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OPTIMAL INITIAL FUNCTIONS OF RETARDED CONTROL SYSTEMS[1]

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This paper deals with optimization problem of initial functions and optimality conditions for retarded functional equations for the given cost functions. Under ensuring the regularity of solution of the retarded system we proceed to necessary optimality condition of the optimal solution for cost function J in set of a admissible controls that is closed and convex.

1. INTRODUCTION

In this paper we deal with optimization problem of initial functions and optimality conditions for retarded functional equations for the given cost functions. Under ensuring the regularity of solution of the retarded system we proceed to necessary optimality condition of the optimal solution for cost function J in set of a admissible controls that is closed and convex.

As for the regularity of solution we deduce the results of G. Di Blasio, K. Kunisch and A. Sinestrari [2] regarding term by term. If the admissible set is closed and convex and the cost function J is strictly convex and coercive then there exists an initial function g for which the cost J is minimized subject the retarded functional differential equation as is in [6]. There exist many literatures which studies optimal control problems of control systems in Banach spaces. However, most studies have been devoted to the systems without delay and the papers treating the optimal initial functions for the retarded system with unbounded operators are not so many.

In Section 2, we present some basic results on existence, uniqueness, and a representation formular functional differential equations in Hilbert spaces. We establish a form of a mild solution which is described by the integral equation in terms of fundamental solution using structural operator. In Section 3, 4, we shall give two forms of quadratic cost functions; one is a quadratic cost criterium in linear dynamic system and the other is a feedback control law for regulator problem. First we consider results on the existence and uniqueness of optimal control in the closed convex admissible set. So we present the necessary conditions of optimality which

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are described by the adjoint state and integral inequality. Maximum principle and bang-bang principle for technologically important costs are also given.

2. FUNCTIONAL DIFFERENTIAL EQUATION WITH TIME DELAY

Let V and H be two real Hilbert spaces. The norm on V (resp. H) will be denoted by $\|\cdot\|$ (resp. $|\cdot|$) and the corresponding scalar products will be denoted by $((\cdot,\cdot))$ (resp. $(\cdot,\cdot)$). Assume $V \subset H$, the injection of V into H is continuous and V is dense in H. H will be identified with its dual space. If V^* denotes the dual space, H may be identified with a subspace of V^* and may write $V \subset H \subset V^*$. Since V is dense in H and H is dense in V^* and the corresponding injections are continuous. If an operator A_0 is bounded linear operator from V to V^* and generates an analytic semigroup, then it is easily seen that

$$H = \left\{ x \in V^* : \int_0^T \|A_0 e^{tA_0} x\|_*^2 \, dt < \infty \right\}, \tag{2.1}$$

for the time $T > 0$ where $\|\cdot\|_*$ is the norm of the element of V^*. The realization of A_0 in H which is the restriction of A_0 to

$$D(A_0) = \{u \in V : A_0 u \in H\}$$

is also denoted by A_0. Therefore, in terms of the intermediate theory from (2.1) we can see that

$$(V, V^*)_{\frac{1}{2},2} = H \tag{2.2}$$

where $(V, V^*)_{\frac{1}{2},2}$ denotes the real interpolation space between V and V^*, and hence we can also replace the intermediate space F in the paper [2] with the space H. Hence, from now on we derive the same results of G. Di Blasio, K. Kunisch and A. Sinestrari [2]. Let $a(u,v)$ be a bounded sesquilinear form defined in $V \times V$ satisfying Gårding's inequality

$$\text{Re } a(u,u) \geq c_0\|u\|^2 - c_1|u|^2, \quad c_0 > 0, \quad c_1 \geq 0.$$

Let A_0 be the operator associated with a sesquilinear form

$$(A_0 u, v) = -a(u,v), \quad u,\ v \in V.$$

Then A_0 generates an analytic semigroup in both H and V^* and so the the following equation may be considered as an equation in both H and V^*:

$$\frac{d}{dt}x(t) = A_0 x(t) + A_1 x(t-h) + \int_{-h}^0 a(s) A_2 x(t+s)\, ds + f(t), \tag{2.3}$$

$$x(0) = g^0, \quad x(s) = g^1(s), \quad s \in [-h, 0). \tag{2.4}$$

Let the operators A_1 and A_2 be bounded linear operators from V to V^*. The function $a(\cdot)$ is assumed to be a real valued Hölder continous in $[-h,0]$. Under these conditions, from (2.2) and Theorem 3.3 of [2] we can obtain the following result.

Proposition 2.1. Let $g = (g^0, g^1) \in H \times L^2(-h, 0; V)$ and $f \in L^2(0, T; V^*)$. Then for each $T > 0$, a solution x of the equation (2.3) and (2.4) belongs to

$$L^2(0,T;V) \cap W^{1,2}(0,T;V^*) \subset C([0,T];H).$$

Let Z denote the product reflexive space $H \times L^2(-h, 0; V)$ with the norm

$$\|g\|_Z = \left(|g^0| + \int_{-h}^{0} \|g^1(s)\|^2 \, ds \right)^{\frac{1}{2}}, \quad g = (g^0, g^1) \in Z.$$

The adjoint space Z^* of Z is identified with the product space $H \times L^2(0, T; V^*)$ via duality pairing

$$(g, h)_Z = (g^0, h^0) + \int_{-h}^{0} (g^1(s), h^1(s)) \, ds, \quad g \in Z, \ f \in Z^*$$

where $(\cdot, \cdot)$ denotes the duality pairing. Let $x(t; g, f)$ be the solution of the equation (2.3) and (2.4) with the initial value $g = (g^0, g^1) \in Z$ and $f \in L^2(0, T; V^*)$. According to S. Nakagiri [7], we define the fundamental solution $W(t)$ for (2.3) and (2.4) by

$$W(t)g^0 = \begin{cases} x(t; (g^0, 0), 0), & t \geq 0 \\ 0 & t < 0 \end{cases}$$

for $g^0 \in H$. Since we assume that $a(\cdot)$ is Hölder continuous the fundamental solution exists as seen in [11]. It is known that $W(t)$ is strongly continuous and $A_0 W(t)$ and $dW(t)/dt$ are strongly continuous except at $t = nr$, $n = 0, 1, 2, \ldots$

For each $t > 0$, we introduce the structual operator $F(\cdot)$ from $H \times L^2(0, T; V)$ to $H \times L^2(0, T; V^*)$ defined by

$$\begin{aligned} &Fg = ([Fg]^0, [Fg]^1), \\ &[Fg]^0 = g^0, \\ &[Fg]^1(s) = F_1 g^1(s) = A_1 g^1(-h - s) + \int_{-h}^{s} a(\tau) A_2 g^1(\tau - s) \, d\tau \end{aligned}$$

for $g = (g^0, g^1) \in H \times L^2(0, T; V)$. Then the adjoint $F^* : Z^* \to Z$ is given by

$$[F^* g]^0 = g^0 \quad [F^* g]^1 = F_1^* g^1 \quad \text{for} \quad g = (g^0, g^1) \in Z^*.$$

The solution $x(t) = x(t; g, f)$ of (2.3) and (2.4) is represented by

$$x(t) = W(t) g^0 + \int_{-h}^{0} W(t + s) F_1 g^1(s) \, ds + \int_{0}^{t} W(t - s) f(s) \, ds$$

for $t \geq 0$.

Let $I = [0,T]$, $T > 0$ be a finite interval. We introduce the transposed system which is exactly the same as in S. Nakagiri [8]. Let $q_0^* \in H$, $q_1^* \in L^1(I;H)$. The retarded transposed system in H is defined by

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} + A_0^* y(t) + A_1^* y(t+h) + \int_{-h}^{0} a(s) A_2^* y(t-s)\,\mathrm{d}s + q_1^*(t) = 0 \text{ a.e. } t \in I, \quad (2.5)$$

$$y(T) = q_0^*, \quad y(s) = 0 \text{ a.e. } s \in (T, T+h]. \quad (2.6)$$

Let $W^*(t)$ denote the adjoint of $W(t)$. Then as proved in S. Nakagiri [8], the mild solution of (2.5) and (2.6) is defined as follows:

$$y(t) = W^*(T-t)\,(q_0^*) + \int_t^T W^*(\xi - t)\, q_1^*(\xi)\,\mathrm{d}\xi,$$

for $t \in I$ in the weak sence. The transposed system will be used to describe a formulation of the optimality conditions for initial function optimization problems.

3. OPTIMALITY CONDITION FOR QUADRATIC COST FUNCTION

Let us assume that

$$G_{ad} = G_{ad}^0 \times G_{ad}^1, \quad G_{ad}^0 \subset H, \quad G_{ad}^1 \subset L^2(-h,0;V)$$

and G_{ad}^0, G_{ad}^1 are closed and convex in H and $L^2(-h,0;V)$, respectively. Let $J = J(g,x)$ be the cost function given by

$$J(g) = \int_0^T \|Cx(t) - z_d(t)\|_X^2\,\mathrm{d}t + (Ng,g)_Z,$$

$$(Ng,g)_Z = (N_0 g^0, g^0) + \int_{-h}^{0} ((N_1 g^1(s), g^1(s)))\,\mathrm{d}s$$

where the operator C is a bounded from H to another Hilbert space X and $z_d \in L^2(I;X)$. Finally we are given N_0, N_1 are self adjoint and positive definite:

$$(N_0 g^0, g^0) \ge c|g^0|^2, \quad \text{and} \quad ((N_1 g^1(s), g^1(s))) \ge c\|g^1(s)\|^2, \quad c > 0,$$

or $(Ng,g)_Z \ge c\|g\|_Z$. Let $x_g(t)$ be a solution of (2.3) and (2.4) associated with the initial function $g \in Z$.

Theorem 3.1. Let the operators C and N satisfy the conditions mentioned above. Then there exists a unique element $g \in G_{ad}$ such that

$$J(g) = \inf_{h \in G_{ad}} J(h). \quad (3.1)$$

Furthermore, it is holds the following inequality:

$$(Ng - F^*(p(0), p(\cdot)), h - g) \ge 0$$

where $p(-t)$ is a solution of (2.5) and (2.6) for initial condition $p(s) = 0$ for $s \in [T, T+h]$ substituting q_1^* by $-C^*\Lambda_X(Cx_g(t) - z_d)$. That is, $p(s) = y(-s)$, $s \in [-h, 0]$, satisfies the following transposed system:

$$\frac{dy(t)}{dt} + A_0^* y(t) + A_1^* y(t+h) + \int_{-h}^{0} a(s)\, A_2 y(t-s)\, ds \tag{3.2}$$

$$-C^*\Lambda_X(Cx_g(t) - z_d) = 0 \quad \text{a.e.} \quad t \in I,$$

$$y(T) = 0, \quad y(s) = 0 \quad \text{a.e.} \quad s \in (T, T+h] \tag{3.3}$$

in the weak sense. Here, the operator Λ_U (resp. Λ_X) is the canonical isomorphism of U (resp. X) onto U^* (resp. X^*).

Proof. Let $x(t) = x(t; 0, f)$. Then it holds that

$$\begin{aligned} J(h) &= \int_0^T \|Cx_h(t) - z_d(t)\|^2\, dt + (Nh, h) \\ &= \int_0^T \|C(x_h(t) - x(t)) + Cx(t) - z_d(t)\|^2\, dt + (Nh, h) \\ &= \pi(h, h) - 2L(h) + \int_0^T \|z_d(t) - Cx(t)\|^2\, dt \end{aligned}$$

where

$$\begin{aligned} \pi(g, h) &= \int_0^T (C(x_g(t) - x(t)), C(x_h(t) - x(t)))\, dt + (Ng, h) \\ L(h) &= \int_0^T (z_d(t) - Cx(t), C(x_h(t) - x(t)))\, dt. \end{aligned}$$

Since

$$x_{g+h}(t) = x(t; g+h, f) = x_g(t) + x_h(t) + x(t; 0, f)$$

the map $h \mapsto x_g(\cdot)$ is an affine map of $Z \to H$. Therefore the form $\pi(g, h)$ is a continuous bilinear form on Z and from assumption of the positive definiteness of the operator N we have

$$\pi(h, h) \geq c\|h\|^2, \quad h \in G_{ad}.$$

Therefore in virtue of Theorem 1.1 of Chapter 1 in [6] there exists a unique $g \in Z$ such that (3.1) holds. If g is an optimal initial function (cf. Theorem 1.3. Chapter 1 in [6]), then from

$$\frac{1}{\theta}[J(g + \theta(h - g)) - J(g)] \geq 0$$

it follows that

$$J'(g)\,(h - g) = 2[\pi(g, h - g) - L(h - g)] \geq 0, \quad h \in G_{ad}, \tag{3.4}$$

where $J'(g)h$ means the Fréchet derivative of J at g, applied to h. It is easily seen that

$$x_g'(t)\,(h - g) = (h - g, x_g'(t)) = x_h(t) - x_g(t).$$

Therefore, (3.4) is equivalent to

$$\begin{aligned}&\int_0^T (Cx_g(t) - z_d(t), C(x_h(t) - x_g(t)))\,dt + (Ng, h-g)\\ =& \int_0^T (C^*\Lambda_X(Cx_g(t) - z_d(t)), x_h(t) - x_g(t))\,dt + (Ng, h-g) \geq 0.\end{aligned}$$

Note that $C^* \in B(X^*, H)$ and for ϕ and ψ in H we have

$$\begin{aligned}(C^*\Lambda_X C\psi, \phi) \text{ (duality } H, H) &= (\Lambda_X C\psi, C\pi) \text{ (duality } X^*, X)\\ &= (C\psi, C\phi) \text{ (scalar product in } X)\end{aligned}$$

where duality pairing is also denoted by $(\cdot,\cdot)$. From Fubini's theorem and

$$x_g(t) - x_h(t) = W(t)\,(g^0 - h^0) + \int_{-h}^0 W(t+s)\,[F_1(g^1 - h^1)]\,(s)\,ds$$

we have

$$\begin{aligned}&\int_0^T (C^*\Lambda_X(Cx_g(t) - z_d(t)), x_h(t) - x_g(t))\,dt + (Ng, h-g)\\ &= \left(\int_0^T W(t)^* C^*\Lambda_X(Cx_g(t) - z_d(T)\,dt, h^0 - g^0\right)\\ &+ \int_{-h}^0 \left([F_1^* \int_{-\cdot}^T W^*(t+\cdot)C^*\Lambda_X(Cx_g(t) - z_d(t))dt](s), h^1(s) - g^1(s)\right) ds + (Ng, h-g)\\ &= (Ng - F^*(p(0), p(\cdot)), h-g) \geq 0\end{aligned}$$

where $p(s)$ is given by (3.2) and (3.3), that is, $y(s)$ is following form:

$$p(s) = -\int_{-s}^T W^*(t+s)\,C^*\Lambda_X(Cx_g(t) - z_d(t))\,dt.$$

□

Remark. Identifying the antidual U with U (and also in case X) we need not use the canonical isomorphism Λ_U. But in case where $U \subset H$ it is difficult to lead the dual space U^* since H has already been identified with its dual.

Corollary 3.1 (Maximum principle). Let G_{ad} be bounded and $N = 0$. If g is an optimal solution for J. Then

$$\max_{h \in G_{ad}} (h, F^*(p(0), p(\cdot))) = (g, F^*(p(0), p(\cdot)))$$

where $p(\cdot)$ is given by in Theorem 3.1.

We remark that if G_{ad} is bounded then the set of elements $g \in G_{ad}$ such that (3.1) is a nonempty, closed and convex set in G_{ad}.

Theorem 3.2 (Bang-Bang Principle). Let F and C be one to one mappings. If there is not the initial function g such that $Cx_g(t) = z_d(t)$ a.e., then the optimal initial function g is a bang-bang initial function, i.e., g satisfies $g \in \partial G_{ad}$ where ∂G_{ad} denotes the boundary of G_{ad}.

Proof. On account of Corollary 3.1 it is enough to show that $F^*(p(0), p(\cdot)) \neq 0$ for almost all s. If $p(s) = 0$ for almost all $s \in [-h, 0)$, then since

$$p(s) = -\int_{-s}^{T} W^*(t+s)C^*\Lambda_X(Cx_g(t) - z_d(t))\,\mathrm{d}t,$$

from assumption and Lemma 5.1 in [8] it follows that

$$Cx_g(t) - z_d(t) = 0 \quad \text{a.e.}$$

It is a contradiction. □

4. OPTIMALITY CONDITION FOR REGULAR COST FUNCTION

In this section, the optimal control problem is to find a initial function g which minimizes the cost function

$$J(g) = (Gx(T), x(T))_H + \int_0^T (D(t)\,x(t),\, x(t))_H\,\mathrm{d}t + (Ng, g)$$

where

$$(Ng, g) = (Rg^0, g^0) + \int_{-h}^{0} (g^1(s),\, Q(t)\,g^1(s))\,\mathrm{d}s$$

and $x(\cdot)$ is a solution of (2.3) and (2.4), $G,\ R \in B(H)$ are self adjoint and nonnegative, and $D \in B(0,T;H,H)$ which is a set of all bounded operators on $(0,T)$ and $Q \in B(0,T;U,U)$ are self adjoint and nonnegative, with $Q(t) \geq m$ for some $m > 0$, for almost all t.

Theorem 4.1. Let G_{ad} be closed convex in Z. Then there exists a unique element $g \in G_{ad}$ such that

$$J(g) = \inf_{h \in G_{ad}} J(h). \tag{4.1}$$

Moreover, it holds the following inequality:

$$(F^*(p(0), p(\cdot)) + Ng, h - g) \leq 0,$$

where $y(t) = p(-t)$ is a solution of (2.5) and (2.6) for initial condition $y(T) = -Gx_g(t)$ and $y(s) = 0$ for $s \in (T, T+h]$ substituting $q_1^*(t)$ by $-D(t)\,x_g(t)$. That is, $y(t)$ satisfies the following transposed system:

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} + A_0^* y(t) + A_1^* y(t+h) + \int_{-h}^{0} a(s)\,A_2 y(t-s)\,\mathrm{d}s - D(t)\,x_g(t) = 0 \text{ a.e. } t \in I, \tag{4.2}$$

$$y(T) = -Gx_g(T), \quad y(s) = 0 \quad \text{a.e. } s \in (T, T+h] \tag{4.3}$$

in the weak sense.

Proof. According to the form of solution of the equation (2.3) and (2.4) the map $g \mapsto x_g(t) = x(t;g,f) = x(t;g,0) + x(t;0,f)$ is an affine map of $Z \to H$. If we set

$$\begin{aligned}\pi(g,h) &= (G(x_g(T) - x(T;0,f)), x_h(T) - x(T;0,f)) \\ &+ \int_0^T (D(t)\,(x_g(t) - x(t;0,f)), x_h(t) - x(t;0,f))\,dt \\ &+ (Gx(T;0,f), x(T;0,f)) + \int_0^T (D(t)\,x(t;0,f), x(t;0,f)) + (Ng,h)\end{aligned}$$

and

$$L(h) = (Gx(T;0,f), x_h(T) - x(T;0,f)) + \int_0^T (D(t)x(t;0,f), x_h(t) - x(t;0,f))\,dt$$

then it is written $J(g)$ in the form

$$J(g) = \pi(g,g) - 2L(g).$$

Under the hypotheses on G, D, and Q, there exists a unique u which minimizes J. Then from

$$\begin{aligned}J'(g)\,(h-g) &= 2[\pi(g,h-g) - L(h-g)] \\ &= 2(Gx_g(T), x_h(T) - x_g(T)) \\ &\quad +2\int_0^T (D(t)x_g(t), x_h(t) - x_g(t))\,dt + 2(Ng, h-g),\end{aligned}$$

(4.1) is equivalent to the fact that

$$\begin{aligned}&\left(Gx_g(T), W(T)\,(h^0 - g^0) + (Gx_g(T), \int_{-h}^0 W(T+s)[F_1(h^1 - g^1)](s)\,ds\right) \\ &+\int_0^T (D(t)x_g(t), W(t)\,(h^0 - g^0))\,dt \\ &+\int_0^T \left(D(t)x_g(t), \int_{-h}^0 W(t+s)[F_1(h^1 - g^1)](s)\,ds\right)\,dt + (Ng, h-g) \\ =\ & (W(T)^* Gx_g(T), h^0 - g^0) + \int_{-h}^0 ([F_1^* W(T+\cdot)\,Gx_g(T)](s), h^1(s) - g^1(s))\,ds \\ &+\left(\int_0^T W(t)^* D(t)\,x_g(t)\,dt, h^0 - g^0\right) \\ &+\int_{-h}^0 \left(\left[F_1^* \int_{-\cdot}^T W(t+\cdot)^* D(t)x_g(t)\,dt\right](s), h^1(s) - g^1(s)\right)\,ds + (Ng, h-g) \\ =\ & ((F^*(W(T)^* Gx_g(T), W(T+\cdot)^* Gx_g(T)), h-g)\end{aligned}$$

$$
\begin{aligned}
&+\left(F^*\left(\int_0^T W(t)^* D(t)x_g(t)\,\mathrm{d}t, \int_{-\cdot}^T W(t+\cdot)^* D(t)x_g(t)\,\mathrm{d}t\right), h-g\right)\\
&+(Ng, h-g)\\
= \;& (F^*(p(0), p(\cdot)) + Ng, h-g) \le 0.
\end{aligned}
$$

Hence

$$p(s) = -W^*(T+s)\,Gx_g(T) - \int_{-s}^T W^*(t+s)\,D(t)\,x_g(t)\,\mathrm{d}t$$

solves (4.2) and (4.3).

Remark. If G_{ad} is bounded and $N = 0$, then for the cost function J in Section 4 we can also obtain the pointwise maximum principle and the bang-bang principle.

From now on, we consider the case where $U_{ad} = L^2(0,T;U)$. Let $x_g(t) = x(t;g,0)+\int_0^t W(t-s)\,f(s)\,\mathrm{d}s$ be solution of (2.3) and (2.4). Define $T \in B(L^2(0,T;H))$ and $T_T \in B(L^2(0,T;H), H)$ by

$$
\begin{aligned}
(T\phi)(t) &= \int_0^t W(t-s)\phi(s)\,\mathrm{d}s,\\
T_T\phi &= \int_0^T W(T-s)\phi(s)\,\mathrm{d}s.
\end{aligned}
$$

We also define solution operators $S \in B(L^2(0,T;Z))$ and $S_T \in B(Z,H)$ by

$$(Sg)(t) = x(t;g,0), \quad S_T g = x(T;g,0).$$

Then we can write the cost function as

$$
\begin{aligned}
J(g) &= (G(x(T;g,0)+T_T f), (x(T;g,0)+T_T f))_H \qquad (4.4)\\
&+ (D(x(\cdot;g,0)+Tf), x(\cdot;g,0)+Tf)_{L^2(0,T;H)} + (Ng,g)_{L^2(0,T;Z)}.
\end{aligned}
$$

The adjoint operators T^* and T_T^* are given by

$$(T^*\phi)(t) = \int_t^T W^*(s-t)\,\phi(s)\,\mathrm{d}s, \qquad (4.5)$$

$$(T_T^*\phi)(t) = W^*(T-t)\,\phi. \qquad (4.6)$$

Theorem 4.2. Let $G_{ad} = L^2(0,T;Z)$. Then there exists a unique initial function g such that (4.1) holds and

$$g(t) = -A^{-1}y,$$

where $A = S_T^* G S_T + S^* D S + N$ and $y = S_T^* G T_T f + S^* D T f$.

Proof. The optimal initial function g for J is the unique solution of

$$J'(g)\,h = 0. \qquad (4.7)$$

From (4.4) we have

$$\begin{aligned} J'(g)h &= 2(G(S_T g + T_T f), S_T h)) + 2(D(Sg + Tf), Sh) + 2(Ng, h) \\ &= 2((N + S_T^* G S_T + S^* DS)g, h) + 2(S_T^* G T_T f + S^* DTf, h). \end{aligned}$$

Hence (4.5) is equivalent to that

$$((Ag + S_T^* T_T f + S^* DTf, h) = 0.$$

Since A is self adjoint and bounded below we have $A^{-1} \in B(0,T;H,U)$ (also see Appendix of [3]). Therefore, the proof is complete. □

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REFERENCES

[1] G. Da Prato and L. Lunardi: Stabilizability of integrodifferential parabolic equations. J. Integral Equations *2* (1990), 2, 281–304.
[2] G. Di Blasio, K. Kunisch and E. Sinestrari: L^2-regularity for parabolic partial integrodifferential equations with delay in the highest-order derivative. J. Math. Anal. Appl. *102* (1984), 38–57.
[3] J.S. Gibson: The Riccati integral equations for optimal control problems on Hilbert spaces. SIAM J. Control Optim. *17* (1979), 4, 537–565.
[4] J.M. Jeong: Stabilizability of retarded functional differential equation in Hilbert space. Osaka J. Math. *28* (1991), 347–365.
[5] J.M. Jeong: Retarded functional differential equations with L^1-valued controller. Funkcial. Ekvac. *36* (1993), 71–93.
[6] J.L. Lions: Optimal Control of Systems Governed by Partial Differential Equations. Springer–Verlag, Berlin – New York 1971.
[7] S. Nakagiri: Structural properties of functional differential equations in Banach spaces. Osaka J. Math. *25* (1988), 353–398.
[8] S. Nakagiri: Optimal control of linear retarded systems in Banach space. J. Math. Anal. Appl. *120* (1986), 169–210.
[9] T. Suzuki and M. Yamamoto: Observability, controllability, and feedback stabilizability for evolution equations I. Japan J. Appl. Math. *2* (1985), 211–228.
[10] H. Tanabe: Equations of Evolution. Pitman, London 1979.
[11] H. Tanabe: Fundamental solution of differential equation with time delay in Banach space. Funkcial. Ekvac. *35* (1992), 149–177

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