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LARGE SCALE DYNAMIC SYSTEM STABILIZATION USING THE PRINCIPLE OF DOMINANT SUBSYSTEMS APPROACH

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This paper considers the problem of stabilizing a large scale dynamic system via decentralized control using the principle of dominant subsystems approach. Sufficient conditions for the existence of local decentralized control laws stabilizing a given large scale dynamic system with dynamic and parametric uncertainties are derived in terms of controller parameters for incompletely known continuous- and discrete-time systems.

1. INTRODUCTION

In order to receive practically applicable control of Large Scale Dynamic Systems (LSDS), the decentralized controllers (DC) have to be used. These controllers consist of several independent control stations, each of which observes only the local output of a subsystem and controls only the local input. It is desired to find the decentralized control of LSDS which satisfies the following requirements [17]:

– The structure and parameters of DC are designed using only the subsystem mathematical models so that the overall system stability is ensured and a desired control performance quality is achieved.

– The DC has to be robust with respect to

1. changes of the interconnections between subsystems,
2. changes of the structure and parameters of other subsystems, and
3. changes of structure and parameters of subsystem itself.

– The decentralized control laws have to be obtained in a decentralized design procedure.

All of these requirements demand the new approches to the design of the decentralized control. In general, the decentralized adaptive control strategies are closely related with the robust SISO adaptive control methods as for example, applications of model reference adaptive controllers [4]. The literature is mainly concerned to the centralized design procedure, in which all DC are determined using the known mathematical model of LSDS, for example, for linear dynamic systems in [1, 12, 10], nonlinear dynamic systems in [5, 13] and adaptive decentralized control in [6, 10].

Decentralized procedures can be found rarely [7, 10]. Various approaches were proposed for the solution of the state control problem for a class of systems stabilizable by the DC. The LSDS subsystems dynamic are supposed to be known while the nonlinear-time-varying interconnections are unknown [4, 10]. Recently [16, 17, 18], we have presented the decentralized adaptive control method stabilizing LSDS using the principle of dominant subsystems. In this paper, we pursue the same idea and the presented results are a generalization of the pole placement method for linear system to the nonlinear-time-varying case.

The remainder of this paper is organized as follows. In Section 2 mathematical description of an investigated system and problem formulation are given. For a given nonlinear-time-varying LSDS with uncertainties, sufficient conditions for the choice of structure and parameters of local continuous and discrete-time controllers which ensure stability of composite systems are derived in Sections 3 and 4.

2. PROBLEM STATEMENT

Consider a nonlinear-time-varying LSDS which can be split into N subsystems:

$$\dot{x}_i = f_i(x_i, t) + b_i(x_i, u_i, t) + h_i(x, t) \qquad i \in \mathcal{N} = \{1, 2, \ldots, N\} \tag{1}$$

where $x_i \in R^{n_i}$, $u_i \in R^{m_i}$ are the local state and control vectors of the ith subsystem, respectively, $n = \sum_{i=1}^{N} n_i$, $m = \sum_{i=1}^{N} m_i$; $f_i(\cdot)$, $b_i(\cdot)$, $h_i(\cdot)$ are continuous and uniformly bounded vector functions, differentiable on the set $R_{\rho_i} \times T \times R^{m_i}$ with respect to variables of the system

$$R_{\rho_i} = \{x_i \in R^{n_i} : \ \|x_i\| \le \rho_i\}, \quad \rho_i > 0$$

and

$$\begin{aligned} & f_i(0, t) = b_i(x_i, 0, t) = h_i(0, t) = 0 \\ & \|h_i(x, t)\| \le \xi_i \|x\|, \quad \forall\, (x, t) \in R^n \times T \\ & T = \langle t_0, \infty), \ x_i(t_0) = x_{i0}, \ i \in \mathcal{N} \end{aligned} \tag{2}$$

The numbers $\xi_i \ge 0$ are supposed to be unknown. The problem of the local control agent is to find adaptive DC of the form:

$$u_i = q_i(x_i, r_i, t) \tag{3}$$

$$\dot{r}_i = g_i(x_i, r_i, t) \tag{4}$$

such that the closed-loop system consisting of the plant (1) and the adaptive decentralized controllers (ADC) (3) and (4) is stable under the disturbances and uncertainties defined below.

In (3) and (4), $r_i \in R^{p_i}$ collects the local controller parameters which will be adapted and $g_i(0, r_i, t) = 0$, $r_i(t_0) = r_{i0}$, $\lim_{t \to \infty} r_i = r_i^* \in R_s$, for $i \in \mathcal{N}$, where $R_s = \{r_i \in R^{p_i}$: system (1) with (3) and (4) is stable in Lyapunov sense, $i \in \mathcal{N}\}$.

3. LARGE SCALE SYSTEM STABILIZATION PROBLEM

3.1. Continuous time case

Recently, we have presented the decentralized adaptive control method stabilizing a LSDS using the principle of dominant subsystem approach. From [3] we recall some preliminary results.

Definition 1. The square matrix $W = [w_{ij}]_{N\times N}$ is diagonally dominant if there exist $d_j > 0$ $(j = 1, 2, \dots, N)$ such that

$$d_i|w_{ii}| > \sum_{j=1, j\neq i}^{N} d_j|w_{ij}|, \quad i = 1, 2, \dots, N$$

or

$$d_j|w_{jj}| > \sum_{i=1, i\neq j}^{N} d_i|w_{ij}|, \quad j = 1, 2, \dots, N. \tag{5}$$

The square matrix W is negative diagonally dominant if it is diagonally dominant and $w_{ii} < 0$, $i \in \mathcal{N}$. The square matrix W is called M-matrix if $w_{ij} \geq 0$ for all off-diagonal elements of W.

Lemma 1. M-matrix is stable if it is negative diagonally dominant.

Let us refer a square matrix W as an aggregation matrix [18, 19] of the investigated system (1), (3) and (4) in the form

$$\frac{\mathrm{d}v_a}{\mathrm{d}t} \leq W v_a \tag{6}$$

where $v_a^{\mathrm{T}} = [v_{1a}, \dots, v_{Na}]$ is a vector Lyapunov function.

The entries of v_a are the Lyapunov functions of isolated subsystems with ADC.

In order to ensure the negative diagonal dominance of the investigated system, the stability measure of the ith subsystem given by the formula

$$\alpha_i = -\frac{\mathrm{d}v_{ia}/\mathrm{d}t}{v_{ia}} \geq -w_{ii} \tag{7}$$

has to be increased when the system (1) with ADC (3) and (4) is not stable and/or a desired control performance quality is not achieved. From Lemma 1, it is obvious that if the LSDS is stabilizable by the supposed ADC there must exist such values of diagonal entries w_{ii} $(i \in \mathcal{N})$ of the matrix W that the investigated system will be stable. The design problem is to find DC (2) and (3) which ensure the negative diagonal dominance of the investigated system.

On the set R_{pi}, determine the function $v_{ia} : R^{n_i} \times T \to R_+$ as a Lyapunov function of the ith isolated subsystem

$$\dot{x}_i = f_i(x_i, t), \quad i \in \mathcal{N}. \tag{8}$$

The entries of vector Lyapunov function (6) can be taken as [8]

$$v_{ia} = v_i + (r_i - r_i^\star)^{\mathrm{T}} (r_i - r_i^\star) \tag{9}$$

The conditions for the change of the stability measure of the i th subsystem as a function of local controller parameters r_i are given by:

$$\frac{\partial \alpha_i}{\partial r_i} = -\frac{\partial}{\partial r_i}\left(\frac{\mathrm{d}v_{ia}/\mathrm{d}t}{v_{ia}}\right) = \frac{-\left\{\frac{\partial}{\partial r_i}[(\operatorname{grad} v_i)^{\mathrm{T}} b_i] + 2\dot{r}_i\right\} v_{ia} + 2\frac{\mathrm{d}v_{ia}}{\mathrm{d}t}(r_i - r_i^\star)}{v_{ia}^2} \neq 0 \tag{10}$$

for $\|x_i\| \neq 0$ and $i \in \mathcal{N}$.
If for all entries of the vector $\frac{\partial \alpha_i}{\partial r_i}$ satisfy the following inequality

$$\frac{\partial \alpha_i}{\partial r_i} \geq 0 \tag{11}$$

then from the equation (10) for the ADC algorithm one has got

$$\dot{r}_i = -\beta_i \frac{\partial}{\partial r_i}[(\operatorname{grad} v_i)^{\mathrm{T}} b_i] \tag{12}$$

with $\beta_i \geq \frac{1}{2}$ and $\dot{r}_i \geq 0$. For

$$\frac{\partial \alpha_i}{\partial r_i} \leq 0 \tag{13}$$

one obtains

$$\dot{r}_i = -\gamma_i \frac{\partial}{\partial r_i}[(\operatorname{grad} v_i)^{\mathrm{T}} b_i] \tag{14}$$

with $\gamma_i \leq \frac{1}{2}$ and $\dot{r}_i \leq 0$.

The proposed ADC algorithm (12) or (14) ensures that the stability measure of all subsystems will increase in the time if $\dot{r}_i$ is not identically equal to zero. Owing to Lemma 1 or the principle of negative diagonal dominance of the investigated system, if the system is stabilizable by the proposed DC, there exist such values of w_{ii} ($w_{ij} = \text{const}$), for $i \in \mathcal{N}$, $j \in \mathcal{N}$, that the system (1) with controller (3) and (4) will be stable. The sufficient conditions of stability of the investigated system are given by the following theorem.

Theorem 1. The equilibrium $z^{\mathrm{T}} = [x^{\mathrm{T}}, (r - r^\star)^{\mathrm{T}}] = 0$ of the system (1), (3) and (12) or (14) is stable and asymptotically stable with respect to variables x (or some part of variables x) on the set $R_{\rho i} \times T$, $i \in \mathcal{N}$, if the following sufficient conditions hold:

(i) For the Lyapunov function of the investigated system

$$V_a = \sum_{i=1}^{N} v_{ia} \tag{15}$$

the following condition holds [11]:

$$a(\|x\|) \leq V_a(x,t).$$

(ii) The conditions given by Eqs. (11), (12) or (13) and (14) are satisfied for the disturbances acting upon the LSDS.

(iii) The investigated system (1) with the controller (2) and (12) or (14) is stabilizable.

Proof. Determine a function $V_a : R^n \times R^p \times T \to R_+$ (cf. (15)) as a Lyapunov function of the investigated system as follows:

$$V_a = \sum_{i=1}^{N} v_i(x_i,t) + (r_i - r_i^\star)^{\mathrm{T}}(r_i - r_i^\star)$$

For the time derivative of the Lyapunov function on the solution of (1), (2) and (3) we can obtain

$$\frac{\mathrm{d}V_a}{\mathrm{d}t} = \sum_{i=1}^{N} \{(\operatorname{grad} v_i)^{\mathrm{T}}[f_i + b_i(x_i, q_i(x_i, r_i, t), t) + h_i] + 2(r_i - r_i^\star)^{\mathrm{T}}\dot{r}_i\} + \frac{\partial V_a}{\partial t} \quad (16)$$

Owing to the condition (12) or (14) the negative definiteness (semidefiniteness) of the second part of Eq. (16)

$$(\operatorname{grad} v_i)^{\mathrm{T}}\, b_i(x_i, q_i(x_i, v_i, t), t), \quad i \in \mathcal{N}$$

become more intensive in the time if $\dot{r}_i$ is not identically equal to zero. Since the stability measure of all subsystems within the dynamic behaviour of the investigated system is almost permanently increasing in the time, owing to the conditions (iii) of Theorem 1 for all controller parameters r_{ij} there exists such instant of time, say $t_1 \in \langle t_0, t_c\rangle$, that $r_{ij} \in R_s$; the last but one term of (16) is at least negative semidefinite then for $t > t_1$ the following inequality holds:

$$\frac{\mathrm{d}V_a}{\mathrm{d}t} \leq 0$$

and the system is stable and asymptotically stable with respect to the part of variables z_1, $z = [x, r - r^\star] = [z_1, z_2]$, for which the following inequality holds:

$$\frac{\mathrm{d}V_a(z,t)}{\mathrm{d}t} \leq -c\,\|z_1\|^2, \quad c > 0.$$

This completes the proof. □

3.2. Discrete–time case

Consider a nonlinear discrete-time-varying LSDS with N subsystems

$$x_i(t+1) = f_i[x_i(t), t] + b_i[x_i(t), u_i(t), t] + h_i[x(t), t] \quad (17)$$

and the adaptive DC

$$u_i(t) = q_i[x_i(t), r_i(t), t] \tag{18}$$
$$\Delta r_i(t) = g_i[x_i(t), r_i(t), t] \tag{19}$$

where $t \in I = \{0, 1, 2, \ldots\}$, $i \in \mathcal{N}$. Determine the Lyapunov function of the ith subsystem with adaptive DC (18) and (19) on the set $R_{\rho i} \times I$ as follows

$$v_{ia}(t) = v_i(t) + (r_i(t) - r_i^\star)^{\mathrm{T}} (r_i(t) - r_i^\star) \tag{20}$$

where $v_i(t)$ is a Lyapunov function of the isolated subsystem $x_i(t+1) = f_i[x_i(t), t]$. Let us take the Lyapunov function of LSDS in the form (15). For the first difference $\Delta V_a(t)$ along the solution of (17), (18) and (19), one can write [9]:

$$\Delta V_a(t) \leq \sum_{i=1}^{N} L_i |h_i[x(t), t]| + \Delta v_i[x_i(t), u_i(t), t] - \Delta r_i^{\mathrm{T}}(t)[2(r_i^\star - r_i(t)) - \Delta r_i(t)] \tag{21}$$

where $\Delta r_i(t) = r_i(t+1) - r_i(t)$, $L_i > 0$ satisfy the following inequality

$$|v_i[x_i(t)', t] - v_i[x_i(t)'']| \leq L_i \|x_i(t)' - x_i(t)''\|$$

for all $x_i(t)'$, $x_i(t)'' \in R^{n_i}$, $t \in I$ and $i \in \mathcal{N}$. In order to ensure the negative diagonal dominance of the investigated system, the stability measure of the ith subsystem defined by the following formula

$$\alpha_i = -\frac{\Delta v_i(t)}{v_{ia}(t)} \tag{22}$$

has to be increased when the system (17) with adaptive DC (18) and (19) is not stable. From Eq. (10), one can obtain the following adaptive decentralized control laws:

$$\frac{\partial \alpha_i}{\partial r_i} \geq 0 \tag{23}$$
$$\Delta r_i(t) = -\beta_i \frac{\partial}{\partial r_i} (\Delta v_i(t)) \tag{24}$$

with $\beta_i \geq \frac{1}{2}$ and $\Delta r_i(t) \geq 0$

$$\frac{\partial \alpha_i}{\partial r_i} \leq 0 \tag{25}$$
$$\Delta r_i(t) = -\gamma_i \frac{\partial}{\partial r_i} (\Delta v_i(t)) \tag{26}$$

with $\gamma_i \leq \frac{1}{2}$ and $\Delta r_i(t) \leq 0$, $i \in \mathcal{N}$.
The stability of the LSDS (17) with (18) and (24) or (26) has to be checked.

Theorem 2. The equilibrium $z(t)^{\mathrm{T}} = [x(t)^{\mathrm{T}}, (r(t) - r^*)^{\mathrm{T}}] = 0$ of the system (17) with adaptive DC (18) and (24) or (26) is stable and asymptotically stable with respect to the variables $x(t)$ (or some part of variables $x(t)$) at the set $R_{\rho i} \times I$, $i \in \mathcal{N}$, if the following sufficient conditions hold:

(i) The Lyapunov function of the investigated system (15) satisfies [11]:

$$a(\|x\|) \leq V_a(t)$$

(ii) The conditions given by Eqs. (23), (24) or (25) and (26) are satisfied for disturbances acting upon the LSDS.

(iii) The investigated system (17) with controller (18) and (19) is stabilizable.

Proof. The proof of Theorem 2 is strictly similar to that of Theorem 1. □

3.3. Simplifications of proposed adaptive decentralized controller

Let us assume that instead of (12) or (14) one may use the following algorithm

$$\dot{r}_i = -Q_i(x_i, r_i, t) \tag{27}$$

where the entries of the vector $Q_i(x_i, r_i, t)$ are positive (negative) definite continuous algebraic functions with $Q_i(0, r_i, t) = 0$, $Q_i(x_i, 0, t) \neq 0$ for $\|x_i\| \neq 0$, and the next matching conditions are valid in Eq. (16)

$$(\operatorname{grad} v_i)^{\mathrm{T}}\, b_i(x_i, q_i(x_i, r_i, t)) = \sum_{j=1}^{m_i} \sum_{v=1}^{\ell_{ij}} r_{ijv}^{2k_{ijv}+1} m_{ijv}(x_i, t) + m_{ij0}(x_i, t) \tag{28}$$

where the indices indicate ith subsystem, jth input of the ith subsystem, and vth controller parameter of the jth input; $k_{ijv} = 0, 1, 2, \ldots$; $p_i = \sum_{j=1}^{m_i} \ell_{ij}$, $i \in \mathcal{N}$. One can rewrite Eq. (16) using Eqs. (27) and (28) in the following form:

$$\begin{aligned} \frac{\mathrm{d}V_a}{\mathrm{d}t} &= \sum_{i=1}^{N} \left\{ H_i(x,t) - \left[\sum_{j=1}^{m_i} \sum_{v=1}^{\ell_{ij}} \left(r_{ijv0} + \right.\right.\right. \\ &+ \left.\left.\left. \int_{t_0}^{t} Q_{ijv}(x_i, r_i, \tau)\, \mathrm{d}\tau \right) r_{ijv}^{2k_{ijv}} m_{ijv}(x_i, t) + 2(r_i - r_i^*)^{\mathrm{T}} Q_i(x_i, r_i, t) \right] \right\} \end{aligned} \tag{29}$$

where

$$H_i(x,t) = (\operatorname{grad} v_i)^{\mathrm{T}} [f_i + h_i] + \frac{\partial v_i}{\partial t} + \sum_{j=1}^{m_i} \sum_{v=1}^{\ell_{ij}} m_{ij0}$$

and we substitute the following equation instead of r_{ijv} :

$$r_{ijv} = -r_{ijv0} - \int_{t_0}^{t} Q_{ijv}(x_i, r_i, \tau)\, \mathrm{d}\tau.$$

The sufficient stability conditions of the investigated system (1) with adaptive DC (3) and (27) are given by the following theorem.

Theorem 3. The equilibrium $z^{\mathrm{T}} = [x^{\mathrm{T}}, (r - r^{\star})^{\mathrm{T}}] = 0$ of the system (1) with adaptive DC (3) and (27) is stable and asymptotically stable with respect to the variables x at the set $R_{\rho i} \times T,\ i \in \mathcal{N}$, if the following sufficient conditions hold:

(i) The following condition holds for the Lyapunov function of investigated system (15) (cf. [11]):

$$a(\|x\|) \leq V_a(x, t).$$

(ii) The investigated system (1) with the controller (3) and (27) is stabilizable, i.e. there exist $r_i^{\star} \in R_s,\ i \in \mathcal{N}$ such that the system is asymptotically stable.

(iii) If the entries of the vector $Q_i(x_i, r_i, t)$ are positive (negative) definite then the corresponding functions $m_{ijv}(x_i, t)$ for $i \in \mathcal{N},\ j = 1, 2, \ldots, m_i,\ v = 1, 2, \ldots, \ell_{ij}$ have to be positive (negative) semidefinite (or definite).

Proof. If there exist some $j = 1, 2, \ldots, m_i,\ v = 1, 2, \ldots, \ell_{ij}$ that the functions $m_{ijv}(x_i, t)$ are definite for all $i \in \mathcal{N}$, then owing to (27) and condition (iii) of Theorem 3 the negative definitness of second part of Eq. (29) become active as for as $\dot{r}_i$ is not identically equal to zero. Since the investigated system is stabilizable and the third part of Eq. (29) is negative definite, there exist instant of time $t_1 \in \langle t_0, t_c \rangle$ and $r_i \in R_s$ such that for $t > t_1$ the

$$\frac{\mathrm{d}V_a}{\mathrm{d}t} < -c\,\|x\|, \quad c > 0$$

and the investigated system is stable and asymptotically stable with respect to the variables x. If for some $i \in \mathcal{N}$ there exists no definite function $m_{ijv}(x_i, t),\ j = 1, 2, \ldots, m_i,\ v = 1, 2, \ldots, \ell_{ij}$, then owing to (27) and conditions (ii) of Theorem 3, there exist an instant of time, say $t_1 \in \langle t_0, t_c \rangle$, and $r_i \in R_s$ such that for $t \geq t_1$ the following inequality holds:

$$\frac{\mathrm{d}V_a}{\mathrm{d}t} < -c\,\|x\|$$

and the system is stable and asymptotically stable with respect to variables x. This completes the proof. □

Consider now that instead of adaptive DC one may use the following algorithms:

$$\begin{aligned} u_i &= q_i(x_i, r_i, t) \\ r_i &= -G_i(x_i, t), \quad i \in \mathcal{N} \end{aligned} \tag{30}$$

Let us take the Lyapunov function of the system (1) with adaptive DC (30) in the form

$$V = \sum_{i=1}^{N} v_i \tag{31}$$

For the time derivative of V along the solution of (1) and (30) with matching condition (28) one may write the following formula:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \sum_{i=1}^{N} \left\{ H_i(x, t) - \left[\sum_{j=1}^{m_i} \sum_{v=1}^{\ell_{ij}} G_{ijv}(x_i, t)\, r_{ijv}^{2k_{ijv}}\, m_{ijv}(x_i, t) \right] \right\} \tag{32}$$

The sufficient stability conditions of the system (1) with adaptive DC (30) are given by the following theorem.

Theorem 4. The equilibrium $x = 0$ of the system (1) with adaptive DC (30) is stable and asymptotically stable with respect to the variables x (or some part of variables x) at the set $R_{\rho i} \times T$, $i \in \mathcal{N}$, if the following sufficient conditions hold:

(i) The following condition holds for the Lyapunov function of investigated system (31) (cf. [11]) $a(\|x\|) \leq V$.

(ii) The investigated system (1) with the controller (30) is stabilizable.

(iii) The sign of the entries of the vector $G_i(x_i,t)$ must be the same as the sign of the corresponding functions $m_{ijv}(x_i,t)$ for all $i \in \mathcal{N}$, $j = 1,2,\ldots,m_i$ and $v = 1,2,\ldots,\ell_{ij}$.

(iv) Assume that there exist some functions $\delta_{ij}(x_i) \in \langle \delta_{ij\,\min}, \delta_{ij\,\max} \rangle$ for $i \in \mathcal{N}$, $j = 1,2,\ldots,m_i$ which characterize the qualitative dynamic properties of the ith subsystem. When, for the some instant of time and $\|x_i\| \neq 0$, one evaluates the dynamic proces of the ith subsystem by $\delta_{ij} = \delta_{ij\,\max}$, for $i \in \mathcal{N}$, $j = 1,2,\ldots,m_i$, then the overall system is on the boundary of stability; for $\delta_{ij} = \delta_{ij\,\min}$ the dynamic behaviour has extremely excellent properties. This like function $\delta_{ij}(x_i)$ was introduced in [8] to design of the adaptive controller. Let there exist positive numbers, $\delta_{ij0} \in \langle \delta_{ij\,\min}, \delta_{ij\,\max} \rangle$, and such properties of scalar continuous functions $G_{ijv}(\delta_{ij}(x_i))$, $i \in \mathcal{N}$, $j = 1,2,\ldots,m_i$, $v = 1,2,\ldots,\ell_{ij}$, that the following conditions hold:

– the function G_{ijv} is increasing on the interval $\delta_{ij} \in \langle \delta_{ij\,\min}, \delta_{ij\,\max} \rangle$, and

– for $\delta_{ij} \geq \delta_{ij0}$ it holds $|r_{ijv}| \geq \sigma_{ijv} > 0$

where σ_{ijv} is a given positive number.

(v) If for some $i \in \mathcal{N}_1 \subset \mathcal{N}$ there does not exist a definite function $m_{ijv}(x_i,t)$, $j = 1,2,\ldots,m_i$, $v = 1,2,\ldots,\ell_{ij}$, then the function $H(x,t)$ must satisfy:

$$H(x,t) = \sum_{i=1}^{N} H_i(x,t) \leq 0 \tag{33}$$

on the set $\{x_i \in R_i^n : \ m_{ijv}(x_i,t) = 0 \text{ and } \|x_i\| \neq 0 \text{ for } i \in \mathcal{N}_1\}$ and $\{x_i \in R_i^n : i \notin \mathcal{N}_1\}$.

Proof. We have to prove the existence of such positive numbers σ_{ijv} that under conditions (i) – (v) of Theorem 4, the investigated system (1) with ADC (30) is stable and asymptotically stable with respect to variables x or part of variables x.

Let us assume that there exist some $j = 1,2,\ldots,m_i$, $v = 1,2,\ldots,\ell_{ij}$ such that the functions $m_{ijv}(x_i,t)$ are definite for all $i \in \mathcal{N}$. Using (1), (2), (29) and (32), we can obtain:

$$\frac{\partial v_i}{\partial t} + (\text{grad } v_i)^{\mathrm{T}} f_i(x_i,t) + \sum_{j=1}^{m_i} \sum_{v=1}^{\ell_{ij}} m_{ij0} \leq \beta_{ii} \|x_i\|^2$$

$$\|(\text{grad } v_i)^{\mathrm{T}} h_i(x,t)\| \leq \|x_i\| \sum_{j=1}^{N} \alpha_{ij} \|x_j\| \tag{34}$$

$$\|m_{ijv}(x_i, t)\| \geq \gamma_{ijv} \|x_i\|^2$$

where α_{ij}, γ_{ijv} are non-negative numbers. The substitution of (34) into Eq. (32) yields to:

$$\frac{\mathrm{d}V}{\mathrm{d}t} \leq \sum_{i=1}^{N} \left(w'_{ii} \|x_i\|^2 + 2 \|x_i\| \sum_{j \neq i}^{N} w'_{ij} \|x_{ij}\| \right) \tag{35}$$

where

$$\begin{aligned} w'_{ii} &= \beta_{ii} + \alpha_{ii} - \sum_{j=1}^{m_i} \sum_{v=1}^{\ell_{ij}} |r_{ijv}|^{2k_{ijv}+1} \gamma_{ijv} \\ w'_{ij} &= \frac{1}{2} \alpha_{ij} \end{aligned}$$

The system (1) with adaptive DC (30) will be stable and asymptotically stable with respect to variables x if the aggregation matrix $W' = [w'_{ij}]_{N \times N}$ is negative definite. From Lemma 1, it follows that there exist such positive numbers σ_{ijv} that for

$$|r_{ijv}| \geq \sigma_{ijv} \quad i \in \mathcal{N},\ j = 1, 2, \ldots, m_i,\ v = 1, 2, \ldots, \ell_{ij},$$

the matrix W' is negative definite. Let us assume that for some $i \in \mathcal{N}_1$ the functions m_{ijv} are indefinite or semidefinite. Due to the condition (iii) of Theorem 4, the second part of Eq. (32) for $i \in \mathcal{N}_1$ is at least positive semidefinite and for $i \notin \mathcal{N}_1$ one is positive definite with respect to the variable x_i. In virtue of the conditions (v) of Theorem 4 and the structure of Eq. (32), there exist positive numbers σ_{ijv}, for $i \in \mathcal{N}$, $j = 1, 2, \ldots, m_i$, and $v = 1, 2, \ldots, \ell_{ij}$ such that

$$\frac{\mathrm{d}V}{\mathrm{d}t} \leq 0$$

and the system is stable and asymptotically stable with respect to variables z_1, $x^{\mathrm{T}} = [z_1^{\mathrm{T}}, z_2^{\mathrm{T}}]$ for which the following inequality holds:

$$\frac{\mathrm{d}V}{\mathrm{d}t} \leq -c \|z_1\|.$$

This completes the proof. □

3.4. Adaptive control of systems with uncertainties

The problem of stability robustness arises from sources such as errors and simplifications in formulating the model of the plant, errors in implementing the controller and the possibility of various sensors or actuator failures. A realistic treatment of modelling uncertainty is to describe the physical plant not by a single model, but a family of possible plant models. Modelling errors in physical systems fall into two broad categories: parametric and dynamic uncertainty [2, 14]. Let us consider a class of non-linear time-varying systems with the known part of LSDS:

$$\dot{x}_i = f_i(x_i, t) + df_i(x_i, t) + b_i(x_i, u_i, t) + db_i(x_i, u_i, t) + h'_i(x_i, t) + dh_i(x, t) + H_{i2}(x, z) \tag{36}$$

and the unknown part:

$$\dot{z}_i = d_i(z_i, t) + H_{i2}(x, z), \quad i \in \mathcal{N}$$

where all functions of (36) are algebraic continuous and bounded, which ensures the unique solution of (36) for both all $x_i(t_0) = x_{i0} \in R^{n_i}$, $z_i(t_0) = z_{i0} \in R^{l_i}$ and continuous input $u_i \in R^{m_i}$. Assume that the functions H_{i1}, H_{i2} and $d_i(z_i, t)$ are unknown. The overall system (36) with the adaptive DC (3) and (27) (for example) can be written in the compact form:

$$\dot{y} = G_1(x, r, t) + H_1(x, z) \qquad \dot{z} = G_2(z, t) + H_2(x, z) \tag{37}$$

where

$$y^{\mathrm{T}} = [x^{\mathrm{T}}, (r - r^{\star})^{\mathrm{T}}], \quad z^{\mathrm{T}} = [z_1^{\mathrm{T}}, \dots, z_N^{\mathrm{T}}] \in R^{\ell}, \quad \ell = \sum_{i=1}^{N} \ell_i.$$

Let us suppose that (37) represents a family of plants with dynamic and parametric modelling uncertainty, where the first isolated subsystem

$$\dot{y} = G_1(x, r, t)$$

describes a nominal plant model with parametric uncertainty [18]. Assume that $G_2(z, t)$ is asymptotically stable with respect to the variables z. Define on the set $R_\rho \times T$ the function $v_1 : R^{n+p} \times T \to R_+$ and on the set $R_\gamma \times T$ $v_2 : R^{\ell} \times T \to R_+$ as Lyapunov functions for the 1st and 2nd isolated subsystem, respectively, where $R_\gamma = \{z \in R^{\ell} : \|z\| \le \gamma > 0\}$. For the time derivative of the Lyapunov function

$$V = v_1 + v_2 \tag{38}$$

along the solution of (37), we obtain

$$\dot{V} = \sum_{i=1}^{2} \left(\frac{\partial v_i}{\partial t} + (\operatorname{grad} v_i)^{\mathrm{T}} (G_i + M_i) \right) \tag{39}$$

If we suppose that the following inequalities hold:

$$0.5 \sum_{i=1}^{2} \|\operatorname{grad} v_i^{\mathrm{T}} H_i\| \le w_{12} \|y\| \, \|z\|$$

$$\frac{\partial v_1}{\partial t} + \operatorname{grad} v_1^{\mathrm{T}} G_1 \le -w_{11}(r) \|y\|^2$$

$$\frac{\partial v_2}{\partial t} + \operatorname{grad} v_2^{\mathrm{T}} G_2 \le -w_{22} \|z\|^2$$

where $w_{ij} \ge 0$, $i, j = 1, 2$, then for (39) it yields

$$\dot{V} \le [\, \|y\| \, \|z\| \,] \begin{bmatrix} -w_{11}(r) & w_{12} \\ w_{12} & -w_{22} \end{bmatrix} \begin{bmatrix} \|y\| \\ \|z\| \end{bmatrix} \tag{40}$$

The sufficient stability conditions of the system (37) are given by the following theorem.

Theorem 5. The equilibrium $[y^{\mathrm{T}}, z^{\mathrm{T}}] = 0$ of the system (37) is stable and asymptotically stable with respect to the variables x on the set $R_\rho \times T$, if the following sufficient conditions hold:

(i) The following condition holds (cf. [11]) for the Lyapunov function of investigated system (38)

$$a(\|x\|) \leq V(x, r, z, t).$$

(ii) The first isolated subsystem of (37) is completely controllable.

(iii) The second isolated subsystem which describes the unmodeled dynamics of the system (37) is asymptotically stable with respect to the variables z.

(iv) The system (37) is stabilizable by the proposed adaptive DC.

(v) The functions H_1 and H_2 are uniformly bounded.

(vi) The following inequality holds:

$$w_{11} w_{22} > w_{12}^2$$

Proof. The prove of this theorem follows immediately from Lemma 1. □

Since we suppose that the conditions of above theorem are fulfiled, there is no problem to stabilize of the LSDS (37) with dynamic uncertainties via in this paper proposed adaptive DC. One may suppose that only parametric uncertainties occur in the first subsystem of (37) for the design of adaptive DC. Let us assume that the parameters of the 1 st subsystem (37) vary over some a priori known compact set C. For the time derivative of the Lyapunov function (15) along the solution of the 1 st subsystem (36) with (28), one may obtain

$$\begin{aligned} \frac{\mathrm{d}V_a}{\mathrm{d}t} &= \sum_{i=1}^{N} \left\{ H_i(x,t) - \left[\sum_{j=1}^{m_i} \sum_{v=1}^{\ell_{ij}} \int_{t_0}^{t} Q_{ijv} \, \mathrm{d}\tau \left(r_{ijv}^{2k_{ijv}} m_{ijv} - 2Q_{ijv} \right) \right] + \right. \\ &+ \left. 2 r_i^{\star T} Q_i + (\operatorname{grad} v_i)^{\mathrm{T}} db_i \right\} \end{aligned} \qquad (41)$$

where (see Eqs. (36) and (29))

$$h_i(x,t) = df_i + dh_i + h_i'$$

and without loss of generality we assume that $r_{ijv0} = 0$. The sufficient stability conditions of the investigated system for $db_i = 0$ are given by Theorem 3.

Let us consider the following three matching conditions:

1. $db_i = \beta_i b_i(x_i, u_i, t)$;

2. db_i has the same structure as $b_i(\cdot)$ and the similar matching condition holds as it is given by Eq. (23);

3. the matrix B_i for the linear LSDS

$$\dot{x}_i = A_{ii} x_i + B_i u_i + \sum_{j=1, j \neq i}^{N} A_{ij} x_j \quad y_i = C_i x_i, \quad i \in \mathcal{N}$$

is given by

$$B_i = B_{i0}(I + G_iS_i)$$

and the adaptive DC

$$u_i = -r_iK_iC_ix_i - K_{p_i}C_{p_i}x_i$$

with

$$\dot{r}_i = -x_i^{\mathrm{T}}Q_ix_i, \; Q_i < 0$$

The robustness properties of the investigated system with above matching conditions can be summarized as follows.

1. $\beta_i > -1$

2. sign $(m_{ijv} + dm_{ijv}) = \mathrm{sign}(m_{ijv})$

3. $C_i^{\mathrm{T}}K_i^{\mathrm{T}}(I + G_iS_i)^{\mathrm{T}}B_{i0}^{\mathrm{T}}P_i + P_iB_{i0}(I + G_iS_i)K_iC_i \geq 0$ if $C_i = I$ and $K_i = B_{i0}^{\mathrm{T}}P_i$ then it yields

$$\lambda_M((G_iS_i)^{\mathrm{T}} + G_iS_i) > -2 \text{ or } \|G_iS_i\| < 1 \text{ for } i \in \mathcal{N}$$

where P_i is the positive definite matrix which can be obtained from the following Lyapunov matrix equation:

$$(A_{ii} - B_iK_{pi}C_{pi})^{\mathrm{T}}P_i + P_i(A_{ii} - B_iK_{pi}C_{pi}) = -M_i, \quad M_i > 0.$$

4. CONCLUSION

In this paper, an original approach to the solution of the LSDS stabilization problem is proposed. Sufficient stabilizability conditions are derived for the nonlinear dynamic system with dynamic and parametric uncertainties. The proposed adaptive decentralized controllers possesses robustness properties with respect to

– changes of the interconnections betwen subsystems,
– changes of the structure and parameters of other subsystems with controllers, and
– the given changes of parameters of subsystem itself.

It is shown that under conditions of Theorem 5, the dynamic uncertainties of the modelled system cannot destabilize the investigated system. From the parametric robustness properties point of view, the parametric changes of input functions $b_i(x_i, u_i, t)$, $i \in \mathcal{N}$ are the most dangerous. It is shown that the proposed adaptive DC possesses robustness properties for a broad class of parametric uncertainties which can be met in practice.

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