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PERFORMANCE OF MULTICHANNEL MULTIACCESS PROTOCOLS WITH RECEIVER COLLISIONS[1]

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The problem of receiver collisions in multichannel multiaccess communication systems is studied in this paper. We develop a Poisson approximation method for the evaluation of the throughput performance measures under receiver collisions consideration assuming receiver buffer with capacity of one packet. Also we calculate the average rejection probability at destination of a packet in order to estimate the effect of receiver collisions on the throughput performance and the total loss probability as a measure of the multichannel system behaviour. The evaluations are carried out for Multichannel Slotted Aloha-type protocols with Poisson arrivals and finite population. Also numerical results are showing the throughput reduction as it compared with the protocol case without receiver collisions.

1. INTRODUCTION

The design and analysis of multichannel networks are becoming increasingly important because they provide facilities for high speed data rates. In high speed networks as wide-band coaxial cables and optical communications, the capacity utilization problem can be faced by multichannel communication techniques. The multichannel solution can be obtained by dividing the bandwidth of a high speed channel or by interconnection of physically separated lower speed channels. Multichannel systems have the added advantages of reliability, adaptability, fault tolerance and performance improvements. There are two causes of packet loss in multichannel systems. First packets are destroyed if two or more stations transmit their packets in the same channel of the multichannel system and they are overlapped in time (channel collisions). Second additional packets are destroyed when a successfully transmitted packet cannot be received by the intended destination because its receiver buffer is 'full'(receiver collisions). In much of the studies related to multichannel multiaccess protocols, the analysis has been done assuming infinite receiver buffer size. However the receiver buffer size used in practice have finite size and implies negative impact on the cost of the required network interfaces for the stations of the system. Thus the choice of the receiver buffer size of a multichannel protocol is widely depended

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on the strictly estimation of the receiver collisions phenomenon and its influence to performance measures. The effect of receiver collisions are rarely studied in the literature. The works reported in [3, 4, 6] are some of these very few studies. In [4] a M-CSMA-IC scheme assumes a simplified model in which the total offered traffic is Poisson and uses the Stirlings numbers of second kind to evaluate the effect of receiver collisions in the system performance for cases of receiver buffer size of one packet. In [3] the M-CSMA/CD protocol is examined for finite number of stations and receiver collisions assumption. In this study simulation techniques are used to estimate the effect of receiver collisions with various receiver buffer sizes. In [6] the M-CSMA/CD protocol with receiver collisions is examined for finite number of stations using discrete time Markov chains in which the probability of correctly received packets at destination is approximately evaluated for cases of receiver buffer size of one packet.

In [2, 5] the concept of receiver collisions is different and it is not related with receiver buffer size. In these cases a receiver collision occurs when a collision-free packet transmission cannot be picked up by the intended destination since the destination's receiver may be tuned to some other channel for receiving data packet from some other source. However in these studies the effect of receiver collisions is ignored.

In this paper, we examine a multichannel model using a) Poisson approximation methods for the total offered traffic to the system b) the number of correctly received packets at destination are also approximated as a Poisson random variable. The material of this paper is organized as follows: In Section 2 the basic assumptions about the examined multichannel multiaccess protocols are given. In Section 3 the analysis of the conventional Multichannel Aloha-type protocol without the effect of receiver collisions is presented assuming Poisson arrivals. Then the throughput performance with effect of receiver collisions is derived based on Poisson approximations statistics. In Section 4 numerical results are presented for various number of channels and stations. Comments on numerical results and explanation of the behaviour are discussed. Also some conclusions are made.

2. MODEL AND ASSUMPTIONS

A multichannel multiaccess communication system consisting of v parallel broadcast channels all of the same capacity is considered. A finite number, M, of stations each one connected by means of separate interfaces to every channel of the system is assumed. The time is slotted on all channels, and these slots are synchronized across all channels. Each station has access to all channels, i. e. it can transmit and/or receive constant length packets that fit to slot size. The round trip propagation delay is small enough (i. e. less than packet transmission time the slot duration) The set of rules that the proposed protocol implies for the stations in the multichannel system are as follows:

1. Every station is equipped with a receiver buffer and a transmitter buffer each one with capacity of one packet. If the transmitter buffer is empty, the station is said to be free, otherwise it is backlogged. If a station is backlogged and generates

a new packet, the packet is lost and never returns.

2. We assume that each packet has a source and a destination address information. A station ready to (re)transmit selects randomly one among the v $(2 \leq v \leq M)$ channels at the beginning of the slot in order to attempt its (re)transmission. Each channel is chosen with equal and constant probability $P_i = 1/v$. If more than one station select the same channel during a time slot to (re)transmit a collision will occur.

3. The successfully (re)transmitted packets are uniformly distributed among the M stations. Thus if two or more stations (re)transmit successfully through different channels during a time slot and their packets destined to same station, the station accepts only one packet in the receiver buffer and rejects all others. This phenomenon is called receiver collision.

4. If a backlogged station retransmits successfully during a time slot and the retransmission is not aborted due to a receiver collision, it becomes free at the next time slot. A free station becomes backlogged in case of a unsuccessful transmission or receiver collisions.

5. The channels are error free and there are no capture phenomena. Thus, packets may be corrupted only because of their concurrent transmission or receiver collisions. We approximate the total number of new transmissions and retransmissions with a Poisson process with mean rate G.

3. ANALYSIS

The throughput reduction induced by receiver collisions is related with the possibility of receiver buffers overflow and this is associated with buffer capacity and the system throughput without the effect of receiver collisions. The possibility of receiver buffer overflow gives rise to rejection probability at destination in steady state which substantiates the throughput loss in quantitative fashion. A more precise way to estimate the impact of packet destruction due to channel collisions and receiver collisions on the throughput performance is the total loss probability. The analysis is composed from two parts. a) Throughput evaluation of conventional multichannel system protocol and b) Throughput analysis of finite receiver buffer size protocol.

3.1. Multichannel slotted ALOHA without receiver collisions

The traffic offered to ith channel is given:

$$G_i = GP_i = G/v. \tag{1}$$

For finite population of stations we adopt the Bertsekas' [1] assumptions for Poisson approximations of the overall traffic G. Thus the throughput per channel in steady state is evaluated as:

$$S_i = G_i e^{-G_i} = \frac{G}{v} e^{-G/v}. \tag{2}$$

Thus the total throughput is:

$$S = \sum_{i=1}^{v} S_i = G e^{-G/v}. \tag{3}$$

3.2. Multichannel slotted ALOHA with receiver collisions

Let

S_{RC} be the multichannel system throughput, conditioning on the receiver collision effect. We still assume that the total offered traffic is Poisson. We define, S_{RC}, as the average number of the correctly received at destination in steady state during a time slot.

S_v be a random variable representing the number of successful (re)transmissions during a time slot from multichannel system.

$H_v(S)$ is a random variable representing the number of different stations selected as destination, given that the mean throughput rate is S packets/slot.

U_n be an indicator function denoting whether a station n ($n = 1, 2, \dots, M$) is selected as destination of packet, i. e.

$$U_n = \begin{cases} +1 & \text{if station } n \text{ is selected during } i\text{th slot} \\ 0 & \text{else.} \end{cases}$$

Let $P_0 = \mathrm{Pr}[U_n = 0]$ in steady state. Consider that $S_v = k$ packets are successfully (re)transmitted from multichannel system during the ith slot. The random distribution of these packets in M stations gives M^k arrangements each with probability M^{-k}. In this case P_0 denotes that no one from k packets are destined to station n. Thus the k packets should be destined to the remaining $M-1$ stations in $(M-1)^k$ different ways. The P_0 conditioning on the $S_v = k$ can be expressed as follows

$$P_0(k) = \frac{1}{M^k}(M-1)^k = [1 - 1/M]^k. \tag{4}$$

Using the approximation $(1-x)^y \approx e^{-xy}$ for small x in (4), we take

$$P_0(k) \approx e^{-k/M}. \tag{5}$$

In steady state $E[S_v = k] = S$ and consequently

$$P_0 = E[P_0(k)] \approx e^{-S/M}. \tag{6}$$

Let P_f be the probability that at least one packet has been destined to station n during a time slot in steady state. Then

$$P_f = \mathrm{Pr}\{Un = 1\} = 1 - P_0 = 1 - e^{-S/M}. \tag{7}$$

The probability $H_v(S) = k$, of finding k different stations that have been selected as destination during a time slot, obeys to binomial probability low.

$$\mathrm{Pr}[H_v(S) = k] = \binom{M}{k} P_f^k P_0^{M-k}. \tag{8}$$

Thus

$$S_{RC} = E\{\mathrm{Pr}[H_v(S) = k]\} = \sum_{k=1}^{M} k \,\mathrm{Pr}[H_v(S) = k] = M(1 - e^{-S/M}). \tag{9}$$

3.2.1. Average rejection probability at destination

The average rejection probability at destination of a packet is evaluated as the ratio of the average number of packet rejection at destination per slot in steady state due to receiver buffer overflow, to the average number of successfully (re)transmitted packets per slot, then

$$P_{\text{rej}} = \frac{S - S_{RC}}{S} \tag{10}$$

An interesting area to evaluate P_{rej} is the point of the maximum stable throughput over multichannel system. It is obvious that this point corresponds to the maximum P_{rej}. If we set the first derivative of the equation (3) with respect to G equal to zero, we find the optimal G that maximize the throughput S of the system. Then we take

$$S_{\max} = v/e, \tag{11}$$

$$G_{\text{opt}} = v. \tag{12}$$

Using the above values we can calculate

$$P_{\text{rej}}(\max) = 1 - \frac{e}{v} M[1 - e^{-v/eM}]. \tag{13}$$

3.2.2. Total loss probability

The total loss probability is defined as the ratio of the average number of packet loss due to transmission collisions over multichannel system and receiver collisions at destination per slot in steady state, to the average number of (re)transmitted packets per slot, then

$$P_{\text{loss}} = \frac{G - S_{RC}}{G}. \tag{14}$$

4. NUMERICAL RESULTS

Figure 1 illustrates the throughput versus the offered traffic G for a $v = 5$, 10, 20 (channel) systems with $M = 50$ stations. It can be observed that throughput measures are on decrease as they compared with the protocol case without receiver collisions for all values of traffic rates. It is interesting to observe that for fixed number of stations, the differences $S - S_{RC}$ are increasing functions of v. The reason is that for a fixed value of G, as v increases, the throughput S increases and consequently the possibility of a packet to be rejected at destination due to receiver collisions is large. For example let $G = 4$, we have for $v = 5$ ($S = 1.797$, $S_{RC} = 1.765$) and $P_{\text{rej}} = 1.776\,\%$), for $v = 10$ ($S = 2.681$, $S_{RC} = 2.610$ and $P_{\text{rej}} = 2.634\,\%$) and for $v = 20$ ($S = 3.275$, $S_{RC} = 3.170$ and $P_{\text{rej}} = 3.204\,\%$).

Figure 2 presents the histogram of the maximum average percent rejection probabilities for $v = 2$, 5, 10, 20 (channel) systems with $M = 50$, 100 stations. It is evident that for fix v as M increases the $P_{\text{rej}}(\max)$ decreases. Also for fix M as v increases $P_{\text{rej}}(\max)$ increases too. In case of $M = 50$ we have for $v{=}5$, $P_{\text{rej}}(\max) = 1.817\,\%$ and for $v = 20$, $P_{\text{rej}} = 7\,\%$. We can say that three parameters characterize the performance behaviour of the multichannel system $v, M, P_{\text{rej}}(\max)$. Figure 3 illustrates the average rejection probabilities versus Traffic G(packets/slot) for $v = 5$, 10, 20 (channel) systems with $M = 50$ stations. For low value of the traffic G, the average rejection probability increases linearly with G (low values of the throughput S). As the G increases approaching G_{opt} and the throughput approaching $S_{\max}$, P_{rej} begins to saturate increasing slowly towards $P_{\text{rej}}(\max)$. For higher values of G ($G > G_{\text{opt}}$) the S is reduced due to channel collisions so P_{rej} decreases because the possibility of collision at destination is low.

Figure 4 depicts the average total loss probabilities versus Traffic G (packets/slot) for $v = 5$, 10, 20 (channel) systems with $M = 50$, 100 stations. It is evident from the figure the dependence of P_{loss} from v. For a given number M of stations as v increases P_{loss} decreases for all values of G. In other words as v increases throughput performance increases too causing lower channel collisions per slot which consequently corresponds to smaller P_{loss} per slot. It is obvious from the figure for fix v and different values of M, the high part of the curves, as G increases P_{loss} curves converge. The reason is that in high load P_{loss} is influenced only from channel collisions because P_{rej} approach zero as Figure 3 indicates.

Figure 5 shows the average total loss probabilities versus Traffic G (packets/slot) for $v = 30$ (channel) system with $M = 50$, 100 stations. It can be seen that P_{loss} take lower values for all values of traffic rates as they compared with P_{loss} without receiver collision consideration. The contribution of receiver collisions on the total loss probability is related with the number of stations of the system. Thus for fix value of v as M increases P_{loss} curves approach the curve of without receiver collision case. The reason as already explained is that as M increases P_{rej} decreases affecting P_{loss} towards lower values. Thus for $v = 30$ and $G = 20$, $P_{\text{loss}} = 48.65\,\%$ without the consideration of receiver collisions. Taking into account receiver collisions we have for $M = 50$, $P_{\text{loss}} = 53.58\,\%$ and for $M = 100$, $P_{\text{loss}} = 51.20\,\%$.

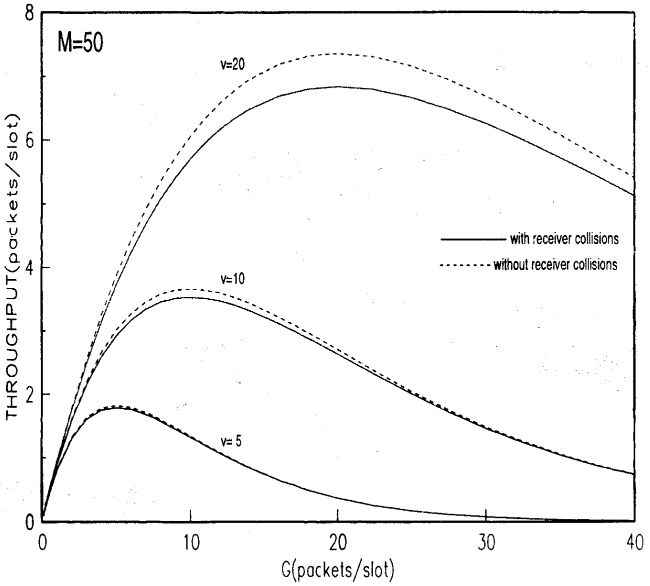

Fig. 1. The throughput versus the offered traffic G characteristics for a $v = 5$, 10, 20 (channel) systems with $M = 50$ stations. Analytical results with and without receiver collisions schemes.

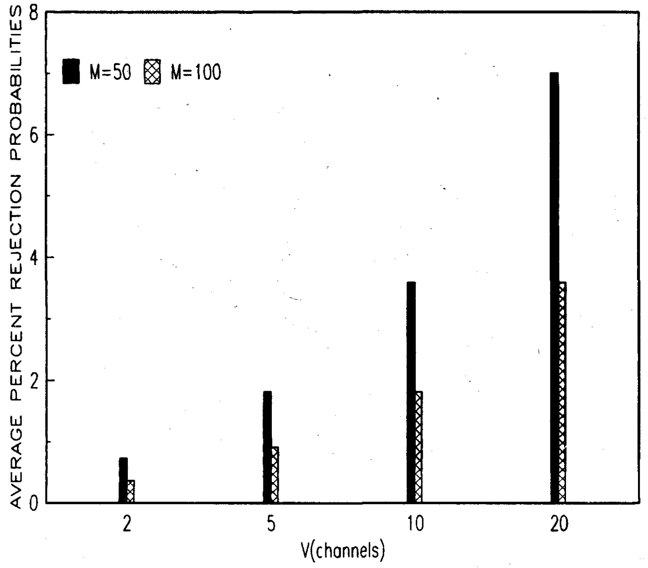

Fig. 2. The histogram of maximum average percent rejection probabilities for $v = 2, 5, 10, 20$ (channel) systems with $M = 50, 100$ stations.

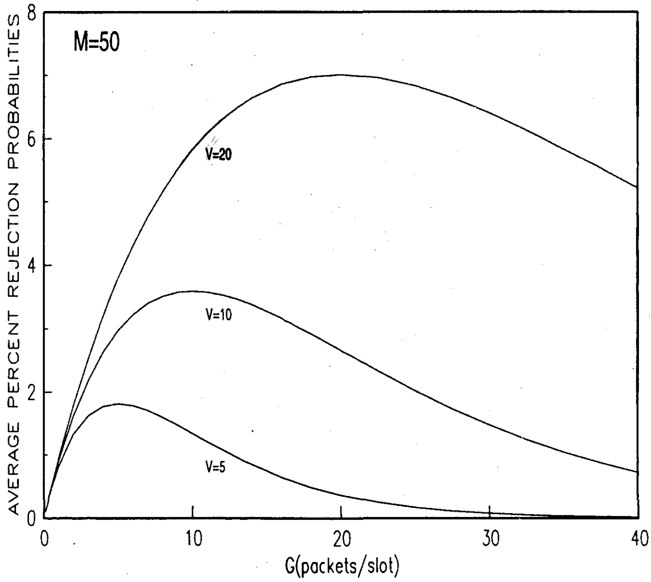

Fig. 3. Average rejection probabilities versus Traffic G (packets/slot) for $v = 5, 10, 20$ (channel) systems with $M = 50$ stations.

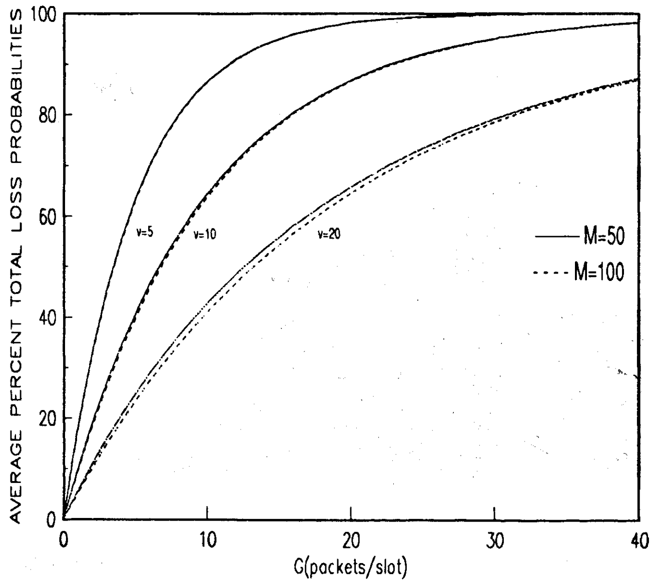

Fig. 4. Average percent total loss probabilities versus Traffic G (packets/slot) for $v = 5, 10, 20$ (channel) systems with $M = 50, 100$ stations.

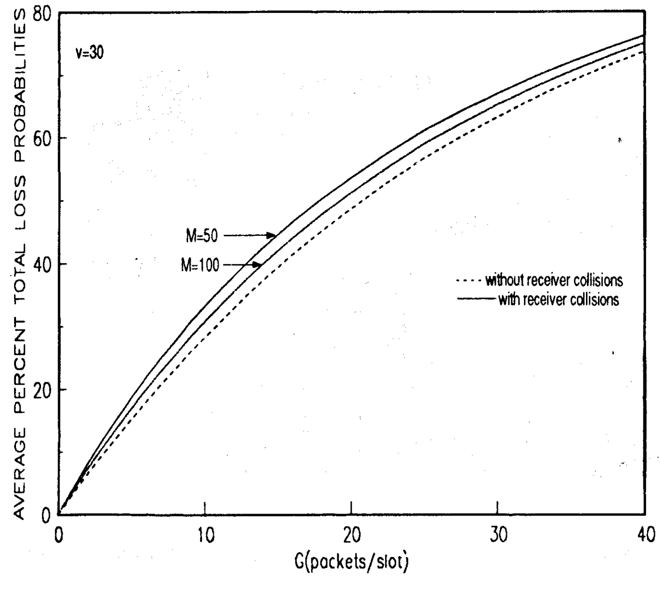

Fig. 5. Average percent total loss probabilities versus Traffic G (packets/slot) for $v = 30$ (channel) system with $M = 50, 100$ stations.

5. CONCLUSIONS

An approximation method which simplifies the evaluation of the throughput performance of a multichannel multiaccess ALOHA-type protocol with receiver collisions is developed. The analysis is based upon Poison assumptions for the total offered traffic over the multichannel system in which each station is equipped with a receiver buffer with capacity of one packet. Also the probability of packet rejection at destination is approximately evaluated using Poisson approximations. An other quantity the total loss probability is studied which presents the total packet loss due to channel collisions and receiver collisions during a time slot. Numerical results prove that three parameters characterize the performance behaviour of the multichannel system $v, M, P_{\text{rej}}(\text{max})$. It was showed that for large population systems the receiver collisions can be ignored with only a small loss of accuracy. In the opposite case in smaller systems the effect of receiver collisions is significant for the performance behaviour which can't be neglected.

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