a function of intereference level for $L = 1$.

Figure 4 shows the influence of the observed data sequence length L for an interference level of 3 dB.

The figure indicates that a window size of about 5 achieves most the possible performance gain.

Figure 5 shows performance for different chip rates. It can be seen that the EM-based algorithm performs well, even at small processing gains.

Figure 6 shows coded performance for a rate 1/2 4-state convolutional code and for both soft and hard-decision decoding. The structure of the EM-based algorithm allows the use of the Viterbi algorithm for efficient decoding.

The EM-cased algorithm was seen to converge mostly within two to three iterations. Results for the random amplitude case have also been obtained and are similar to those presented above for random phase.

Finally, we compare the performance of the EM algorithm with that of a notch filter, implemented as a two-sided transversal filter and optimized as described in [10]. Figure 7 compares the performance of the notch filter receiver and the EM-based algorithm for both the random phase and amplitude cases.

Here the filter is implemented using 13 taps, the processing gain is 31, and the interference to signal ratio is 10 dB. The observation window length for the EM algorithm is 5. It can be seen that the performance is improved by using the EM algorithm, but at the cost of increased computational complexity.

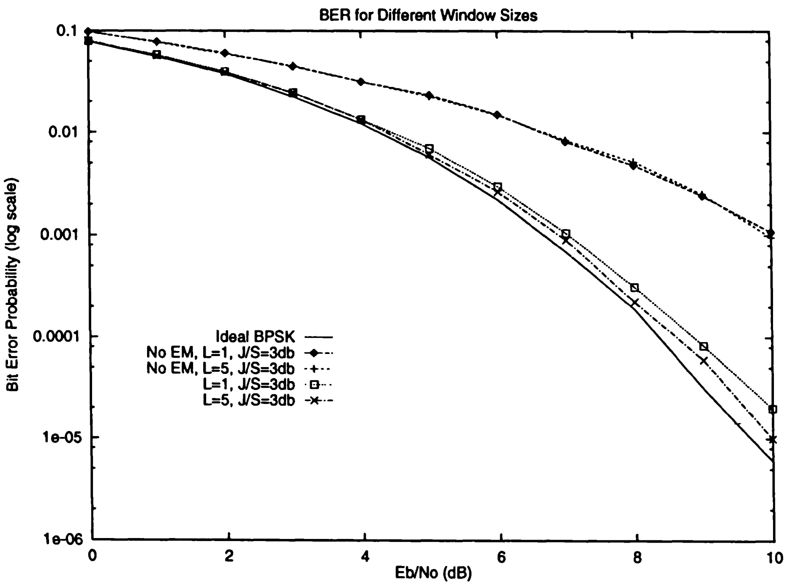

Fig. 4. BER as a function of window length.

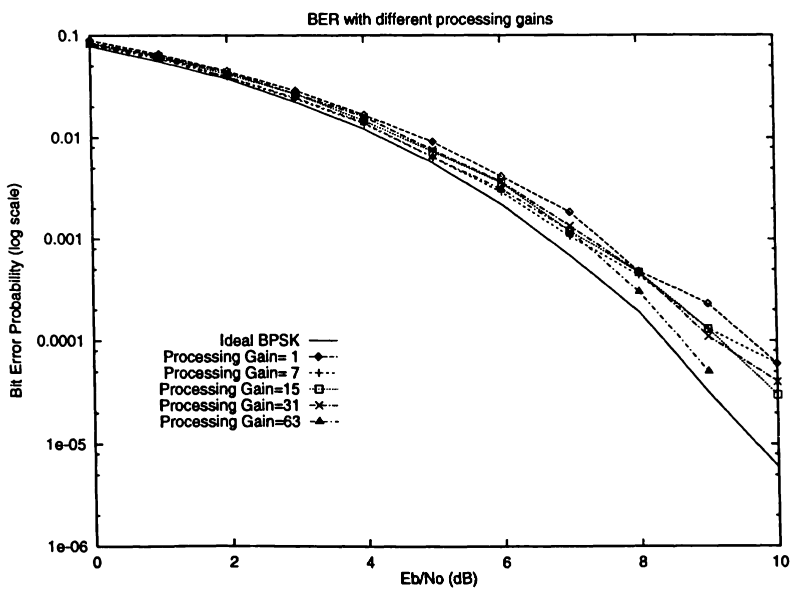

Fig. 5. Performance comparison under different precessing gains. The $L = 1$ and $J/S = 10\,\text{dB}$.

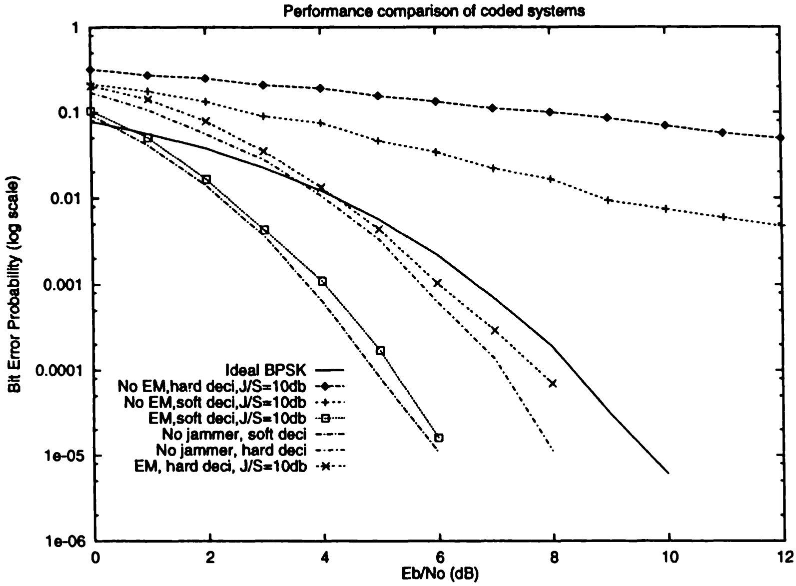

Fig. 6. Performance for coded systems.

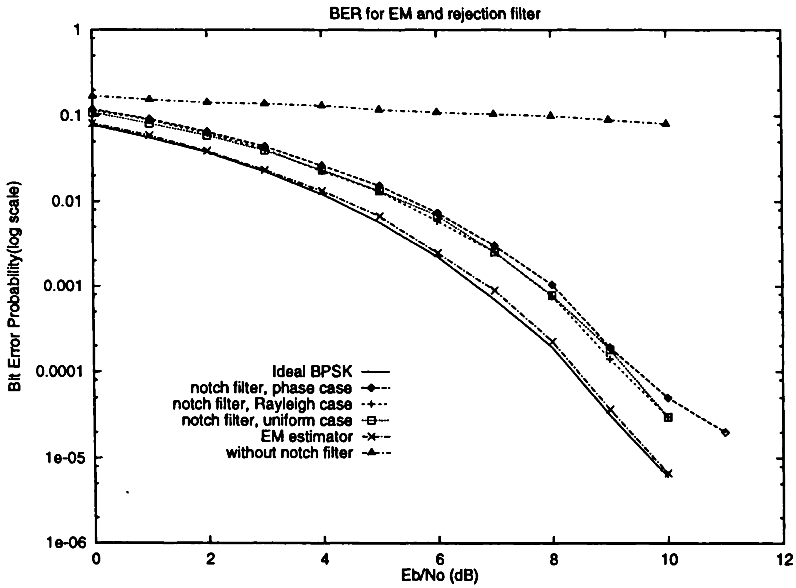

Fig. 7. Comparison between the EM algorithm and the notch filter receiver.

4. CONCLUSIONS

We have applied the EM algorithm to the problem of sequence estimation in the presence of narrowband interference. The EM-based algorithm performed very well, achieving near-optimal performance for a large range of interference levels, at the cost, however, of increased complexity. This increased complexity probably means that the EM-based algorithm will not replace the simple notch filter algorithm for rejecting tone interference. However, the overall success of the EM algorithm does provide motivation for applying it to more general and realistic models of interference, where the increased complexity may be justified by the improved performance compared to alternative algorithms.

(Received April 8, 1998.)

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