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DECOUPLING IN NONLINEAR SINGULAR SYSTEMS WITH APPLICATIONS IN AUTONOMOUS VEHICLES

MANOLIS A. CHRISTODOULOU

In this paper the problem of input-output decoupling in nonlinear singular systems is solved via the application of a particular nonlinear PD feedback control law. An invertible transformation is provided, which converts the closed loop singular system into an equivalent regular, to which a static state feedback law is applied. This transformation, hence, allows the theory that exists for the control of regular nonlinear systems to be applied in the nonlinear singular systems. At the end an application of the theory is tested on a nonlinear singular model of an autonomous excavator.

1. INTRODUCTION

Consider the non-linear, time invariant dynamical system of the nth order

$$E(x)\,\dot{x} = A(x)\,x + B(x)\,u \tag{1a}$$

$$y = C(x)\,x \tag{1b}$$

$$x(t_0) = x_0 \tag{1c}$$

Where x is an n-vector of state variables, u is an l vector of control inputs y is an m vector of output measurements, $x(t_0)$ is the initial state at time t_0 and $E(x)$, $A(x)$, $B(x)$, $C(x)$ are nonlinear matrix functions of x, of appropriate dimensions. In this paper it is assumed that both $E(x)$ and $A(x)$ are $n \times n$ square matrices and $B(x)$ is an $n \times l$ rectangular matrix. Here it is also assumed that the matrix $E(x)$ is singular in non-isolated points.

Systems of the form (1) are generally equivalent to systems of the form:

$$E(x)\,\dot{x} = f(x) + B(x)\,u \tag{2}$$

together with (1b) and (1c), where $f(x)$ is a nonlinear in x vector function. This function can always be written in the form $f(x) = A(x)\,x$ as in (1) with an infinite of possible selections for the matrix $A(x)$.

If the matrix $E(x)$ in (1) is regular, then system (1) is called a regular non-linear dynamical system. Otherwise, system (1) is called *singular* or *generalized state space* or *descriptor or semistate.* Moreover if the matrics $E(x)$, $A(x)$ and $B(x)$ are constant and independent of x, then equation (1) becomes the well known linear singular system. An extensive review literature on linear singular systems is provided by Lewis [1].

Dynamical systems of the form (1) appear in many practical applications, as for example in Leontieff models in multisector economy [2], in large scale systems [3], in singularly perturbed systems and in nonlinear electric circuits [4]. It is also noted that nonlinear systems of the form (1) appear in various robotics applications. For example robots performing a certain task, i.e., holding an object or cooperating robots executing a joint task are considered as a dynamical system to which equality constraints are imposed. Those algebraic constraints represent the physical interaction between the various moving objects. The collection of both synamic and algebraic equations has a unified framework, called a singular system (1). It is explained with many practical applications by McClamroch [5]. Furthermore, autonomous vehicles, guided on prespecified paths have the above dynamic singular representation (Christodoulou and Isik [6]). Those interesting practical applications provide important grounds for the study and control of the above systems.

Find, if possible, a feedback control law (static or dynamic), which will totally decouple the inputs and outputs of system (1), i.e., if $l = m$, the feedback must yield a system in which each input u_i is going to affect only one output y_i, $i = 1, 2, \ldots, m$.

The linear regular version for this problem was originally solved by Falb and Wolovich [7] and later a geometric solution was presented by Wonham and Morse [8], [9]. Mufti [10] presented an interesting note in the frequency domain, and Hautus and Heymann [11] gave a transfer function solution. Work on the nonlinear regular systems has been done by Mikhail and Wonham [12]. The geometric theory in the nonlinear systems has been introduced by Isidori et al., [13] and by Hirschorn [14]. They provided the necessary and sufficient conditions for the existence of a solution via static state feedback. Dynamic feedback control laws have also been investigated in recent years [15–17].

Recently there have been numerous attempts in trying to solve the problem of simultaneous decoupling and internal stability [18–21]. In [21] Isidori and Grizzle present a solution for the non-linear systems which in analogous to that of Gilbert [22] for the linear case.

The problem of noninteracting control for the linear singular systems of the form (1) has been treated by Zhou et al., [23] where they used a constant ratio proportional and derivative feedback law, which under certain assumptions transforms the closed loop system into a regular one. This method has been further improved and enhanced in [24, 25]. Independently and slightly ahead of time, Christodoulou et al. [26] presented the same constant ratio proportional-derivative feedback which transforms the closed loop system into a regular one and provided solution to the input output decoupling problem of singular systems. This theory appeared also in [27]. Mertzios and Christodoulou [28] presented a solution to the decoupling and pole placement in linear singular systems with dynamic state feedback. They also used output state feedback [29] and finally provided solution to the simultaneous decoupling and data sensitivity problem [30].

In the present paper we provide necessary and sufficient conditions for the existence of local solutions to the nonlinear singular decoupling problem. We apply a particular feedback law which transforms the closed loop system into a regular one. The transformation we supply is non-singular, thus invertible.

Using the inverse map, we go back to the original system. Thus, all solutions obtained for the regular case can be transformed into algorithms which are applicable to the singular case.

2. BACKGROUND AND DEFINITIONS

For a non-linear system of the form (1) we introduce the following definitions.

Definition 2.1. Define by $\hat{S}(x)$ the set of all nonlinear functions in x matrix triples of the form

$$(E(x), A(x), B(x)) \in \mathbb{R}^{n \times n} \times \mathbb{R}^{n \times n} \times \mathbb{R}^{n \times m}$$

Definition 2.2. Let $S(x)$ be the subset of $\hat{S}(x)$ defined via

$$S(x) = \{(E(x), A(x), B(x)) \in \hat{S}(x) \colon \det (s\, E(x) - A(x)) \neq 0\}$$

It is here noted, according to Campell [2], that the regularity (i.e., $= 0$) of the determinant at Definition 2.2, at the point x_0, does not guarantee the uniqueness of the solution, if x_0 is an admissible initial condition, as is the case for linear systems.

Definition 2.3. For each $\theta(x) \colon \mathbb{R}^n \to \mathbb{R}$, define by $S_\theta(x)$ the subset of $S(x)$ as follows

$$S_\theta(x) = \{(E(x), A(x), B(x)) \in S(x) \colon \det (\cos \theta(x)\, E(x) - \sin \theta(x)\, A(x)) \neq 0\}$$

The following proposition can be deduced based on the previous definition.

Proposition 2.1. For each x the following three proposition are true.

a) $S_\theta(x)$ is an open and dense subset of $S(x)$

b) $S_{\theta+\pi}(x) = S_\theta(x)$

c) $S(x) = \bigcup_{\theta \in \Theta} S_\theta(x)$

Where Θ is the set of all functions θ which take values on the whole interval $(0, \pi)$. Next, define a map $R_\varphi \colon \hat{S}(x) \to \hat{S}(x)$, by

Definition 2.4.

$$R_\varphi(E(x), A(x), B(x)) = (\cos \varphi(x)\, E(x) + \sin \varphi(x)\, A(x)\,,\\ - \sin \varphi(x)\, E(x) + \cos \varphi(x)\, A(x), B(x))$$

where $\varphi(x)$ is an arbitrary nonlinear function $\varphi \colon \mathbb{R}^n \to \mathbb{R}$.

Here we note that if

$(\hat{E}(x), \hat{A}(x), \hat{B}(x)) = R_\varphi(E(x), A(x), B(x))$, then it is true that

$$\begin{bmatrix} \hat{E}(x) \\ \hat{A}(x) \end{bmatrix} = \begin{bmatrix} \cos\varphi(x) I & \sin\varphi(x) I \\ -\sin\varphi(x) I & \cos\varphi(x) I \end{bmatrix} \begin{bmatrix} E(x) \\ A(x) \end{bmatrix}$$

and $\hat{B}(x) = B(x)$.

Proposition 2.2. a) R_0 is the identity map of $\hat{S}(x)$

b) $R_{\varphi_1} \circ R_{\varphi_2} = R_{\varphi_1 + \varphi_2}$

c) $R_\varphi(S(x)) = S(x)$

d) $R_\varphi(S_\theta(x)) = S_{\varphi+\theta}(x)$

3. FEEDBACK LAW FOR THE NONLINEAR SYSTEM

Here we introduce a feedback law of the form

$$u = F(x)\,[\cos\theta(x)\, x - \sin\theta(x)\, \dot{x}] + v \tag{3}$$

which is applied to systems belonging in the set $S_\theta(x)$. This is a Proportional-Derivative state feedback law of a nonlinear form, where $F(x)$ is a nonlinear matrix in x of appropriate dimensions. This feedback law is preserving the properties of the set $S_\theta(x)$, as explained in the following theorem.

Theorem 3.1. Let $(E(x), A(x), B(x)) \in S_\theta(x)$ and let $(\hat{E}(x), \hat{A}(x), B(x))$ be the system which is obtained after the application of the feedback law (3). Then $(\hat{E}(x), \hat{A}(x), B(x)) \in S_\theta(x)$.

Proof. See [31]

Assume that f_θ is a map provided by the law (4), i.e.,

$$\begin{aligned} f_\theta(F)\,(E(x), A(x), B(x)) &= (E(x) + \sin\theta(x)\, B(x)\, F(x), \\ &\quad A(x) + \cos\theta(x)\, B(x)\, F(x), B(x)) \end{aligned} \tag{4}$$

then the following theorem is true:

Theorem 3.2. The rotation map commutes with the feedback law as follows:

$$R_\varphi \circ f_\theta(F) = f_{\theta+\varphi}(F) \circ R_\varphi$$

Proof. See [31].

This Theorem provides the following diagram:

$$\begin{array}{ccc} S_\theta(x) & \xrightarrow{f_\theta(F)} & S_\theta(x) \\ {\scriptstyle R_\varphi}\downarrow & & \downarrow{\scriptstyle R_\varphi} \\ S_{\theta+\varphi}(x) & \xrightarrow{f_{\theta+\varphi}(F)} & S_{\theta+\varphi}(x) \end{array}$$

Assume now that $\varphi = -\theta$. Then, it is obvious that the rotated system becomes a regular nonlinear one. Thus the following result is true.

Corollary 3.1. The following diagram is commutative

$$\begin{array}{ccc} S_\vartheta(x) & \xrightarrow{f_\theta(F)} & S_\vartheta(x) \\ R_{-\theta}\downarrow & & \downarrow R_{-\theta} \\ S_0(x) & \xrightarrow{f_{\theta+\varphi}(F)} & S_0(x) \end{array}$$

This is a very important corollary because it allows the nonlinear singular systems to which a feedback law (3) is applied, to be transformed locally under an invertible rotation into equivalent regular nonlinear ones to which a static nonlinear state feedback law is applied of the form:

$$u = F(x)\, x + v \tag{5}$$

Thus the theory from regular nonlinear systems may be applied to the nonlinear singular systems, under the specific feedback law (3). Next the control algorithm is presented.

Control Algorithm. According to the previous corollary, the following steps are followed in solving a nonlinear singular control problem.

Step 1: The original singular system $(E(x), A(x), B(x))$ is rotated by $R_{-\theta}$, so as a regular one $(E_0(x), A_0(x), B(x))$ is obtained.

Step 2: The feedback control problem for the regular system is solved under (5).

Step 3: The solution to the singular system will be provided similarly, via a control law of the form (3).

4. DECOUPLING LAW

In this section, we are using a control algorithm to solve the single-input, single-output decoupling problem for a system of the form (1) using a feedback law of the form (3). To this end we assume that the system possesses the same number of inputs and outputs, i.e., $m = l$.

For the case, where (1) is regular, i.e., E is nonsingular, we know that the input-output decoupling problem is solved via the following theorem.

Theorem 4.1. (Isidori, et al., [13]). Let the dimensions of u and y be the same, and equal to m. Then the static, state feedback, noninteracting control is solvable locally in a regular fashion if

a) The $m \times m$ matrix $A^*(x)$ defined by

$$a_{ij}(x) = L_{(E^{-1}B)_j} L_{E^{-1}Ax}^{\varrho_i} c_i(x)$$

is nonsingular for every x.

b) For each i, the dimension of the codistribution $\pi_i(x)$ defined by

$$\pi_i(x) = \operatorname{span}\{dc_i(x), dL_{E^{-1}Ax}\, c_i(x), \ldots, dL^{\varrho_i}_{E^{-1}Ax}\, c_i(x)\}$$

is constant for all x.

c) For any disjoint nonempty subjects I and J of $\{1, 2, \ldots, m\}$

$$(\sum_{i \in I} \pi_i) \cap (\sum_{j \in J} \pi_j) = \emptyset$$

Here by L_T we denote the Lie derivative with respect to a vector field T as:

$$L_T = \sum_{i=1}^{n} T_i(x) \frac{\partial}{\partial x_i}$$

and $c_i(x)$ denotes the ith row of the matrix $C(x)$ in (1b). Also the numbers ϱ_i are defined as the largest integers, such that for all $r < \varrho_i$ and $x \in \mathbb{R}^n$

$$L_{E^{-1}B} L^r_{E^{-1}Ax}\, c_i(x) = 0$$

According to Singha [32], a solution to the decoupling problem based on an algebraic setup is given as follows:

Theorem 4.2 (Singha [32]). A necessary and sufficient condition that there exists a feedback law (static) which decouples the regular system (1) (under the assumption $\det(E) \neq 0$), is that $B^*(x)$ is nonsingular, where:

$$B^*(x) = \begin{bmatrix} \dfrac{dD_{i,\varrho_1}(x)}{dx} E^{-1}B \\ \vdots \\ \dfrac{dD_{m,\varrho_m}(x)}{dx} E^{-1}B \end{bmatrix}$$

and ϱ_i and D_{i,ϱ_i} are defined as follows:

$$\varrho_i = \begin{cases} \min k \colon \dfrac{dD_{i,k}(x)}{dx} E^{-1}B \neq 0 & k \in (0, 1, \ldots, n-1) \\ n-1 \text{ if } \dfrac{dD_{i,k}(x)}{dx} E^{-1}B = 0\,, & \text{for every } k \end{cases}$$

$$D_{i,k}(x) = \frac{dD_{i,k-1}(x)}{dx} E^{-1}A$$

$$D_{i,0}(x) = c_i(x)$$

Then a static state feedback control law that decouples the system is provided as follows:

$$u(t) = F^*\, x(t) + G^*\, v(t) \tag{6}$$

where

$$F^*(t) = -(B^*(x))^{-1}\, D^*(x)$$

$$G^*(x) = (B^*(x))^{-1}$$

and

$$D^*(x) = \begin{bmatrix} D_{1,\sigma_1}(x) \\ \vdots \\ D_{m,\varrho_m}(x) \end{bmatrix}$$

Now the system (1) is represented by the quadruple (E, A, B, C). A generalization of the previous Theorem 4.2 can be derived by following the next synthesis algorithm.

1) Given $(E, A, B, C) \in S(x)$, choose θ such that $(E, A, B, C) \in S_\theta(x)$.
Let $(E_0, A_0, B, C) = R_{-\theta}(E, A, B, C)$.

2) Let $(E_0, A_0, B, C) \in S_0(x)$ and $(\hat{E}, \hat{A}, B, C) = R_\theta(E_0, A_0, B, C)$. The system $(\hat{E}, \hat{A}, B, C)$ is locally input-output decoupled if the conditions of the previous theorem are met.

3) A state feedback gain matrix F needs to be chosen, so that the closed loop system satisfies the rotation specifications.

4) If such an F exists, then the feedback law of the form (3) decouples the original system.

From all the above, the next very important theorem follows:

Theorem 4.3. A necessary and sufficient condition that there exists a feedback law of the form (3), which decouples locally the singular system (1), is that $B^*(x)$ as defined below is nonsingular.

$$B^*(x) = \begin{bmatrix} \dfrac{\mathrm{d}D_{1,\varrho_1}(x)}{\mathrm{d}x} (\cos\theta E - \sin\theta A)^{-1} B \\ \vdots \\ \dfrac{\mathrm{d}D_{m,\varrho_m}(x)}{\mathrm{d}x} (\cos\theta E - \sin\theta A)^{-1} B \end{bmatrix}$$

where the ϱ_i and D_{i,ϱ_i} are defined as follows

$$\varrho_i = \begin{cases} \min k\colon \dfrac{\mathrm{d}D_{i,k}(x)}{\mathrm{d}x} (\cos\theta E - \sin\theta A)^{-1} B \neq 0 & k(0, 1, \ldots, n-1) \\ n-1 \text{ if } \dfrac{\mathrm{d}D_{i,k}(x)}{\mathrm{d}x} (\cos\theta E - \sin\theta A)^{-1} B = 0\,, & \text{for every } k \end{cases}$$

$$D_{i,k}(x) = \frac{\mathrm{d}D_{i,k-1}(x)}{\mathrm{d}x} (\cos\theta E - \sin\theta A)^{-1} A$$

$$D_{i,0}(x) = c_i(x)$$

Then a static feedback control law that decouples the system is provided by

$$u(t) = F^*(\cos\theta x - \sin\theta x) + G^* v \tag{7}$$

where F^* and G^* are given as in (6).

According to Theorem 4.1, an analogous is presented here for the singular case.

Theorem 4.4. The static, state feedback noninteracting control for the singular system (1) is solvable in a regular fashion if

a) The $m \times m$ matrix $A^*(x)$ defined by

$$a_{ij}(x) = L_{[(\cos\theta E - \sin\theta A)^{-1}B]_j} L^{\varrho_i}_{(\cos\vartheta E - \sin\vartheta A)^{-1}Ax} c_i(x)$$

is nonsingular for every x.

b) For each i, the dimension of the codistribution $\pi_i(x)$ defined by

$$\pi_i(x) = \operatorname{span}\{dc_i(x), dL_{(\cos\vartheta E - \sin\vartheta A)^{-1}Ax}\, c_i(x), \ldots, dL^{\varrho_i}_{(\cos\vartheta E - \sin\vartheta A)^{-1}Ax} c_i(x)\}$$

is constant for all x.

c) For any disjoint nonempty subsets I and J

$$\Big(\sum_{i\in I} \pi_i\Big) \cap \Big(\sum_{j\in J} \pi_j\Big) = \emptyset$$

the operator L is defined as before. The integers ϱ_i are defined as in Theorem 4.1 with the mpdification that the matrix $\cos\theta E - \sin\theta A$ replaces E.

It is clear here that the solution to the problem depends on the selection of the arbitrary nonlinear function $\theta(x)$ (which should make the system to belong to $S_\theta(x)$). This provides extra degrees of freedom in searching for decoupled nonlinear systems, which do posess other properties too.

5. ILLUSTRATIVE EXAMPLE

Consider the following nonlinear model of an autonomous excavator [6].

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -x_2 \cos x_4 \\ (1/m)\, u_2 - (k_b/m)\, x_2 x_5 \\ x_2 u_1 \\ x_2 u_1 + (x_2/x_1) \sin x_4 \\ k_B x_5 x_2 - f(x_1, x_3) \end{bmatrix}$$

Where the state and input variables are defined as follows:

$x_1 = R$ = length of the position vector from the origin
$x_2 = v$ = velocity magnitude
$x_3 = \varphi$ = position angle, from positive x-axis
$x_4 = \alpha$ = angle of approach
$x_5 = F_z$ = verticle force during excavation
$u_1 = k$ = curvature of the path
$u_2 = F_t$ = tangential driving force

The constants are as follows:

m = vehicle mass

k_B = proportionality constant

Whereas the function $f(x_1, x_3) = f(R, \varphi)$ is the generalized constraint imposed on the viscous force during the excavation process. Here without loss of generality, we assume $f(x_1, x_3) = x_1$ and $m = k_B = 1$.

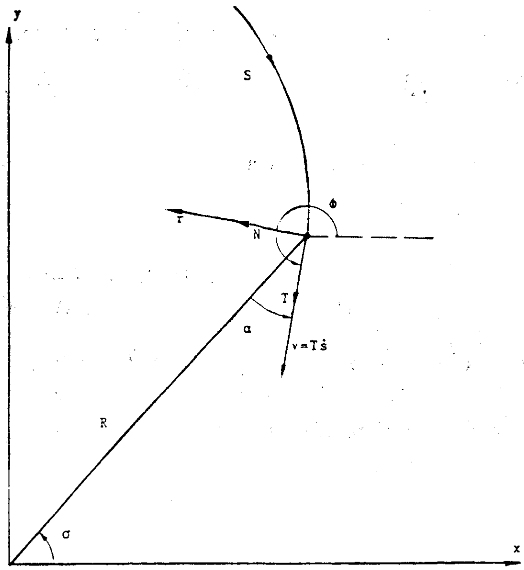

Fig. 1. Mobile Robot.

A schematic diagram of the above model is depicted in Figure 1. This model may have a representation of the form (1) as follows:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 & -\cos x_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x_2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & (\sin x_4)/x_1 & 0 & 0 & 0 \\ -1 & x_5 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ x_2 & 0 \\ x_2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Originally, we want to make the above system belong in $S_\theta(x)$. Thus we form the matrix

$$\cos\theta E - \sin\theta A = \begin{bmatrix} \cos\theta & \sin\theta\cos x_1 & 0 & 0 & 0 \\ 0 & \cos\theta & 0 & 0 & x_2\sin\theta \\ 0 & 0 & \cos\theta & 0 & 0 \\ 0 & -(\sin\theta\sin x_4)/x_1 & 0 & \cos\theta & 0 \\ \sin\theta & -x_5\sin\theta & 0 & 0 & 0 \end{bmatrix}$$

Next we find its determinant:

$$\det(\cos \theta E - \sin \theta A) = x_2 \cos^2 \theta \sin \theta [\sin \theta \cos \theta x_5 + \sin^2 \theta \sin x_1]$$

From the determinant we get the conditions $x_2 \neq 0$, $\theta \neq 0$, $\theta \neq \pi/2$, and from the term inside the parenthesis

$$\theta \neq \tan^{-1} \frac{-x_5}{\sin x_1}$$

Then the local decoupling problem is solvable if the matrix $B^*(x)$ is of full rank and the control law is of the proportional derivative feedback form (3).

6. CONCLUSIONS

In this paper, a unified theory which is applicable to all singular and nonsingular nonlinear systems is presented. The control law that is applied here is of the nonlinear PD feedback form, where an arbitrary nonlinear function $\theta(x)$ is introduced ($\theta(x)$ satisfies some mild constraints). Then a generalized rotation is introduced. It transforms the closed loop nonlinear singular system, to which a law of the form (3) is applied, into a regular one. A static state feedback law is, in turn, applied to the regular system. Thus, all control problems for nonlinear singular systems are reduced to those of regular nonlinear systems under static state feedback.

The theory is applied to the decoupling problem of singular systems, and the necessary and sufficient conditions for the local solvability of the above problem are presented. Finally the theory is applied to an example of an autonomous excanator, where the model is assumed to be of the generalized nonlinear singular form.

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Prof. Dr. Manolis A. Christodoulou, Department of Electronic & Computer Engineering, Technical University of Crete, 73100 Chania, Crete. Greece.