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CONFIDENCE INTERVALS FOR RELIABILITY FUNCTIONS OF AN EXPONENTIAL DISTRIBUTION UNDER RANDOM CENSORSHIP

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The asymptotic normality of Bayes estimators of the reliability function of an exponential distribution based on randomly censored data is studied. A Monte-Carlo simulation is used to examine how well two large-sample confidence bands for Bayes estimators do in small and moderate samples. The results are compared with the confidence intervals for the maximum likelihood estimator.

1. INTRODUCTION

Arbitrarily right censored data arise commonly in industrial life testing and medical follow-up studies. In reliability testing some objects are removed from the experiment before they fail. In medical research we find there are some individuals who die by reasons which are desirable to exclude from consideration, or may themselves decide to leave and move elsewhere. The model of random censorship is useful for analysing these data. Let $X_1, \ldots, X_n$ be independent identically distributed (i.i.d.) random variables with the distribution function F, and let $T_1, \ldots, T_n$ be i.i.d. random variables which are independent of X_j's and possess the distribution function G. In our model X_j's represent times to failure, while T_j's are times censors, G being a nuisance distribution. Under random censorship we can only observe the pairs

$$(W_1, I_1), \ldots, (W_n, I_n)$$

where

$$W_j = \min(X_j, T_j),$$

$$I_j = I(X_j \leqq T_j) = 1 \quad \text{if} \quad X_j \leqq T_j\,, \quad \text{that is}\,, \quad X_j \text{ is uncensored}\,,$$

$$= 0 \quad \text{if} \quad X_j > T_j\,, \quad \text{that is,} \quad X_j \text{ is censored.}$$

Let X_1 and T_1 have the Lebesgue densities f and g, respectively. Then (W_1, I_1) has the density

$$h(w, i) = \{(f(w)[1 - G(w)]\}^i \{g(w)[1 - F(w)]\}^{1-i}, \quad w \in \mathbb{R}, \quad i = 0, 1,$$

with respect to Lebesgue times counting product measure.

The unknown parameters are often estimated by the method of maximum likelihood. If there is no functional dependence between parameters of F and G, it is sufficient, to obtain estimators of the parameters of F, to maximize the sub-likelihood functions

$$L_F = \prod_{j \in U} f(X_j) \prod_{j \in C} [1 - F(T_j)]$$

where $U = \{j: I_j = 1\}$ is the set of the indices of uncensored observations, while $C = \{j: I_j = 0\}$ is the set of indices of censored observations.

Here we deal with the case of exponentially distributed X_j's. Suppose that

$$\begin{aligned} F(x) &= 1 - \exp(-x/\theta), \quad x > 0, \\ &= 0 \quad \text{otherwise.} \end{aligned} \tag{1.1}$$

We assume that the censoring distribution is a Weilbull one with parameters $k\theta$ and β, i.e.,

$$\begin{aligned} G(t) &= 1 - \exp\{-[t/k\theta]^\beta\}, \quad t > 0, \\ &= 0 \quad \text{otherwise.} \end{aligned}$$

This assumption is a slight extension of the usual Koziol-Green model under which the times censors are also exponentially distributed. In that case, $\beta = 1$.

The reliability corresponding to (1.1) taken at mission time $x \geqq 0$ is

$$R(x) = \exp(-x/\theta).$$

Without any loss of generality, after a proper change of the time scale, we can restrict ourselves to $x = 1$ so that we shall study

$$R = \exp(-1/\theta).$$

We shall use the following notation:

$$W = W_1 + \ldots + W_n, \quad I = I_1 + \ldots + I_n, \quad Y = I/W, \quad \overline{W} = W/n,$$
$$\bar{I} = I/n.$$

Since engineering designs are rather evolutionary than revolutionary processes it is often useful to utilize a priori information on reliability of the current design to get more reasonable conclusions on the future device. Bayes approach is a simple way how to impose a priori knowledge of the subject. The results of the present paper are directly applicable in various engineering problems as well as in biometrical research.

2. ESTIMATION

The maximum likelihood estimator of R is

$$R_1 = \exp(-Y).$$

To obtain the Bayes estimator we suppose that the hazard rate $\lambda = 1/\theta$ is a random variable distributed according to the Gamma a priori distribution with the density function

$$q(\lambda) = \frac{a^p}{\Gamma(p)} e^{-a\lambda} \lambda^{p-1} \quad \lambda > 0,$$
$$= 0 \quad \text{otherwise}.$$

Then the prior density function of R is

$$s(r) = \frac{a^p}{\Gamma(p)} r^{a-1}(-\ln r)^{p-1} \quad 0 < r < 1,$$
$$= 0 \quad \text{otherwise}$$

and the posterior distribution of R given $(W_1, I_1), \ldots, (W_n, I_n)$ has now the density function

$$\varphi(r, W_1, I_1, \ldots, W_n, I_n) = \frac{(a + W)^{I+p}}{\Gamma(I + p)} r^{a+W-1}(-\ln r)^{I+p-1}, \quad 0 < r < 1.$$

Taking the expectation of R with respect to the posterior distribution we get the Bayes estimator optimal with respect to the quadratic loss function

$$R_2(a, p) = \left(\frac{W + a}{W + a + 1}\right)^{I+p}.$$

In case $p \geqq 1$ an alternative Bayes estimator may be obtained by maximizing the posterior density function:

$$R_3(a, p) = \exp\left(-\frac{I + p - 1}{W + a - 1}\right) \quad \text{if} \quad W + a - 1 > 0,$$
$$= 0 \quad \text{otherwise}.$$

3. ASYMPTOTIC DISTRIBUTION

The maximum likelihood estimator for the hazard rate $\lambda = 1/\theta$ is $Y = I/W$. For the convergence of this estimator we can use the result from Miller:

$$Y \sim N(\lambda, \lambda^2/I), \quad \text{i.e.:}$$
$$n^{1/2}(Y - \lambda) \to N(0, \lambda^2/\mathrm{E}I_1) \tag{3.1}$$

where I = No. of uncensored observations, may be replaced by $\mathsf{E}I$ if the latter is available. (The notation $\sim$ denotes "is asymptotically distributed as".) In our model:

$$\mathsf{E}I = n\mathsf{E}I_1 = \theta^{-1}n \int_0^\infty \exp(-x/\theta) \exp(-(x/k\theta)^\beta)\,\mathrm{d}x =$$
$$= n \int_0^\infty (\exp(-y) \exp(-y/k)^\beta)\,\mathrm{d}y\,.$$

For $\beta = 1$, we have the result obtained in [3]:

$$n^{1/2}(Y - \lambda) \to N(0, (\theta\delta)^{-1}) \quad \text{with} \quad 1/\delta = 1/\theta + 1/k\theta\,.$$

For $\beta \neq 1$, $\mathsf{E}I$ must be calculated by numerical integration. For each value of β we can choose k to achieve the desired expected proportion of uncensored observations.

Theorem 1. For $n \to \infty$ we have:

$$n^{1/2}(R_1 - R) \to N(0, \lambda^2 R^2/\mathsf{E}I_1) \tag{3.2}$$
$$n^{1/2}(R_2 - R) \to N(0, \lambda^2 R^2/\mathsf{E}I_1) \tag{3.3}$$
$$n^{1/2}(R_3 - R) \to N(0, \lambda^2 R^2/\mathsf{E}I_1)\,. \tag{3.4}$$

Proof. Let $g(t) = \exp(-t)$; $g'(t) = -\exp(-t)$. Using (3.1) and (6a. 2.1) in [6] we have (3.2). In the case that the censoring distribution is also exponential ($\beta = 1$), this formula coincides with that one in [2]. □

The formula (6a. 2.1) is not generally applicable if instead of g, we have a function g_n depending on n explicitly. This is the case for R_2 and R_3. After some algebra R_2 and R_3 can be written in the following forms:

$$R_2 = [1 + 1/(W + a)]^{-(I+p)} = [1 + n^{-1}(\overline{W} + a/n)^{-1}]^{-(\bar{I}n+p)} =$$
$$= \mathrm{e}^{-Y}\left[1 + \frac{1}{2n\overline{W}}(Y(1 + 2a) - 2p) + O_p(n^{-2})\right],$$
$$R_3 = \mathrm{e}^{-Y}\left[1 + \frac{1}{n\overline{W}}(Y(a - 1) - p + 1) + O_p(n^{-2}))\right].$$

Now we want to find the asymptotic distribution of R_2 and R_3. We have $R_2 \approx = g(Y, n)$; $\partial g/\partial Y$ exists and converges to $\exp(-\lambda)$ as $n \to \infty$, $Y \to \lambda$. With this condition, we can use (6a. 2.5) in [6] which together with (3.1) gives

$$\sqrt{(n)}\,(R_2 - g(\lambda, n)) \to N(0, \lambda^2 R^2/\mathsf{E}I_1)\,.$$

Furthermore, $g(\lambda, n)$ can be replaced by $R = \exp(-\lambda)$ because

$$\sqrt{(n)}\,(g(\lambda, n) - R) \to 0 \quad \text{as} \quad n \to \infty$$

and so we have (3.3).

The proof of (3.4) is similar to that of (3.3). □

4. CONFIDENCE INTERVALS

We study two classes of confidence intervals for R with each of R_1, R_2, R_3 estimators. The first ones are constructed using the $\log(-\log)$ transformation that usually improves the convergence to normality because the asymptotic variance does not depend on the unknown parameter. Consider

$$Y \sim N(\lambda, \lambda^2/\mathsf{E}I)$$

and put $g(t) = \log t$ in (6a. 2.1), [6]. Thus we have

$$\log Y \sim N(\log \lambda, 1/\mathsf{E}I)\,.$$

Hence, the confidence interval for $R = \exp(-\lambda)$ is

$$\exp(-\exp(\log Y \pm Z_{\alpha/2}\, 1/\sqrt{\mathsf{E}I}))$$

where $Z_{\alpha/2}$ is the $(1 - \alpha/2)$th quantile of the standard normal distribution.

Note that $\mathsf{E}I = n\,.\,\mathsf{E}I_1 = n\,.$ expected proportion of uncensored observations.

For R_2, we first apply (6a. 2.1) with $g(t) = -\log t$ to the formula (3.3), and we have:

$$n^{1/2}(-\log R_2 - \lambda) \to N(0, \lambda^2/\mathsf{E}I_1)\,.$$

Applying now (6a. 2.1) with $g(t) = \log t$ we have

$$\log(-\log R_2) \sim N(\log \lambda, 1/(n\, \mathsf{E}I_1))\,.$$

Hence, the confidence interval for $R = \exp(-\lambda)$ is

$$\exp(-(\exp(\log(-\log R_2) \pm Z_{\alpha/2}(1/n\, \mathsf{E}I_1)^{1/2})))\,.$$

With R_3 we obtain a similar interval. We shall denote these intervals as R_1, R_2, and R_3.

The other class of intervals with confidence coefficient $1 - \alpha$ is based on the asymptotic normality of R_i $(i = 1, 2, 3)$. They are given by

$$R_i \pm Z_{\alpha/2} R_i Y/(n\, \mathsf{E}I_1)^{1/2}\,, \quad i = 1, 2, 3\,,$$

where instead of R and λ we use their estimators R_i and Y. We shall denote these intervals as $R_1(b)$, $R_2(b)$, and $R_3(b)$.

5. A SIMULATION STUDY

To examine how well the intervals based on asymptotic distributions do in finite sample situations, a Monte Carlo simulation was performed to determine the achieved levels of the bands under several situations.

We consider mission times corresponding to some chosen percentiles. We performed simulation for percentiles of order $q = 0{\cdot}05$, $0{\cdot}5$ and $0{\cdot}95$, which cover the part of the distribution with high reliability $(R = 0{\cdot}95)$ as well as the tail $(R = 0{\cdot}05)$. We use four values for the shape parameters of the censoring Weilbull distribution:

$\beta = 0{\cdot}5$, 1, $1{\cdot}5$, 2. For each value of β, we take the corresponding scale parameter k to achieve an expected proportion of uncensored observation of $\mathsf{E}I_1 = 0{\cdot}2$, $0{\cdot}5$, $\frac{2}{3}$, and $0{\cdot}8$. The sample sizes were $n = 30$ and 50, and the confidence coefficient $0{\cdot}95$. The prior parameters in $R_2(a, p)$ and $R_3(a, p)$ were chosen $a = 4\theta$, $a = 8\theta$, $a = 2\theta$, $p = 4$, according to two principles: (i) the standard error of the a priori distribution should be half of the prior expected value; (ii) the first prior distribution has the expectation equal to the hazard rate $\lambda = 1/\theta$, the second one equal to half of it, and the third one equal to double of it.

For each combination of the various specifications, 400 data sets and their corresponding confidence intervals were generated. The observed coverage probability was calculated as the fraction of 400 confidence bands containing the true reliability $R = \exp(-1/\theta)$. This number of replicas provides a standard deviation in the estimated coverage probability of about $0{\cdot}01$. All of the simulation results were computed on the IBM-AT computer using the uniform random number generator which is in the Turbo Pascal library. The numerical results are given in Tables 1–4.

6. CONCLUSIONS

First, R_1 and R_2 behave better than $R_1(b)$ and $R_2(b)$, respectively. They have a superior performance because the asymptotic variance of the $\log(-\log)$ transformation of R_1 and R_2 does not depend on the unknown parameter θ. It is an empirical fact, confirmed in this study, that transforming an estimate to remove the dependence of the variance on the unknown parameter tends to improve the convergence to normality by reducing the skewness.

R_3 and $R_3(b)$ are rather sensitive to the choice of the prior parameters and above all to the chosen percentile. Furthermore as q increases, the effects on R_3 and $R_3(b)$ are opposite. So for $a = 2\theta$ and $a = 4\theta$, R_3 is better if $q = 0{\cdot}95$ and $R_3(b)$ is better if $q = 0{\cdot}05$ or $0{\cdot}5$. For $a = 8\theta$, they behave almost equally, however.

The R_1 interval appears slightly anticonservative giving less than the desired coverage. However, R_2 is conservative giving more than the desired coverage except for $a = 8\theta$. Hence R_2 has a superior performance to R_1 except for $a = 8\theta$, that it's similar. We can conclude, too, that we obtain excellent results if the choice of the prior parameters is perfect ($a = 4\theta$), good results if we underestimate the hazard rate ($a = 2\theta$), and worse results (but not too bad) if we overestimate it ($a = 8\theta$).

Under almost all circumstances (except for $q = 0{\cdot}95$ and $a = 8\theta$) R_2 behave quite better than R_3 and $R_3(b)$. In some cases R_3 and $R_3(b)$ give very poor coverage probabilities, and so they are not recommended.

The level of censoring has not a clear effect on the coverage. On one hand, as the proportion of uncensored observations (I_1) increases the estimators are more reliable and the coverage should increase. On the other hand when I_1 increases the size

Table 1. Observed coverage for Weibull censoring distribution with $\beta = 0{\cdot}5$.

q	0·05				0·5				0·95			
u	0·2	0·5	$\frac{2}{3}$	0·8	0·2	0·5	$\frac{2}{3}$	0·8	0·2	0·5	$\frac{2}{3}$	0·8
					$n = 30$	$a = 4\theta$	$p = 4$					
R_1	·935	·955	·967	·965	·955	·930	·940	·955	·965	·957	·942	·960
$R_1(b)$	·907	·945	·962	·945	·932	·920	·942	·957	·825	·877	·867	·912
R_2	·999	·987	·975	·985	·997	·987	·970	·980	·995	·985	·982	·980
$R_2(b)$	·945	·967	·977	·970	·970	·965	·980	·982	·992	·997	·982	·975
R_3	·750	·720	·732	·715	·887	·915	·930	·957	·887	·975	·990	·985
$R_3(b)$	·927	·965	·970	·960	·965	·965	·977	·975	·650	·765	·772	·817
					$n = 30$	$a = 2\theta$	$p = 4$					
R_2	·992	·960	·942	·962	·997	·950	·940	·947	·999	·987	·977	·965
$R_2(b)$	·980	·990	·970	·975	·992	·940	·932	·940	·952	·915	·902	·927
R_3	·452	·557	·527	·532	·647	·775	·845	·895	·587	·897	·952	·967
$R_3(b)$	·977	·987	·965	·965	·962	·917	·915	·930	·255	·487	·610	·622
					$n = 30$	$a = 8\theta$	$p = 4$					
R_2	·977	·955	·947	·957	·969	·977	·955	·940	·957	·925	·927	·940
$R_2(b)$	·907	·922	·917	·922	·925	·972	·950	·940	·982	·990	·992	·995
R_3	·930	·870	·885	·877	·932	·967	·947	·937	·980	·967	·977	·987
$R_3(b)$	·872	·882	·897	·892	·907	·967	·945	·937	·982	·977	·967	·975
					$n = 50$	$a = 4\theta$	$p = 4$					
R_1	·947	·962	·940	·937	·947	·965	·938	·957	·938	·955	·943	·951
$R_1(b)$	·925	·955	·957	·932	·962	·957	·932	·950	·907	·942	·938	·952
R_2	·990	·977	·970	·960	·985	·977	·976	·980	·982	·980	·972	·975
$R_2(b)$	·965	·975	·970	·952	·983	·962	·971	·975	·932	·942	·950	·945
R_3	·697	·682	·690	·677	·840	·900	·897	·947	·897	·982	·985	·992
$R_3(b)$	·947	·962	·962	·940	·967	·965	·962	·970	·885	·875	·875	·880
					$n = 50$	$a = 2\theta$	$p = 4$					
R_2	·980	·965	·935	·940	·990	·957	·962	·947	·999	·972	·982	·955
$R_2(b)$	·930	·917	·912	·915	·985	·952	·960	·945	·970	·972	·970	·960
R_3	·437	·512	·547	·595	·632	·842	·860	·960	·677	·940	·982	·977
$R_3(b)$	·960	·952	·925	·945	·967	·942	·952	·940	·899	·892	·892	·877
					$n = 50$	$a = 8\theta$	$p = 4$					
R_2	·952	·960	·960	·955	·952	·950	·952	·957	·917	·912	·932	·950
$R_2(b)$	·905	·940	·942	·945	·942	·945	·942	·957	·922	·920	·925	·930
R_3	·882	·862	·837	·860	·942	·930	·937	·942	·970	·955	·975	·977
$R_3(b)$	·880	·912	·932	·930	·927	·935	·942	·955	·892	·877	·867	·862

Table 2. Observed coverage for Weibull censoring distribution with $\beta = 1$.

q	0·05				0·5				0·95			
u	0·2	0·5	$\frac{2}{3}$	0·8	0·2	0·5	$\frac{2}{3}$	0·8	0·2	0·5	$\frac{2}{3}$	0·8
					$n = 30$	$a = 3\theta$	$p = 4$					
R_1	·902	·955	·945	·957	·925	·935	·935	·940	·937	·940	·940	·950
$R_1(b)$	·860	·942	·927	·960	·887	·925	·942	·930	·865	·870	·902	·897
R_2	·997	·992	·965	·977	·997	·985	·987	·972	·990	·972	·982	·977
$R_2(b)$	·927	·962	·965	·985	·940	·960	·962	·972	·980	·982	·972	·980
R_3	·740	·702	·700	·727	·905	·922	·935	·935	·937	·980	·990	·997
$R_3(b)$	·897	·952	·947	·970	·932	·950	·960	·970	·662	·792	·777	·802
					$n = 30$	$a = 2\theta$	$p = 4$					
R_2	·987	·972	·967	·965	·999	·972	·970	·967	·999	·977	·975	·977
$R_2(b)$	·970	·982	·970	·992	·975	·962	·955	·967	·927	·940	·905	·945
R_3	·440	·477	·522	·572	·600	·772	·842	·887	·557	·940	·965	·985
$R_3(b)$	·937	·975	·965	·990	·952	·947	·945	·960	·252	·550	·610	·712
					$n = 30$	$a = 8\theta$	$p = 4$					
R_2	·965	·925	·945	·950	·955	·930	·945	·942	·937	·930	·925	·922
$R_2(b)$	·862	·880	·917	·932	·905	·920	·932	·937	·972	·992	·992	·997
R_3	·890	·827	·862	·870	·925	·917	·922	·930	·975	·967	·967	·985
$R_3(b)$	·825	·855	·887	·910	·897	·920	·925	·932	·975	·992	·982	·977
					$n = 50$	$a = 4\theta$	$p = 4$					
R_1	·930	·950	·950	·957	·955	·945	·942	·925	·927	·950	·937	·950
$R_1(b)$	·902	·935	·935	·942	·930	·942	·932	·922	·870	·925	·935	·942
R_2	·997	·967	·970	·975	·997	·980	·972	·960	·970	·967	·960	·980
$R_2(b)$	·962	·952	·957	·962	·982	·980	·970	·955	·902	·925	·927	·960
R_3	·732	·690	·670	·720	·835	·907	·920	·912	·910	·980	·980	·990
$R_3(b)$	·906	·952	·960	·967	·937	·947	·954	·950	·807	·852	·865	·885
					$n = 50$	$a = 2\theta$	$p = 4$					
R_2	·975	·980	·937	·947	·990	·950	·935	·970	·999	·965	·970	·967
$R_2(b)$	·927	·940	·925	·932	·975	·950	·930	·970	·927	·915	·932	·915
R_3	·457	·540	·572	·630	·622	·802	·812	·907	·670	·942	·982	·987
$R_3(b)$	·960	·972	·930	·945	·940	·937	·925	·967	·385	·707	·720	·747
					$n = 50$	$a = 8\theta$	$p = 4$					
R_2	·935	·922	·947	·957	·917	·932	·922	·955	·867	·922	·930	·922
$R_2(b)$	·925	·910	·935	·947	·925	·927	·915	·936	·890	·915	·917	·925
R_3	·840	·792	·832	·847	·895	·910	·905	·942	·950	·975	·980	·982
$R_3(b)$	·830	·862	·885	·901	·903	·925	·928	·934	·976	·974	·983	·985

Table 3. Observed coverage for Weibull censoring distribution with $\beta = 1{\cdot}5$.

q	0·05				0·5				0·95			
u	0·2	0·5	$\frac{2}{3}$	0·8	0·2	0·4	$\frac{2}{3}$	0·8	0·2	0·5	$\frac{2}{3}$	0·8
					$n = 30$	$a = 4\theta$	$p = 4$					
R_1	·960	·937	·970	·950	·965	·925	·945	·932	·960	·937	·970	·950
$R_1(b)$	·902	·912	·972	·940	·927	·907	·930	·922	·905	·897	·905	·910
R_2	·999	·970	·987	·980	·999	·975	·985	·967	·997	·972	·990	·977
$R_2(b)$	·957	·952	·980	·967	·965	·950	·972	·962	·995	·967	·990	·980
R_3	·772	·725	·692	·717	·895	·897	·937	·930	·957	·972	·999	·990
$R_3(b)$	·935	·932	·975	·952	·960	·942	·962	·955	·760	·777	·767	·835
					$n = 30$	$a = 2\theta$	$p = 4$					
R_2	·997	·967	·965	·965	·997	·962	·970	·965	·999	·970	·987	·982
$R_2(b)$	·977	·977	·967	·992	·985	·950	·957	·960	·960	·910	·922	·925
R_3	·440	·500	·512	·570	·647	·770	·860	·890	·730	·912	·980	·987
$R_3(b)$	·965	·962	·962	·987	·970	·935	·937	·952	·327	·495	·577	·632
					$n = 30$	$a = 8\theta$	$p = 4$					
R_2	·975	·920	·950	·940	·982	·917	·955	·965	·940	·890	·915	·917
$R_2(b)$	·887	·892	·920	·925	·920	·910	·955	·962	·980	·985	·992	·992
R_3	·902	·822	·867	·870	·952	·895	·942	·955	·977	·962	·977	·965
$R_3(b)$	·857	·847	·897	·900	·905	·902	·947	·957	·980	·982	·982	·992
					$n = 50$	$a = 4\theta$	$p = 4$					
R_1	·962	·950	·940	·955	·962	·950	·940	·955	·965	·932	·952	·960
$R_1(b)$	·942	·940	·925	·962	·957	·950	·935	·957	·917	·910	·950	·942
R_2	·995	·977	·965	·982	·992	·972	·962	·982	·987	·965	·975	·970
Rå(b)	·985	·975	·955	·972	·986	·980	·964	·975	·937	·907	·947	·940
R_3	·730	·672	·755	·692	·877	·885	·895	·942	·937	·967	·997	·985
$R_3(b)$	·942	·937	·968	·958	·954	·952	·971	·956	·789	·796	·785	·842
					$n = 50$	$a = 2\theta$	$p = 4$					
R_2	·985	·960	·942	·945	·999	·955	·970	·955	·990	·960	·972	·957
$R_2(b)$	·947	·935	·922	·905	·995	·952	·967	·955	·972	·942	·977	·960
R_3	·482	·535	·587	·525	·677	·822	·872	·860	·777	·945	·982	·987
$R_3(b)$	·960	·952	·937	·932	·967	·950	·962	·947	·960	·970	·970	·985
					$n = 50$	$a = 8\theta$	$p = 4$					
R_2	·957	·932	·930	·937	·947	·905	·972	·957	·930	·915	·902	·932
$R_2(b)$	·997	·975	·967	·967	·987	·947	·955	·967	·765	·822	·762	·852
R_3	·862	·810	·800	·817	·920	·870	·930	·940	·980	·965	·952	·967
$R_3(b)$	·967	·952	·955	·957	·960	·920	·950	·930	·910	·917	·900	·930

Table 4. Observed coverage for Weibull censoring distribution with $\beta = 2$.

q	0·05				0·5				0·95			
u	0·2	0·5	$\frac{2}{3}$	0·8	0·2	0·5	$\frac{2}{3}$	0·8	0·2	0·5	$\frac{2}{3}$	0·8
					$n = 30$	$a = 4\theta$	$p = 4$					
R_1	·967	·907	·965	·945	·952	·950	·937	·960	·967	·907	·965	·945
$R_{\mathrm{i}}(b)$	·922	·905	·960	·932	·935	·925	·935	·957	·922	·872	·895	·892
R_2	·999	·970	·992	·975	·992	·987	·975	·982	·999	·950	·987	·977
$R_2(b)$	·970	·937	·980	·962	·960	·975	·967	·980	·977	·967	·992	·977
R_3	·785	·720	·705	·725	·905	·915	·910	·945	·962	·972	·997	·987
$R_3(b)$	·935	·920	·967	·945	·957	·972	·962	·975	·795	·775	·790	·830
					$n = 30$	$a = 2\theta$	$p = 4$					
R_2	·990	·962	·957	·975	·997	·962	·962	·970	·999	·987	·977	·980
$R_2(b)$	·990	·972	·972	·970	·997	·937	·957	·955	·957	·922	·925	·932
R_3	·505	·525	·535	·547	·690	·795	·865	·867	·735	·907	·972	·987
$R_3(b)$	·975	·965	·975	·970	·972	·917	·947	·932	·332	·472	·590	·830
					$n = 30$	$a = 8\theta$	$p = 4$					
R_2	·990	·897	·952	·962	·982	·887	·935	·957	·957	·880	·880	·940
$R_2(b)$	·915	·842	·895	·945	·947	·877	·930	·957	·972	·987	·995	·992
R_3	·947	·782	·822	·895	·960	·855	·922	·950	·972	·965	·957	·970
$R_3(b)$	·887	·790	·857	·927	·937	·865	·930	·952	·972	·997	·985	·977
					$n = 50$	$a = 4\theta$	$p = 4$					
R_1	·975	·947	·930	·947	·975	·947	·930	·947	·975	·947	·930	·947
$R_1(b)$	·975	·935	·930	·952	·985	·945	·935	·957	·915	·885	·927	·940
R_2	·965	·925	·940	·950	·960	·973	·935	·945	·895	·885	·917	·965
$R_2(b)$	·995	·977	·952	·972	·999	·980	·970	·980	·940	·885	·925	·935
R_3	·717	·667	·722	·667	·887	·875	·910	·937	·960	·980	·982	·987
$R_3(b)$	·942	·930	·970	·592	·963	·972	·962	·978	·892	·870	·872	·897
					$n = 50$	$a = 2\theta$	$p = 4$					
R_2	·992	·955	·960	·955	·990	·962	·967	·957	·995	·962	·985	·967
$R_2(b)$	·970	·940	·940	·922	·982	·955	·965	·957	·975	·937	·970	·965
R_3	·467	·530	·542	·522	·692	·822	·875	·860	·802	·940	·995	·980
$R_3(b)$	·975	·952	·965	·942	·980	·950	·960	·947	·787	·832	·878	·892
					$n = 50$	$a = 8\theta$	$p = 4$					
R_2	·965	·905	·942	·967	·967	·915	·920	·947	·935	·860	·912	·947
$R_2(b)$	·935	·872	·890	·923	·957	·915	·945	·962	·977	·982	·967	·958
R_3	·867	·792	·830	·837	·940	·895	·902	·937	·977	·947	·960	·980
$R_3(b)$	·892	·865	·877	·930	·920	·915	·930	·952	·872	·883	·887	·912

of the intervals decreases and so does the coverage. None of the two effects is dominant.

With respect to the shape parameter β of the Weilbull distribution, the results are very similar for the four values of β. It is logical because for each value of β, we obtain its corresponding confidence interval. The results would have been different if we had constructed an interval with exponential censoring and studied the robustness with respect to the assumed censoring distribution.

The achieved confidence level of R_2 is very similar for $n = 30$ and $n = 50$. For R_1 this level is slightly higher for $n = 50$. Anyway we can conclude that the large sample band based on the log $(-\log)$ transformation of the asymptotic distribution of R_2 does very well with small and moderate samples.

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REFERENCES

[1] J. D. Emerson: Effects of censoring on the robustness of exponential-based confidence intervals for medial lifetime. Commun. Statist. *B10* (1981), 6, 617—627.

[2] J. Hurt: On estimation in the exponential case under random censorship. In: Proc. Third Pannonian Symp. on Math. Statist., Visegrad, Hungary, Akademiai Kiadó, Budapest 1982.

[3] J. Hurt: Comparison of some reliability estimators in the exponential case under random censorship. In: Proc. Fifth Pannonian Symposium on Math. Statist., Visegrad, Hungary, Akademiai Kiadó, Budapest 1986.

[4] J. A. Koziol and S. B. Green: A Cramer - von Mises statistic for randomly censored data. Biometrika *63* (1976), 465—474.

[5] R. G. Miller, Jr.: Survival Analysis. Wiley, New York 1981.

[6] C. R. Rao: Linear Statistical Inference and Its Applications. Wiley, New York 1973.

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