

# ON-LINE PARAMETER ESTIMATION AND TWO-LEVEL CONTROL OF LARGE-SCALE DISCRETE TIME SYSTEMS

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This paper introduces a new approach for parameter estimation and two-level control of large-scale discrete systems using input-output data. Parallel recursive least square estimation algorithms based on special observable canonical innovation model are considered and subsequently, parallel state estimation schemes are developed by partitioning the conventional Kalman-filter equation. Two-level controller is implemented by using estimated parameters. Global output feedback controller is designed to reduce the effect of state interaction between the subsystems and subsequently, a numerically reliable and stable local controller based on singular value decomposition (SVD) technique is considered to stabilize and improve the performance of the composite system. Simulation studies for parameter estimation and control of discrete time systems are carried out by considering a numerical example of flight control problem.

## 1. INTRODUCTION

A variety of different identification and parameter estimation algorithms for dynamic processes have been received much attention recently [1], [8], [13]. Among the identification algorithms, the ones that perform on-line by sequentially updating the parameter estimates from noisy measurements in a stochastic environment are the most important issue for engineering application. Many of the powerful results of modern stochastic control and estimation theory have been derived on the basis of a state space representation of a system. The state space model usually belongs to the research area of engineering and control. Often such a model is not available a priori and must be deduced from the system in operation. On-line identification of state variable model has been attempted by several research workers. Identification of a nonparametric model and realization of a state space model through transformation was first introduced by Saridis [15]. Identification of an input-output representation, the realization of block-observable canonical state space model was first proposed by Tse and Weinert [19]. Early eighties El-Sherief [3] has proposed an efficient but simpler form of a combined three-stage algorithm in a bootstrap manner for simultaneous state estimation, direct identification and control of a state space model. The state estimates are obtained using the innovation model of Kailath [7]

where the Kalman gain and innovation terms are estimated together with the system parameters. Recently, Ray and Rao [14] have extended the work of El-Sherief [3] to reduce further computational burden and also developed parallel algorithms in order to estimate state and parameters of the system by exploiting a special observable canonical state space model. Algorithm proposed by these authors require the information of state estimates in the parameter estimation stage which causes the additional telemetry costs and moreover, this may leads to converge parameters in wrong values. A recent paper by McElveen and Lee [11] discusses a stochastic approximation technique for multi-input multi-output system identification which is based on state space formulization of a system. Algorithm proposed by them do not require any information of state estimates for parameter estimation but on the other hand, it requires significant computational burden and accordingly it becomes unattractive for large scale systems.

Extensive work has been done in the past 20 years on the subject of system identification and several books have been written, for example, Ljung [9], Eykhoff [5], and Soderstrom and Stoica [18]. Also several survey or review papers has been written on the field: Aström and Eykhoff [2], Young [20], and El-Sherief and Sinha [4]. A class of research workers has pointed to a number of techniques by which the accuracy of an estimated model parameters can be judged and the role of model validation procedures for assessing different kinds of errors can be found in [10].

The aim of this paper is to employ a special observable canonical model in order to reduce computational burden and also an attempt is made to develop a numerically reliable combined parameter estimation and two-level control scheme to regulate the system performance. This paper is organized as follows:

A formal statement of the problem is given in Section 2. In Section 3, we consider the development of proposed parameter estimation based two-level control algorithm for linear time-invariant discrete time systems. In Section 4, we illustrate the application of the proposed scheme by simulating a flight control problem and study the effectiveness of the proposed algorithm. Finally, some conclusions are made in Section 5.

## 2. PROBLEM STATEMENT

Let us consider the following  $m$ -input,  $p$ -output,  $n$ th-order linear time-invariant discrete-time multivariable stochastic system

$$X(k+1) = AX(k) + BU(k) + \tau\omega(k) \quad (1)$$

$$Y(k) = CX(k) + \eta(k) \quad (2)$$

where  $X(k)$  is the  $n$ -dimensional state vector,  $U(k)$  is the  $m$ -dimensional feedback control vector,  $Y(k)$  is the  $p$ -dimensional measured output vector,  $\omega(k)$  is the  $m$ -dimensional input noise vector and  $\eta(k)$  is the  $p$ -dimensional measurement noise vector. It is assumed that the system (1) and (2) is completely controllable and observable and the system matrices are constant but unknown. The sequences  $\omega(k)$  and  $\eta(k)$  are assumed to be white noises and have Gaussian distribution with the

following statistics

$$\begin{aligned} E\{\omega(k)\} &= E\{\eta(k)\} = 0 \\ E\{\omega(k)\omega(l)'\} &= \Omega \delta_{kl} \\ E\{\eta(k)\eta(l)'\} &= \Lambda \delta_{kl} \\ E\{\omega(k)\eta(l)'\} &= 0 \end{aligned}$$

where  $\delta_{kl}$  is the Kronecker delta. It is also assumed that the initial state  $X(0)$  is a Gaussian random variable with known mean  $\bar{X}(0)$  and covariance  $\bar{P}(0)$ .

The above system model (1) and (2) can be transformed into a one-way observable canonical form using a suitable transformation matrix  $M$  [6] and it is given by

$$\bar{X}(k+1) = \bar{A}\bar{X}(k) + \bar{B}U(k) + \bar{\tau}\omega(k) \tag{3}$$

$$Y(k) = \bar{C}\bar{X}(k) + \eta(k) \tag{4}$$

with

$$X(k) = M\bar{X}(k) \text{ and } \bar{A} = M^{-1}AM, \bar{B} = M^{-1}B,$$

$$\bar{\tau} = M^{-1}\tau \text{ and } \bar{C} = CM.$$

The matrices  $A$  and  $C$  have the following special structures.

$$\bar{A} = \begin{bmatrix} \bar{A}_{11} & 0 & \cdots & 0 \\ \bar{A}_{21} & \bar{A}_{22} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \bar{A}_{p1} & \bar{A}_{p2} & \cdots & \bar{A}_{pp} \end{bmatrix}$$

with

$$\bar{A}_{ii} = \begin{bmatrix} -a_{ii}(1) & 1 & 0 & \cdots & 0 \\ -a_{ii}(2) & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots & \cdots & 1 \\ -a_{ii}(n_i) & 0 & 0 & \cdots & 0 \end{bmatrix}_{n_i \times n_i}$$

$$\bar{A}_{ij} \underset{(i>j)}{=} \begin{bmatrix} -a_{ij}(1) & 0 & 0 & \cdots & 0 \\ -a_{ij}(2) & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots & \cdots & 0 \\ -a_{ij}(n_i) & 0 & 0 & \cdots & 0 \end{bmatrix}_{n_i \times n_j}$$

and

$$\bar{C} = \begin{bmatrix} 1 & 0 & 0 \cdots & 0 & 0 \cdots & 0 \cdots & 0 \cdots & 0 \\ 0 & 0 & 0 \cdots & 0 & 1 \cdots & 0 \cdots & 0 \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 \cdots & 0 & 0 \cdots & 0 \cdots & 1 \cdots & 0 \end{bmatrix}_{p \times n}$$

where  $n_i$ ,  $i = 1, 2, \dots, p$  are the observability structural parameters or indices of the system that are assumed, in the present situation, to be known. In general, if structural parameters are not known, they can be estimated in advance from the input-output records [19]. It follows from the observability property assumed for the systems, that the order "n" of the system is related to the set of indices  $n_i$  for the canonical model as follows

$$n = \sum_{i=1}^p n_i.$$

It is necessary to mention here that the matrices  $\bar{B}$  and  $\bar{\tau}$  do not have any special structures and the number of parameters involves in  $\bar{A}$  and  $\bar{B}$  can be expressed as

$$\sum_{j=1}^p (j \times n_i) + n \times m$$

which is much less in number compared to the original system parameters. This transformation in turn reduces the computational burden further for the case where all the system parameters of the model are unknown and need to be estimated on-line to obtain appropriate feedback controller parameters. To ensure identifiability, we transform the system (3) and (4) into the following equivalent steady state innovations model with one noise source  $e(k)$  [7].

$$\bar{X}(k+1) = \bar{A}\bar{X}(k) + \bar{B}U(k) + \bar{K}e(k) \quad (5)$$

$$Y(k) = \bar{C}\bar{X}(k) + e(k) \quad (6)$$

where  $\bar{K}$  is the steady-state Kalman gain matrix of dimension  $(n \times p)$  and  $e(k)$  is innovation sequence with  $E\{e(k)\} = 0$  and  $E\{e(k)e'(l)\} = E_1 \delta_{kl}$  respectively. The aim of the paper is to design an efficient and numerically reliable linear regulator for the system (1) and (2) using the on-line parameter estimation and two-level control scheme that is based on the innovation model (5) and (6).

Exploiting the observable canonical structure, we develop 'p'-parallel recursive least square estimators to estimate the transform system parameters  $\bar{A}$ ,  $\bar{B}$  along with the Kalman filters gain  $\bar{K}$ . After estimating the system parameters, the composite system states  $\bar{X}(k)$  can be estimated directly by partitioning the conventional Kalman filter equation. Subsequently, the updated system parameters are then used to implement a two-level controller and it is discussed in detail in the next section.

### 3. ON-LINE PARAMETER ESTIMATION AND TWO-LEVEL CONTROLLER

Let us consider the system described by the equations (5) and (6), the structures of  $\bar{A}$  and  $\bar{C}$  for the canonical model have already given in Section 2 and the structures

of  $\bar{B}$  and  $\bar{K}$  are given below

$$\bar{B} = \begin{bmatrix} b_{11}(1) & b_{12}(1) & \dots & b_{1m}(1) \\ b_{11}(2) & b_{12}(2) & \dots & b_{1m}(2) \\ \vdots & \vdots & & \vdots \\ b_{11}(n_1) & b_{12}(n_1) & \dots & b_{1m}(n_1) \\ \dots & \dots & & \dots \\ \vdots & \vdots & & \vdots \\ \dots & \dots & & \dots \\ b_{p1}(1) & b_{p2}(1) & \dots & b_{pm}(1) \\ b_{p1}(2) & b_{p2}(2) & \dots & b_{pm}(2) \\ \vdots & \vdots & & \vdots \\ b_{p1}(n_p) & b_{p2}(n_p) & \dots & b_{pm}(n_p) \end{bmatrix}, \bar{K} = \begin{bmatrix} k_{11}(1) & k_{12}(1) & \dots & k_{1m}(1) \\ k_{11}(2) & k_{12}(2) & \dots & k_{1m}(2) \\ \vdots & \vdots & & \vdots \\ k_{11}(n_1) & k_{12}(n_1) & \dots & k_{1m}(n_1) \\ \dots & \dots & & \dots \\ \vdots & \vdots & & \vdots \\ \dots & \dots & & \dots \\ k_{p1}(1) & k_{p2}(1) & \dots & k_{pm}(1) \\ k_{p1}(2) & k_{p2}(2) & \dots & k_{pm}(2) \\ \vdots & \vdots & & \vdots \\ k_{p1}(n_p) & k_{p2}(n_p) & \dots & k_{pm}(n_p) \end{bmatrix}$$

Due to the special structure of the innovation model (5) and (6), one can express the  $i$ th subsystem output  $y_i(k)$  in terms of past outputs, inputs and innovation sequences and obtained the following ARMAX model.

$$y_i(k) = \alpha'_i(k) \theta_i(k-1) + e_i(k), \quad \text{for } i = 1, 2, \dots, p \tag{7}$$

where

$$\begin{aligned} \theta_i(k-1) &= [a_{i1}(1) \dots a_{i1}(n_i) | \dots | a_{ii}(1) \dots a_{ii}(n_i) | \\ & b_{i1}(1) \dots b_{i1}(n_i) | \dots | b_{im}(1) \dots b_{im}(n_i) | d_{i1}(1) \dots d_{i1}(n_i) | \dots | d_{ii}(1) \dots d_{ii}(n_i) | \\ & k_{i,(i+1)}(1) \dots k_{i,(i+1)}(n_i) | \dots | k_{ip}(1) \dots k_{ip}(n_i)]' \quad \text{for } i \leq p \tag{8} \\ d_{ij}(l) &= a_{ij}(l) + k_{ij}(l) \quad \text{for } i = 1, 2, \dots, p; j = 1, 2, \dots, i; l = 1, 2, \dots, n_i \end{aligned}$$

and

$$\begin{aligned} \alpha'_i &= [-y_i(k-1), -y_1(k-2) \dots -y_1(k-n_i) | \dots | -y_i(k-1) \\ & -y_i(k-n_i) | u_1(k-1) \dots u_1(k-n_i) | \dots | u_m(k-1) \dots \\ & \dots u_m(k-n_i) | e_i(k-1) \dots e_i(k-n_i) | \dots | e_p(k-1) \dots e_p(k-n_i) ] \tag{9} \end{aligned}$$

By using a recursive least-square algorithm, one may obtain an unbiased estimate of the parameter vector  $\theta_i$ . More interestingly, equation (7) indicates that the parameter vector  $\theta_i$  for each subsystem can be estimated independently using  $p$ -parallel estimators. The following self-tuning control scheme may be used for identification and control of a multivariable discrete time systems.

*Stage 1:* Parameter estimation based on recursive least-square algorithm [5].

The parameters of the system matrices  $\bar{A}$ ,  $\bar{B}$  and  $\bar{K}$  of (5) can be obtained using the following set of equations for  $i = 1, 2, \dots, p$ .

$$\hat{\theta}_i(k) = \hat{\theta}_i + g_i [y_i(k) - \alpha'_i(k) \hat{\theta}_i(k-1)] \tag{10}$$

$$g_i(k) = P_i(k-1) \alpha_i(k) [1 + \alpha'_i(k) P_i(k-1) \alpha_i(k)]^{-1} \tag{11}$$

$$P_i(k) = P_i(k-1) - \frac{P_i(k-1) \alpha_i(k) [P_i(k-1) \alpha_i(k)]'}{[1 + \alpha_i(k)' P_i(k-1) \alpha_i(k)]} \tag{12}$$

The matrix  $P_i(k)$  is a symmetric matrix and  $\hat{\theta}_i(k)$  is defined as the estimate of  $\theta_i(k)$  at the  $k$ th iteration. The parameter estimation algorithm may be expected to converge to the correct parameter values under certain conditions [6], [9]. It can be noted that the parameter estimates  $\hat{\theta}_i(k)$ , for  $i = 1, 2, \dots, p$ , yield the estimates of the matrices  $\bar{A}$ ,  $\bar{B}$  and  $\bar{K}$  of the innovation model. Estimated parameters can then be used to obtain the states of the system using the following 'p' partitioned Kalman-filter equations.

$$\begin{aligned} \widehat{X}_i(k+1/k) &= \widehat{A}_{ii}(k) \widehat{X}_i(k/k-1) + \sum_{j=1}^{i-1} \widehat{A}_{ij}(k) \widehat{X}_j(k/k-1) \\ &\quad + \sum_{j=1}^m \widehat{B}_{ij} U_j(k) + \sum_{j=1}^p \widehat{K}_{ij}(k) e_j(k) \end{aligned} \tag{13}$$

$$e_i(k) = y_i(k) - \bar{C}_{ii} \widehat{X}_i(k/k-1), \quad i = 1, 2, \dots, p. \tag{14}$$

Stage 2: Design of two-level controller for interconnected discrete time system.

(i) Design procedure of global controller ( $U_g(k)$ ):

Let us rewrite the equations (5) and (6) in the following form

$$\begin{aligned} \bar{X}(k+1) &= \widehat{A}_d(k) \bar{X}(k) + \widehat{A}_{of}(k) \bar{X}(k) + \widehat{B}(k) U(k) + \widehat{K}(k) e(k) \tag{15} \\ Y(k) &= \bar{C} \bar{X}(k) + e(k) \tag{16} \end{aligned}$$

where

$$\begin{aligned} \widehat{A}_d(k) &= \text{diag. block of matrix } \widehat{A} \\ \widehat{A}_{of}(k) &= \text{off diag. block of matrix } \widehat{A} \end{aligned}$$

and

$$\begin{aligned} U(k) &= U_g + U_l(k) \\ &= \text{global output feedback controller} \\ &\quad + \text{local state feedback controller.} \end{aligned}$$

One can select the global output feedback controller gain  $L_g(k)$  in the following manner

$$\begin{aligned} \widehat{B}(k) U_g(k) + \widehat{A}_{of}(k) \bar{X}(k) &= 0 \\ \widehat{B}(k) [-L_g(k) Y(k)] &= -\widehat{A}_{of}(k) \bar{X}(k) \\ \widehat{B}(k) L_g(k) \bar{C} &= \widehat{A}_{of}(k) \\ L_g(k) &= [\widehat{B}'(k) \widehat{B}(k)]^{-1} \widehat{B}'(k) \widehat{A}_{of}(k) \bar{C}' [\bar{C} \bar{C}']^{-1} \end{aligned} \tag{17}$$

to reduce the effect of state interaction between the subsystems [16]. It can be noted that the matrix  $\bar{C} \bar{C}'$  is unity matrix due to the special structure of  $\bar{C}$  and the

rank of the matrix  $\widehat{B}'(k)\widehat{B}(k)$  is 'm). Using the expression (17), the global output feedback control signal  $U_g = -L_g(k)Y(k)$  is generated to reduce the effect of state interactions which in turn makes the composite system weakly coupled.

(ii) Design procedure for local controller ( $L_i(k)$ ):

Let us consider interaction-free  $i$ th subsystem and each decoupled subsystem dynamics is described by the following set of equations (for  $i = 1, 2, \dots, p$ )

$$\overline{X}_i(k+1) = \widehat{A}_{ii}\overline{X}_i + \widehat{B}_{ii}(k)U_{li}(k) + \omega_i^*(k) \tag{18}$$

$$Y_i(k) = \overline{C}_{ii}\overline{X}_i(k) + \eta_i(k) \tag{19}$$

where

$$\omega_i^* = \sum_{j=1}^p \widehat{K}_{ij} e_j(k).$$

It is assumed that the pair  $(\widehat{A}_{ii}(k), \widehat{B}_{ii}(k))$  is controllable. Note that the above two equations are obtained while the global control signal is employed to the transformed system model (3) and (4) and the effect of input interaction terms are not taken into consideration in equation (18). This assumption will simplify the development of a local controller. Local controllers are used to regulate the system performance and also to stabilize the composite system.

Given the system (18) and (19), our problem is now to design a local controller

$$U_{li}(k) = -L_i(k)\widehat{X}_i(k/k-1) \quad \text{for } i = 1, 2, \dots, p \tag{20}$$

so as to minimize the performance index

$$J_i [U_{li}(k)] = E \left[ \overline{X}_i'(k_f) \overline{N}_i \overline{X}_i(k_f) + \sum_{k=0}^{k_f-1} \overline{X}_i'(k) \overline{Q}_i \overline{X}_i(k) + U_{li}'(k) \overline{R}_i U_{li}(k) \right] \tag{21}$$

We have not offered any criterion on the choice of  $\overline{Q}_i$  and  $\overline{R}_i$  matrix in equation (21). A methodology for selection of these weights can be found in standard text book [17]. The optimal control sequences  $\{U_{li}(k)\}$  for  $i = 1, 2, \dots, m$  are obtained using the following procedures (see the Appendix):

The local control law for the  $i$ th subsystem is given by

$$U_{li}(k) = -L_i(k)\widehat{X}_i(k/k-1) \tag{22}$$

and the local controller gain  $L_i(k)$  is computed by the following backward recursion for  $j = k_f - 1, k_f - 2, \dots, 1$ . Now, we construct the following matrix

$$T_i^{(j)} \begin{bmatrix} \sqrt{Q_i^{(j)}} \\ \varphi_i^{(j+1)} \end{bmatrix} = \begin{bmatrix} S_i^{(j)} \\ 0 \end{bmatrix}, \quad \varphi_i^{(k_f)} = \overline{N}_i. \tag{23}$$

Note that the superscript indicates the backward recursion and  $T_i^{(j)}$  is a Householder matrix which transform the above matrix into special form.

Define the following matrices

$$L_i(j) = \begin{bmatrix} S_i(j) \overline{A_{ii}}(j) \\ 0 \end{bmatrix}_{(n_i+m_i) \times n_i} \quad \text{and} \quad E_i(j) = \begin{bmatrix} S_i(j) \overline{B_{ii}}(j) \\ \sqrt{R_i}(j) \end{bmatrix}_{(n_i+m_i) \times m_i} \quad (24)$$

and the singular value decomposition of the real matrix  $E_i(j)$  is a factorization of  $E_i(j)$  into a product of the matrices

$$E_i(j) = P_i(j) \begin{bmatrix} V_i(j) \\ 0 \end{bmatrix} M_i'(j) \quad (25)$$

where  $P_i(j)$  and  $M_i'(j)$  are the orthogonal matrices of dimension  $(m_i+n_i) \times (m_i+n_i)$  and  $(m_i \times m_i)$  respectively. The product of the matrices  $P_i'(j)$  and  $L_i(j)$  can be partitioned in the following manner:

$$P_i'(j) L_i(j) = \begin{bmatrix} \phi_i(j) \\ \varphi_i(j) \end{bmatrix}, \quad \phi_i(j) \in R^{n_i \times n_i}; \quad \varphi_i(j) \in R^{n_i \times n_i}. \quad (26)$$

The stationary values of  $V_i(j)$ ,  $M_i(j)$  and  $\varphi_i(j)$  at  $k$ th time index can be obtained by simply iterating the above equations (23)–(26) in backward recursion and their stationary values are denoted as  $V_i(k)$ ,  $M_i(k)$  and  $\phi_i(k)$  respectively. Then the local controller gain matrix at time index ' $k$ ' is given by

$$L_i(k) = M_i(k) [V_i(k)]^{-1} \phi_i(k) \quad (27)$$

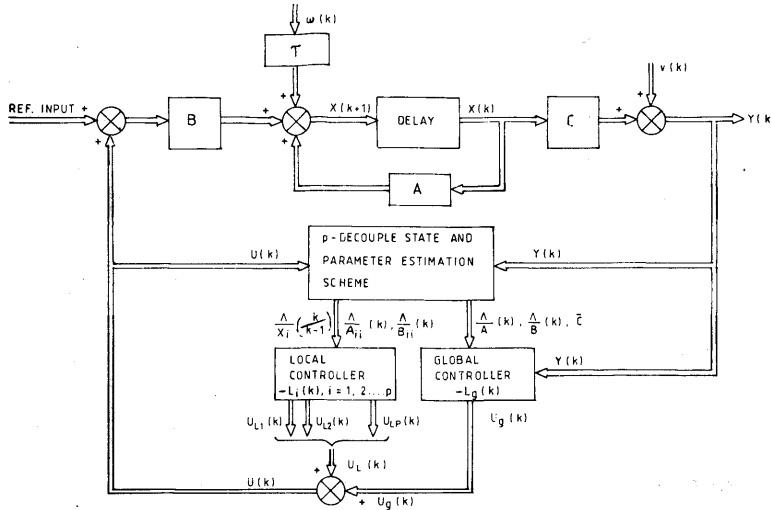


Fig. 1. Block diagram for two level control scheme for unknown systems.

Local controller design based on SVD technique (equations (23)–(26)) exhibits numerically more stable and reliable. Effectiveness of the proposed algorithm will be considered by demonstrating with a numerical example of flight control problem. A block diagram of the proposed control scheme for multivariable unknown discrete time systems is shown in Figure 1.



## 4. RESULTS AND SIMULATION STUDY

In order to study the effectiveness of the proposed scheme, we have considered multi-input multi-output (MIMO) linearized flight control problem [12]. The linearized continuous model is discretized with a sampling period  $T = 0.01$  sec for the MACH=0.9, 10,000 ft MSL flight condition. The discretized model is given below.

$$X(k+1) = \begin{bmatrix} 1.00 & -2.694e-8 & 2.77e-4 & 9.95e-3 \\ -3.216e-1 & 0.9998 & 0.4337 & 0.236 \\ -7.8426e-6 & -3.83e-7 & 0.979 & 9.84e-3 \\ 3.63e-6 & -5.383e-5 & 5.51e-2 & 0.99 \end{bmatrix} X(k) + \begin{bmatrix} 1.59e-3 & -4.97e-4 \\ 3.13e-4 & 2.05e-1 \\ -3.66e-3 & -4.15e-3 \\ -3.18e-1 & -9.92e-4 \end{bmatrix} U(k) + \begin{bmatrix} 1.59e-3 & -4.97e-4 \\ 3.13e-2 & 2.05e-1 \\ -3.66e-1 & -1.15e-3 \\ -3.18e-1 & -9.92e-2 \end{bmatrix} \omega(k) \quad (28)$$

$$y(k) = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 1.0 \\ 1.0 & 0.0 & -1.0 & 1.0 \end{bmatrix} X(k) + \eta(k). \quad (29)$$

The states represents the pitch angle (deg), forward velocity (fps), angle of attack (deg) and pitch rate (deg/sec) respectively. The control vector represents the elevator deflection and flaperon deflection commands.

System (28) - (29) is transformed into the following observable canonical form [6].

$$\bar{X}(k+1) = \begin{bmatrix} 1.9798 & 1 & 0 & 0 \\ -0.9695 & 0 & 0 & 0 \\ -0.0002 & 0 & 1.9524 & 1 \\ 0.0 & 0 & -0.95 & 0 \end{bmatrix} \bar{X}(k) + \begin{bmatrix} -0.3179 & -0.0992 \\ 0.3112 & 0.097 \\ 0.0021 & 0.0037 \\ -0.0020 & -0.0036 \end{bmatrix} U(k) + \begin{bmatrix} -0.3179 & -0.0992 \\ 0.3112 & 0.097 \\ 0.0021 & 0.0037 \\ -0.0020 & -0.0036 \end{bmatrix} \omega(k) \quad (30)$$

$$Y(k) = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix} \bar{X}(k) + \eta(k). \quad (31)$$

The proposed algorithm has been tested by using the following initial data:

- Input noise covariance matrix

$$\Omega = \begin{bmatrix} 0.40 & 0.0 \\ 0.0 & 0.40 \end{bmatrix}$$

- Measurement noise covariance matrix

$$\Lambda = \begin{bmatrix} 0.01 & 0.0 \\ 0.0 & 0.01 \end{bmatrix}$$

- Initial value of state  $X(0) = [0 \ 0 \ 0 \ 0]'$ .

1st stage: Parameter estimation part

- Subsystem 1:

$$\begin{aligned} P_1(0) &= 10^6 \times I_{10 \times 10}, \quad \widehat{X}_1(1/0) = [0 \ 0]' \\ \widehat{\theta}_1(0) &= [0 \ 0 \mid 0.1 \ 0.1 \mid 0.1 \ 0.1 \mid 0 \ 0 \mid 0 \ 0]' \end{aligned}$$

- Subsystem 2:

$$\begin{aligned} P_2(0) &= 10^6 \times I_{12 \times 12}, \quad \widehat{X}_2(1/0) = [0 \ 0]' \\ \widehat{\theta}_2(0) &= [0 \ 0 \mid 0 \ 0 \mid 0.1 \ 0.1 \mid 0.1 \ 0.1 \mid 0 \ 0 \mid 0 \ 0]' \end{aligned}$$

2nd stage: The following initial data are considered for the design of controller.

- i) Centralized controller:

$$Q = \text{diag}[30.0 \ 30.0 \ 9.0 \ 9.0] \quad R = \begin{bmatrix} 15.0 & 0.0 \\ 0.0 & 15.0 \end{bmatrix}$$

- ii) Two-level controller:

Choice for weighting matrices are given below:

- Subsystem 1:

$$Q_1 = \text{diag}[1.0 \ 1.0] \quad \text{and} \quad R_1 = 1.0.$$

- Subsystem 2:

$$Q_2 = \text{diag}[1.0 \ 1.0] \quad \text{and} \quad R_2 = 1.0.$$

Effectiveness of the proposed method is verified by simulating the flight control problem and the results are compared with the centralized scheme. Convergence of model and controller parameter estimates are presented in Figures 2–5. It can be observed from the figures that the performance of the proposed scheme is very close to the centralized method. Tables 1–4 shows the parameter estimates at different iteration stages and error norms of the parameter estimates are also included in the tables.

Table 1. Error Norms =  $\sqrt{\sum (\text{Actual value} - \text{Estimated value})^2}$ ,  
S1 stands for centralized scheme and S2 stands for proposed method.

Scheme used	Transformed system parameters of matrix $\bar{A}$						
		$a_{11}(1)$	$a_{11}(2)$	$a_{21}(1)$	$a_{21}(2)$	$a_{22}(1)$	$a_{22}(2)$
True Value		-1.9798	0.9695	-0.0002	0.0	-1.9524	0.9519
Estimate after 25 iterations	S1	-1.9770	0.9696	0.0006	-0.0008	-1.9815	0.9815
	S2	-1.9700	0.9697	-0.0922	0.0910	-1.7330	0.7333
Estimate after 50 iterations	S1	-1.9690	0.9694	-0.0002	-0.0007	-1.9793	0.9793
	S2	-1.9700	0.9696	-0.0003	0.0	-1.9793	0.9792
Estimate after 75 iterations	S1	-1.9694	0.9694	-0.0002	0.0	-1.9793	0.9793
	S2	-1.9699	0.9695	-0.0002	0.0	-1.9793	0.9793
Error norm after 100 iterations	S1	49.82	17.19	2.39	3.08	3.78	6.42
	S2	51.97	19.91	1.96	1.61	3.94	3.78

**Table 2.** Error Norms =  $\sqrt{\sum (\text{Actual value} - \text{Estimated value})^2}$ .

Scheme used	Transformed system parameters of matrix $\bar{B}$				
	$b_{11}(1)$	$b_{11}(2)$	$b_{12}(2)$	$b_{21}(2)$	$b_{22}(2)$
True Value	-0.3179	0.3112	0.0970	-0.0020	-0.0036
Estimate after 25 iterations	S1 -0.3179	S1 0.3112	S1 0.0970	S1 -0.0023	S1 -0.0037
Estimate after 50 iterations	S2 -0.3176	S2 0.3112	S2 0.0970	S2 0.0277	S2 0.0064
Estimate after 75 iterations	S1 -0.3177	S1 0.3112	S1 0.0969	S1 -0.0021	S1 -0.0037
Error norm after 100 iterations	S2 0.653	S2 11.154	S2 7.047	S2 0.727	S2 0.451
	S1 0.613	S1 16.586	S1 19.235	S1 0.6085	S1 0.7428

**Table 3.**

Centralized controller parameters

	$l_{11}$	$l_{12}$	$l_{13}$	$l_{14}$	$l_{21}$	$l_{22}$	$l_{23}$	$l_{24}$
True Value	3.896	2.558	0.291	0.272	1.230	0.813	-0.341	-0.307
Estimate after 25 iterations	3.436	2.135	0.313	0.293	1.090	0.685	-0.443	-0.399
Estimate after 50 iterations	3.412	2.112	0.305	0.285	1.113	0.707	-0.432	-0.390
Estimate after 75 iterations	3.415	2.115	0.305	0.285	1.110	0.705	-0.433	-0.390
Estimate after 100 iterations	3.414	2.114	0.305	0.285	1.111	0.705	-0.432	-0.390

**Table 4a.**

Global controller parameters (proposed scheme)

	$l_{g,11}$	$l_{g,12}$	$l_{g,13}$	$l_{g,14}$
True Value	0.0102	0.0	-0.0328	0.0
Estimate after 25 iterations	10.6200	0.0	-34.0600	0.0
Estimate after 50 iterations	0.0156	0.0	-0.0501	0.0
Estimate after 75 iterations	0.0123	0.0	-0.0395	0.0
Estimate after 100 iterations	0.0129	0.0	-0.0414	0.0

**Table 4b.**

Local controller parameters (proposed scheme)

	Subsystem 1		Subsystem 2	
	$l_1$	$l_2$	$l_1$	$l_2$
True Value	4.237	3.127	-0.8777	-0.7971
Estimate after 25 iterations	3.530	2.485	-2.7590	-2.6940
Estimate after 50 iterations	3.530	2.482	1.0970	-0.9972
Estimate after 75 iterations	3.530	2.482	-1.0850	-0.9860
Estimate after 100 iterations	3.530	2.482	-1.0870	-0.9878

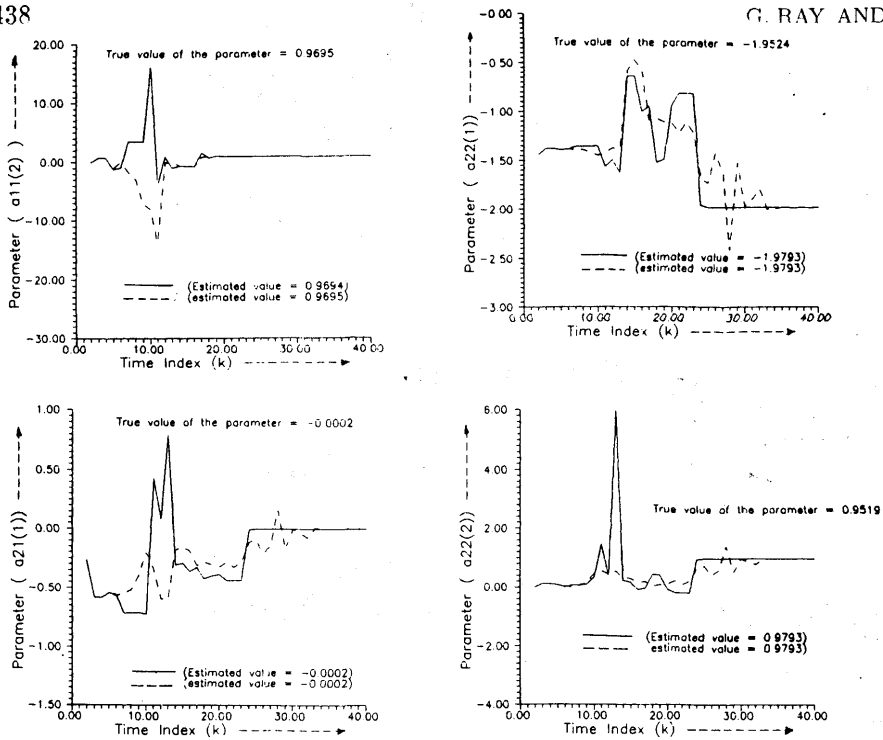


Fig. 2. Convergence of estimates parameters. — Centralized scheme --- Proposed Scheme.

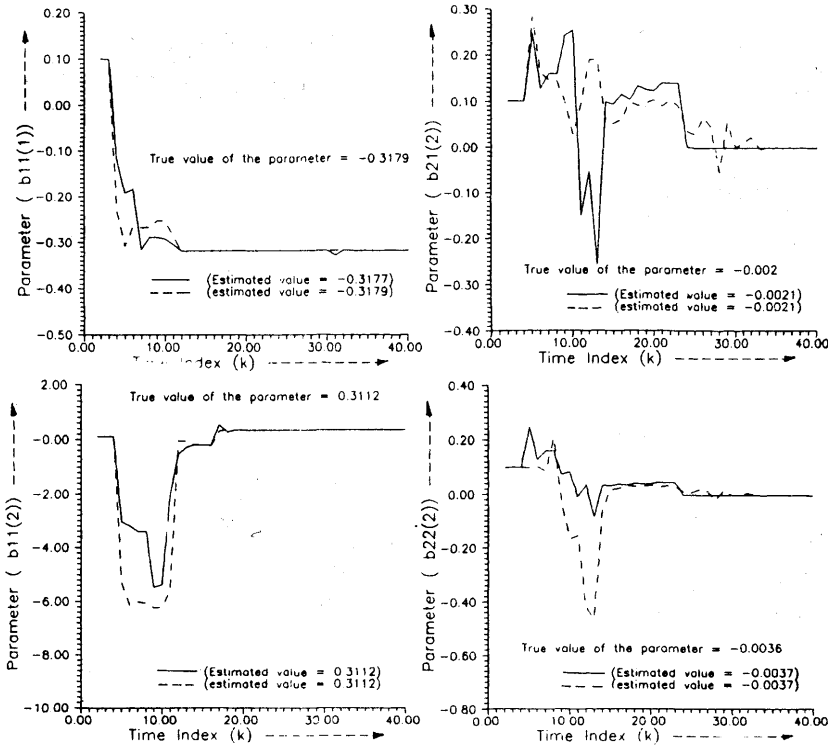


Fig. 3. Convergence of estimates parameters. — Centralized scheme --- Proposed Scheme.

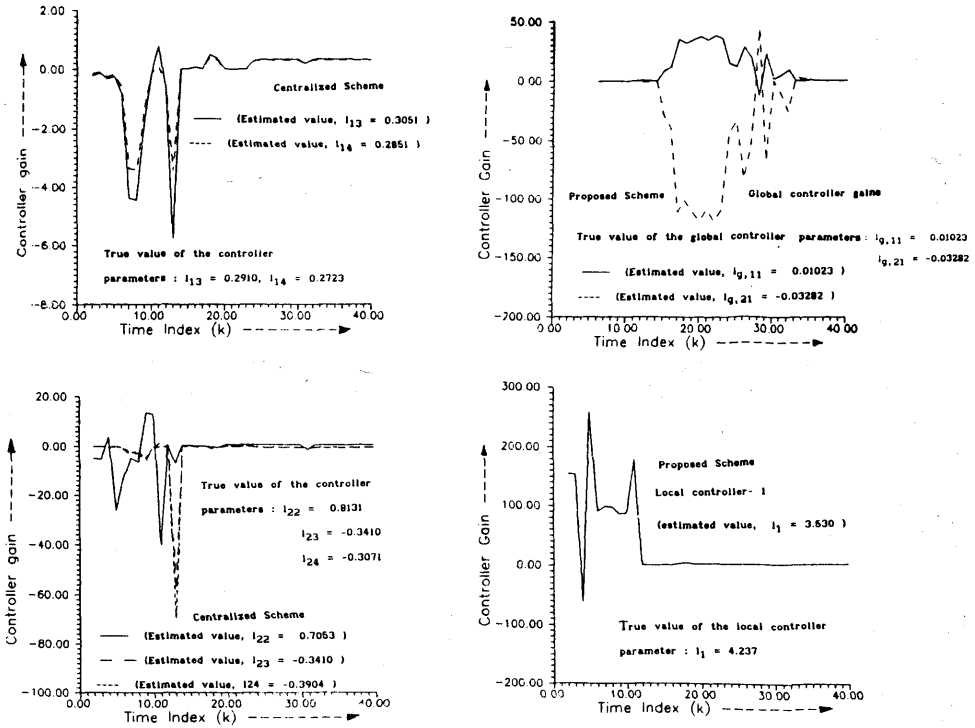


Fig. 4. Convergence of estimates parameters.

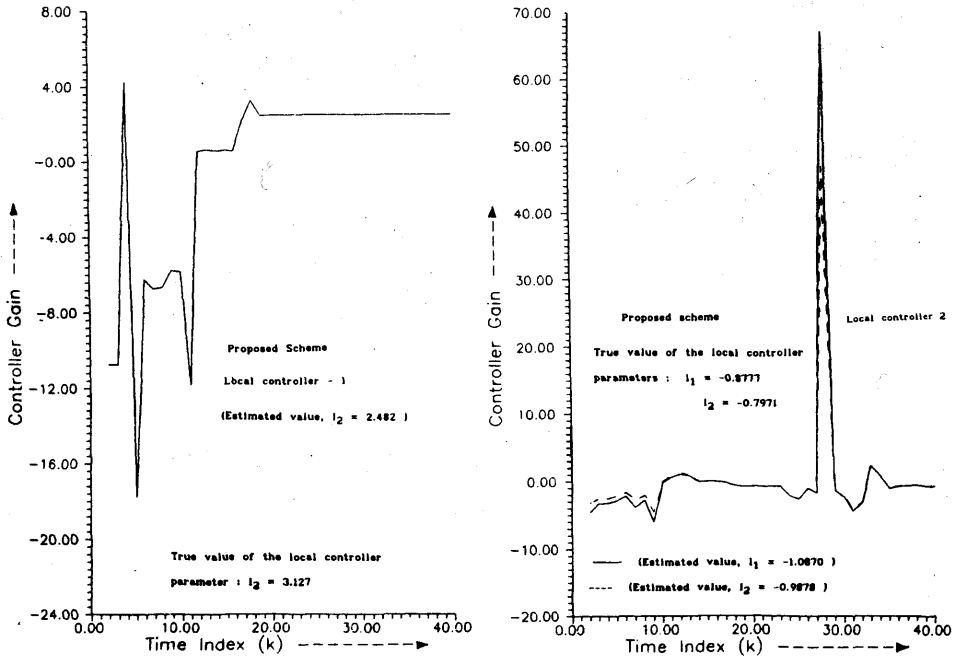


Fig. 5. Convergence of estimates parameters.

## 5. CONCLUSIONS

A new form of the on-line parameter estimation and two-level control of unknown large scale discrete time system is presented in this paper based on the use of an innovation model. Proposed scheme is based on a special observable canonical model (one-way coupling form) and considerably less number of parameters ( $\sum_{i=1}^p (i \times n_i) + n \times m + n \times p$ ) are involved in the parameter estimation algorithm compared to the other existing bootstrap algorithms. It must be noted that the ' $p$ '-parallel parameter estimation schemes and subsequently, ' $p$ '-parallel state estimators are implemented by exploiting the observable canonical structure of the innovation model. Two-level controller is developed for a large scale systems with a view to reduce the effect of state interaction between the subsystem while the global output feedback control law is used and subsequently, numerically sound and accurate method in the formulation of local control law based on SVD technique is designed to achieve regulated system response. It is observed that the conventional linear-quadratic regulator (LQR) based local controller leads to serious numerical problems while the transformed innovation model is weakly controllable and the elements of input matrix  $\bar{B}(k)$  are relatively small. It is also observed that the local controller based on SVD technique increases the computational burden but, on the other hand, the proposed method possess good numerical properties. Results of the proposed method is compared with the centralized controller based on SVD technique. Simulation results are presented in Figures 2-5, which indicates that the proposed method works quite satisfactorily.

## APPENDIX

Let us consider a linear discrete time system is described by

$$X(k+1) = AX(k) + BU(k) \quad (32)$$

$$Y(k) = CX(k) \quad (33)$$

The matrices have the proper dimensions and it is assumed that the  $(A, B)$  is controllable pair. Our aim is to develop an numerically reliable control law that minimizes the quadratic performance index

$$J[U(k)] = E \left[ X'(k_f) N X(k_f) + \sum_{k=0}^{k_f-1} \{X'(k) Q X(k) + U'(k) R U(k)\} \right] \quad (34)$$

Let us assume that the optimal control actions  $\{U^0(0), U^0(1) \dots U^0(k_f - 2)\}$  have already obtained by adopting Bellman's dynamic programming principle of optimality and the cost function that is to be minimized by  $U^0(k_f - 1)$  at the last stage is

$$\begin{aligned} J(k_f - 1) &= X'(k_f) N X(k_f) + U'(k_f - 1) R(k_f - 1) U(k_f - 1) \\ &= [AX(k_f - 1) + BU(k_f - 1)]' N [AX(k_f - 1) + BU(k_f - 1)] \\ &\quad + U'(k_f - 1) R(k_f - 1) U(k_f - 1) \end{aligned}$$

$$= \left\| \begin{bmatrix} \sqrt{N}' A & \sqrt{N}' B \\ 0 & \sqrt{R}'(k_f - 1) \end{bmatrix} \begin{bmatrix} X(k_f - 1) \\ U(k_f - 1) \end{bmatrix} \right\|^2 \tag{35}$$

$$= \|LX(k_f - 1) + Z(k_f - 1)U(k_f - 1)\|^2 \tag{36}$$

where

$$\begin{bmatrix} \sqrt{N}' A \\ 0 \end{bmatrix} = L \quad \text{and} \quad \begin{bmatrix} \sqrt{N}' B \\ \sqrt{R}'(k_f - 1) \end{bmatrix} = Z(k_f - 1).$$

We perform a singular value decomposition on

$$Z(k_f - 1) = P(k_f - 1) \begin{bmatrix} V(k_f - 1) \\ 0 \end{bmatrix} M'(k_f - 1). \tag{37}$$

Using the expression (35), we obtain the following relation from equation (34)

$$\begin{aligned} J(k_f - 1) &= \| V(k_f - 1) M'(k_f - 1) U(k_f - 1) + \phi(k_f - 1) X(k_f - 1) \|^2 \\ &+ \| \varphi(k_f - 1) X(k_f - 1) \|^2 \end{aligned} \tag{38}$$

where

$$P'(k_f - 1)L = \begin{bmatrix} \phi(k_f - 1) \\ \varphi(k_f - 1) \end{bmatrix}.$$

Equation (36) indicates that the minimum of  $J(k_f - 1)$  with respect to  $U(k_f - 1)$  is obtained for

$$U^*(k_f - 1) = -M(k_f - 1)[V(k_f - 1)^{-1}]\phi(k_f - 1) \tag{39}$$

and the corresponding minimum residual is

$$J^*(k_f - 1) = \min_{U(k_f - 1)} J(k_f - 1) = \|\varphi(k_f - 1) X(k_f - 1)\|^2. \tag{40}$$

It is assumed that all control actions  $U(k)$  prior to  $k$ th sequence have been determined, so that  $\{U(k), \dots, U(k_f - 1)\}$  are the only control actions yet to be exerted. Using the principle of optimality, the optimal control action at stage 'k' is obtained by minimizing the following cost function with respect to  $U(k)$ .

$$J(k) = \|X(k + 1)\|_{Q(k)} + \|U(k)\|_{R(k)} + J_1(k + 1). \tag{41}$$

Using the relation (36), we obtained

$$\begin{aligned} J(k) &= \|X(k + 1)\|_{Q(k)} + \|U(k)\|_{R(k)} + \|\varphi(k + 1) X(k + 1)\|^2 \\ &= \left\| \begin{bmatrix} \sqrt{Q(k)'} \\ \varphi(k + 1) \end{bmatrix} X(k + 1) \right\|^2 + \|\sqrt{R(k)'} U(k)\|^2 \\ &= \left\| T(k) \begin{bmatrix} \sqrt{Q(k)'} \\ \varphi(k + 1) \end{bmatrix} X(k + 1) \right\|^2 + \|\sqrt{R(k)'} U(k)\|^2 \\ &= \|\sqrt{S(k)} X(k + 1)\| + \|\sqrt{R(k)} U(k)\|. \end{aligned} \tag{42}$$

Where  $T(k)$  is a Householder matrix such that

$$T(k) \begin{bmatrix} \sqrt{Q(k)}' \\ \varphi(k+1) \end{bmatrix} = \begin{bmatrix} S(k) \\ 0 \end{bmatrix}$$

Now, we can express  $X(k+1)$  in terms of  $X(k)$  and  $U(k)$  using the expression (30) and obtained the following relation

$$\begin{aligned} J(k) &= \left\| \begin{bmatrix} S(k)A & S(k)B \\ 0 & \sqrt{R(k)}' \end{bmatrix} \begin{bmatrix} X(k) \\ U(k) \end{bmatrix} \right\|^2 \\ &= \|L(k)X(k) + Z(k)U(k)\|^2. \end{aligned} \quad (43)$$

The equation has the same form as that of equation (A.5) and using the equations (35)–(37), we can write the control law as

$$U^*(k) = -M(k)V(k)^{-1}\phi(k) \quad (44)$$

The minimum value of  $J$  at  $k$ th stage is thus obtained as

$$J(k) = \min_{U(k) \dots U(k_f-1)} J(k) = \|\varphi(k)X(k)\|^2.$$

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