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CONTRIBUTION TO PRIOR TUNING OF LQG SELFTUNERS

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A prior prediction of control quality achievable by the optimally tuned LQG selftuner of a fixed structure is proposed. The prediction helps to judge in advance the usefulness of the intended selftuner implementation. The proposed algorithmization makes also possible the off-line tuning of particular penalties in accordance with user's wishes. In this way, the used Bayesian methodology (together with existing solution of structure determination problem) provides algorithmic tools for systematic pre-tuning of majority of user's knobs.

1. INTRODUCTION

Selftuning controllers have made substantial progress both in the underlying theory and in the algorithmization [12]. In spite of success in some applications attempted, a significant market penetration has not been reached yet. There is a couple of causes of this state. In our opinion, relatively high complexity of the "tuning of selftuners" is one of them. Current design methodologies leave too many "knobs" to be committed by a user which cannot have deep knowledge of these controllers.

The situation calls for a computer-based support for implementing selftuners. The need has been recognized by many investigators in the field and specialized expert systems have been proposed [13]. As a rule, selftuners are used in a substantially simpler informational environment than that met in medicine, economy etc. This fact offers a chance to build up the discussed computer support in a traditional, algorithmically oriented way. For instance, the choice of a structure of linear regression models which are used in majority of selftuners is essentially resolved [4]. Recently, a promising procedure for data-based selection of a sampling period has been proposed [7].

The paper contributes to a dreamed up CAD by theoretically supported algorithms which transform the user's wishes and the data collected on the process into the structure and initial conditions of the designed selftuner. Specifically, a prior prediction of control quality achievable by the optimally tuned selftuner of a fixed

structure is proposed, helping to judge in advance the usefulness of the intended selftuner implementation. As a by-product some important user's knobs and initial conditions can be systematically pre-tuned.

The layout of the paper is as follows: The general problem addressed is formalized and its Bayesian solution given. Then the application to the linear normal regression model and to the multistep quadratic criterion is given. A description of algorithmic aspects and possible ways of pretuning of some user's knobs is included. The exposition is illustrated by a simulated example.

In conclusions, the proposed algorithm is shown to complete the list of tools formerly developed for prior tuning of LQG selftuners.

2. NOTATION

In the paper, the following notation is used:

$'$ transposition (column vectors are assumed);
t discrete time $(t \in \{1, 2, \ldots, T\})$;
T control horizon $(T \in \{1, 2, \ldots, \infty\})$;
$y(t)$ system output at time t;
$u(t)$ system input as time t;
$d'(t) = [y'(t), u'(t)]$ measured data;
$d(1 \ldots t) = \{d(1), d(2), \ldots, d(t)\}$;
$z'(t) = [u'(t), d'(t-1), \ldots, d'(t-n), 1]$ regression vector (n is system order);
Θ unknown parameters in system description (finite-dimensional case is assumed only);
P matrix of regression coefficients;
R covariance matrix of the stochastic component of the regression model;
ω design (multivariate) parameter (typically penalty on input increments);
$L_\omega(T, d(1 \ldots T))$ loss function specified by the fixed design parameter ω;
C control law (mapping C (available information): $d(1 \ldots t) \to u(t+1)$);
(c.) p.d.f. (conditional) probability density function;
$p(a \mid b, c)$ c.p.d.f. of random variable a conditioned on b, c (no distinction is made between random variables, their realizations and arguments of $p(\cdot \mid \cdot)$);
$p(a \mid t; b) = p(a \mid d(1 \ldots t), b)$;
$\mathsf{E}^C[a \mid b, c]$ conditional expectation determined by the c.p.d.f. $p(a \mid b, c)$ and a control law C;
$\Theta_i, q_i, i = 1, 2, \ldots, N$ random samples generated in Monte Carlo study;
$R^{1/2}$ a square root of the matrix R;
$\|x\|^2_{H_x} = \mathrm{tr}\,(x'H_x'H_x x)$ square of the weighted quadratic norm of a matrix x (if x reduces to a column vector trace operation $\mathrm{tr}(\cdot)$ is superfluous);
$S_z(\Theta)$ extended stationary Riccati matrix related to the regression model specified by the parameter Θ.

3. GENERAL PROBLEM FORMULATION AND SOLUTION

The relation of the system output $y(t)$ at the time t to the system input $u(t)$ and to the "past" input-output data $d(1 \ldots t-1) = \{d(1), d(2), \ldots, d(t-1)\}$ (with $d' = [y', u']$) is assumed to be describable by a *system model* [11] determining the c.p.d.f.

$$p(y(t) \mid t-1; u(t), \Theta) \tag{1}$$

parametrized by a finite-dimensional unknown parameter Θ. Available knowledge about the unknown parameter is expressed in terms of the *prior p.d.f.*

$$p(\Theta) = p(\Theta \mid \textit{preliminary data}) \tag{2}$$

which has been constructed using Bayesian methodology [11] from the model (1), expert knowledge and data measured on the system within a preliminary identification stage.

The system (1) will be controlled by a (selftuning) controller the control strategy of which is specified by a sequence of control laws, i.e. by a sequence of mappings

$$C\,(\textit{information available}): d(1 \ldots t) \to u(t+1) \quad t = 1, 2, \ldots, T-1 \tag{3}$$

where the control horizon T is assumed to be potentially infinite, i.e. $T = \infty$ is achieved by continuous extension of the finite horizon case [5].

The performance of the control strategy to be designed is measured by the *loss function*

$$L_\omega(T, d(1 \ldots T)) \tag{4}$$

where ω is a design parameter (typically, penalty on input increments).

The loss function (4) does not suit for a prior evaluation of control strategies as it depends on the random constituents of the input-output relation and on the uncertain parameter Θ. Instead of the loss function, if the parameter Θ is known, the (conditional) expectation

$$\mathsf{E}^C[L_\omega(T, d(1 \ldots T)) \mid \Theta] \tag{5}$$

is minimized with respect to admissible control strategies $C(\cdot)$.

To our purposes, we can restrict to the loss functions which can be interpreted as sample moments of the measured data (typically, variance). For them, if the controller stabilizes the system characterized by the parameter Θ, it can often be shown that

$$L_\omega(T, d(1 \ldots T)) - \mathsf{E}^C[L_\omega(T, d(1 \ldots T)) \mid \Theta] \to 0 \quad \text{as} \quad T \to \infty \tag{6}$$

in some sense.

Assuming validity of the asymptotic equality (6) the (expected) control quality can be predicted for a chosen control strategy if *the complete system description is known*. If, however, the parameter Θ is incompletely known the conditional expectation $\mathsf{E}^C[\cdot \mid \Theta]$ is uncertain, too.

The paper answers the question

> What prediction of the expected loss (5) can be made (at least for $T \to \infty$) using knowledge of the c.p.d.f.'s (1), (2) only?

in a special practically important case of the selftuners based on the enforced separation of identification and control synthesis. The controllers of this type

- construct recursively estimated $\hat{\Theta}(t)$ of the unknown parameter Θ using the observed data $d(1 \dots t)$;
- determine the control law $C_0(\hat{\Theta}(t))$ minimizing the criterion (5) under the assumption that $\Theta = \hat{\Theta}(t)$, i.e.

$$C_0(\hat{\Theta}(t)) = \arg\min_{C} \mathsf{E}^C[L_\omega(T, d(1 \dots T)) \mid \hat{\Theta}(t)] \quad (\text{as } T \to \infty) \tag{7}$$

- generate the new system input using this control law

$$C_0(\hat{\Theta}(t)): d(1 \dots t) \to u(t+1) \tag{8}$$

Ideally, it holds

$$C_0(\hat{\Theta}(t)) - C_0(\Theta) \to 0 \quad \text{as} \quad t \to \infty \tag{9}$$

where $C_0(\Theta)$ is the control law minimizing the loss function (5).

We shall predict the control performance under the idealized situation assuming that the asymptotic relations (6), (9) hold. For the *fixed design parameter* ω, the *predicted quantity* will be

$$q_\omega(\Theta) = \min_{C(\Theta)} \lim_{T \to \infty} \mathsf{E}^C[L_\omega(T, d(1 \dots T)) \mid \Theta] \tag{10}$$

This equation can be taken (formally) as the definition of the mapping

$$\mathscr{R}: \Theta \to q \tag{11}$$

Within Bayesian framework, prediction of the uncertain quantity $q_\omega(\Theta)$ is given if its p.d.f. $p(q)$ is specified. As the relation of the uncertain parameter Θ and of the minimum loss $q(\Theta)$ is deterministic, the solution is conceptually trivial:

> Find the p.d.f. $p(q)$ if the transformation $\mathscr{R}: \Theta \to q$ and the p.d.f. $p(\Theta)$ are given.

Remarks.

- The stationary expected loss is predicted under the ideal situation described by the assumptions (6), (9). In this sense, an "optimistic" lower bound of the practically achievable loss is predicted.
- For brevity, technical details related to the symbols like arg min, min ... are omitted.

- The design parameter ω has been introduced in order to facilitate the subsequent task of its automated prior tuning (see the discussion in Section 6). In the body of the paper, it will not be used and will be hidden.
- Our formulation could be generalized by analyzing the function defined as follows: Choose the control law

$$C_0(\hat{\Theta}) = \arg \min_{C(\hat{\Theta})} \lim_{T \to \infty} \mathsf{E}^{C(\hat{\Theta})}[L(T, d(1 \dots T)) \mid \hat{\Theta}]$$

  and define, *for this fixed control law*, the function of the system parameter Θ and its estimate $\hat{\Theta}$

$$q(\Theta, \hat{\Theta}) = \lim_{T \to \infty} \mathsf{E}^{C(\hat{\Theta})}[L(T, d(1 \dots T)) \mid \Theta]$$

  For the problem treated, practical significance of such sensitivity study is, however, restricted.
- The prediction problem has been formulated for loss functions depending on observable data. An extension to a state dependent loss is straightforward.

4. CONCEPTUAL SOLUTION FOR LINEAR NORMAL REGRESSION MODEL AND QUADRATIC LOSS

This section describes a specific choice of the p.d.f.'s (1), (2) and of the loss function (4). The prediction problem is then solved conceptually.

4.1 Problem ingredients

The c.p.d.f. (1) specifying *linear normal regression model* of a finite order n takes the form

$$p(y(t) \mid t-1; u(t), \Theta) = p(y(t) \mid z(t), P, R) = N_{y(t)}(P' z(t), R) \tag{12}$$

where we have denoted

$N_y(m, R)$ normal p.d.f. of y specified by the mean m and the covariance R;
P matrix (of appropriate dimensions) of regression coefficients;
$z(t)$ regression vector consisting of linear functions (e.g. differences) of several delayed inputs and outputs. Typically,

$$z'(t) = [u'(t), y'(t-1), u'(t-1), \dots, y'(t-n), u'(t-n), 1];$$

with n denoting the order of the system.
$\Theta = (P, R)$ unknown parameters of the model.

The model (12) serves as the basic one for majority of practically used selftuners. Its numerical feasibility and relatively high flexibility are the strongest reasons for its use. This also explains why the algorithmization has primarily been developed for this model in [3]. The present paper reports the achieved results adding to them

a parametric approximation of the required p.d.f. (see Section 5) and extending the exploitation of the evaluation performed.

It can be shown [11] that Gauss-inverse-Wishart distribution is the self-reproducing prior p.d.f. $p(\Theta) = p(P, R)$ for the linear normal regression model (12). For this prior p.d.f., Bayesian identification which determines the p.d.f. (2) reduces algorithmically to recursive least squares.

The p.d.f. (2) will be (approximately) Gaus-inverse-Wishart if any prior p.d.f. (nonzero for possible values of Θ) has been modified by sufficiently informative preliminary data. Thus, without a substantial loss of generality, we can restrict to the *prior p.d.f. of Gauss-inverse-Wishart form*. Then any available algorithmization of recursive least squares can be used for specifying the needed statistics from preliminary data. A factorized version [1] is preferable as its results fit to our next treatment.

The predicted quantity will be the achievable control quality of selftuners optimizing the expectation of the *multistep quadratic loss*

$$L_\omega(T, d(1 \ldots T)) = \frac{1}{T}\{\|z(T)\|^2_{H_z} + \sum_{t=1}^{T}[\|y(t)\|^2_{H_y} + \|u(t)\|^2_{H_u}]\} \tag{13}$$

where

$\|x\|^2_{H_x} = x'H'_xH_xx$ is square of the quadratic norm (of a vector x) with square-root weights stored in the matrix H_x;

Thus the design parameter ω consists of the matrices H_y, H_u, H_z. For stability reasons, the matrix H_z should have sufficient rank [5].

4.2 Solution

It is well known that the minimal loss for the system with fixed parameters P, R can be found by solving the discrete-time Riccati equation or by algebraic methods [8]. We have used the ready efficient algorithm for solving a factorized version of the Riccati equation as it fits to the overal design of the selftuners [5]. The specific way of the evaluation is, however, irrelevant to the next treatment. The existence of a numerically reliable algorithm which is able to evaluate the strongly nonlinear ("Riccati") mapping

$$\mathscr{R}: P, R \to q \in [0, \infty) \tag{14}$$

is of importance for the approach.

Numerical realization of the mapping prevents us a priori from evaluating the p.d.f. $p(q)$ analytically. A version of the Monte Carlo method has been found [3] the only procedure without excessive requirements on computational power. The idea is quite straightforward: Selecting N independent random samples of Θ, say Θ_i $i = 1, 2, \ldots, N$, distribution according to the p.d.f. $p(\Theta)$ we find N independent samples of q, say q_i, $i = 1, 2, \ldots, N$, taking simply

$$q_i = \mathscr{R}(\Theta_i) \tag{15}$$

The solved problem is then formulated as the estimation of an unknown p.d.f. $p(q)$ from N independent samples having this distribution.

Couple of methods have been elaborated for solving the task. As it was little known about possible shape of the searched for p.d.f. $p(q)$, the most flexible nonparametric Bayesian approach to the p.d.f. estimation in the vein [2] was adopted in [3]. The experience gained from the nonparametric estimation has given us the chance to search for a widely valid parametric version of this estimation part. The search has been motivated pragmatically: the nonparametric approach is time consuming. With surprising precision, the *searched for p.d.f. has been found to be log-normal.* Its estimation and other algorithmic details will be given in next section.

Remarks.

- The chosen loss function corresponds to the regulation problem with penalized inputs. It has been chosen because of notational simplicity. An extension to a nonzero setpoint and to the practically more sensible penalty on input increments is straightforward.
- In the loss (13), LD or UD factorizations of the weighting matrices [1] are practically used. Simpler notation is the only reason for the description in terms of the square-root version. The same simplification will be applied to recursive least squares discussed in next section.
- The need for numerical reliability of the algorithm realizing (14) can be recognized if we recall that the minimal loss q is finite only if the coefficients P specify a stabilizable system. The measure assigned by Gauss-inverse-Wishart distribution to the "unstabilizable" P's is zero. Formally, such coefficients make no harm when transforming the c.p.d.f. $p(\Theta)$ to the p.d.f. $p(q)$. However, the probability that we shall select poorly stabilizable P's is always nonzero.

5. ALGORITHMIC DETAILS

The evaluation sketched in preceding section will be summarized here in a commented "flowchart".

1. Data checking and structure determination

Serious outliers are removed from the collected data set, the algorithms for structure determination [4] and sampling determination [7] are used for specifying the regression model structure. This point is mentioned here for completeness only.

2. Construction of prior p.d.f. $p(\Theta \mid \textit{preliminary data})$

Recursive least squares are applied to the available data under the assumption that the structure of the regressor has been selected. The initial conditions either reflect poor information before data exploiting or expert knowledge quantified in the way described in [6].

Gauss-inverse-Wishart p.d.f. $p(P, R)$ can be characterized uniquely by

d_f = number of data pairs + nonnegative initial condition;

$\hat{P} = \mathsf{E}[P \mid \textit{preliminary data}]$ = least-squares estimate of the coefficients P;

$$\hat{R} = \mathsf{E}[R \mid \textit{preliminary data}] = \frac{\text{remainder of least squares}}{d_f - \textit{dimension of output} - 1};$$

$\hat{R}_{ij}GG'$ = covariance of ith columns of P with G being square-root of the regression matrix produced by least squares.

3. Generating independent samples of parameters

For Gauss-inverse-Wishart p.d.f. $p(P, R)$, the coefficients P are normally distributed on the noise covariance R. Thus, the required samples of P_i conditioned on R_i can be found by affinely transforming mutually independent, zero-mean and unit variance normal entries of a matrix E (of the same dimensions as P) available at any computer. Denoting $R_i^{1/2}$ a square root of ith realization of the covariance matrix R, the transformation can be written

$$P_i = \hat{P} + GE_iR_i^{1/2}$$

The generating R_i, which has inverse-Wishart distribution, is a bit more complicated. A transformation of normally distributed variables recommended in [14] is used.

4. Generating independent samples of achievable loss

For a fixed parameter $\Theta = (P, R)$, the transformation (15) has the following structure

$$q(\Theta) = \|R^{1/2}\, S_y(\Theta)\|^2 \tag{16}$$

The matrix $S_y(\Theta)$ and related control law $C(\Theta)$, determining new input $u(t+1)$ as a solution of the linear relation $C'(\Theta)\, z(t+1) = 0$, are contained in the square root of the (extended) stationary Riccati matrix

$$S_z(\Theta) = \begin{bmatrix} & 0 & 0 \\ C(\Theta) & S_y(\Theta) & 0 \\ & \cdot & \cdot \end{bmatrix} \tag{17}$$

A detailed structure of the matrix $S_z(\Theta)$ and an efficient algorithmization is described in [5]. For our presentation, it is important that the algorithm can be interpreted as successive approximations for constructing the matrix $S_z(\Theta)$. A dynamic-program-

ming interpretation shows that the weighting matrix H_z is the proper initial condition of the performed iterations.

The relation of null-space of H_z and properties of the system coefficients is decisive for achieving convergence to the optimal solution. For unknown parameters, the matrix H_z has to have sufficient rank. Specific values of H_z influence rate of convergence achieved (sometimes substantially). The latter property is recalled for stressing importance of the H_z choice. Other details or appropriate references can be found in [5].

5. Estimation of the p.d.f. $p(q)$

The experience gained from the nonparametric estimation of the p.d.f. $p(q)$ leads us to hypothesis that samples q_i are log-normally distributed. Since that, the nonparametric estimation has served us for comparison only. For this reason, the parametric version is sketched only.

Log-normality of q means that the quantity $\zeta = \ln(q)$ is normally distributed with an (unknown) constant mean $\bar{\zeta}$ $(= \ln(\bar{q})$ for some $\bar{q} > 0)$ and a variance s. The Bayesian estimation of the assumed trivial model

$$p(\zeta) = N_\zeta(\bar{\zeta}, s) \tag{17}$$

reduces to recursive least squares. Using ready formulae given e.g. in [11] we find

$$p(\zeta \mid \zeta_1, \ldots, \zeta_N) = p(\zeta \mid q_1, \ldots, q_N) = St_\zeta(\hat{\zeta}, \hat{s}, N + 1) \tag{19}$$

where

$St_\zeta(m, s, k)$ denotes Student p.d.f. of ζ specified by the expectation m, the variance s and the number of degrees of freedom k;

$\hat{\zeta}$ is the arithmetic mean of the random samples ζ_i;

$\hat{s}$ is the remainder of least squares divided by $N + 1$;

N is the number of samples used for constructing the predictive p.d.f. $p(\zeta \mid q_1, \ldots, q_N)$.

Transforming the p.d.f. ζ back to the quantity of interest we find the final form of the searched for p.d.f.

$$p(q \mid q_1, \ldots, q_N) = \frac{c}{q\left(1 + \frac{1}{(N+1)\,\hat{s}} \ln^2\left(\frac{q}{\hat{q}}\right)\right)^{\frac{N+3}{2}}} \tag{20}$$

where

c is an appropriate constant normalizing the integral of the p.d.f. to unity;

$\hat{q}$ is the *geometric mean* of the random samples $q_1, \ldots, q_N$.

Remarks.

– The heavy tail of log-normal distribution reflects the expected loss when the selected P_i's describe poorly stabilizable systems.

- The geometric mean is known to be always smaller than the arithmetic one. It characterizes a "center" of samples from a heavy-tailed distribution better than the arithmetic mean.
- At present, a fixed number of the random samples is used. Of course, other stopping rules can be applied.

6. OTHER OUTPUTS OF DATA ANALYSIS

Our central problem, i.e. the prediction of achievable control quality, has been resolved in preceding sections. The evaluations performed can be (with little extra effort) used for deciding among options available for tuning of the controller. Possibilities are judged in this section.

Two types of the options can be distinguished:

- *Objective* options reflect degree of knowledge available about the process to be controlled. Their choice can be and should be done automatically.
- *Subjective* options correspond to user's wishes and preferences. They are true user's knobs. CAD should aid in expressing the requirements quantitatively or in transforming them from a user-friendly representation to a selftuner-friendly representation.

The identification options belong to the objective class in the above dichotomy. Bayesian identification provides the needed (conceptual) algorithm, i.e. it gives rule how to combine prior knowledge and (preliminary) data for the controlled design. The information is compressed into the p.d.f. (2), which can be evaluated in the particular case treated: the statistics needed for the assumed Gauss-inverse-Wishart distribution are gained by recursive least squares applied in the preliminary stage.

The design parameters determining the used loss function are subjective in general. For the quadratic loss, the factors of the penalization matrices in (13) should be selected. The subsequent discussion of the terminal penalty H_z and of the pair H_y, H_u will be separated as H_z can be given objective character.

1. Choice of H_z as initial guess of Riccati matrix.

The selftuner working in real-time is able to search for the minimizing argument in (7) within a limited horizon only. It makes a quantitative choice of H_z important (because of the influence of H_z on the rate of convergence, see the first remark in Section 4.2 and step 4 in Section 5).

The highest rate is achieved if H_z in (13) is close to the matrix $S_z(\Theta)$ (17) which is composed of the optimal stationary control law and of the square root Riccati matrix. It is thus reasonable to choose for real-time use

$$H_z = \hat{S}_z = \text{a preliminary-data-based point estimate of } S_z(\Theta)$$

We have chosen

$$\hat{S}_z = \frac{1}{N} \sum_{i=1}^{N} S_z(\Theta_i)$$

where $S_z(\Theta_i)$ is the stationary value of the matrix $S_z(\Theta)$ (17) determined for *i*th parameter sample. This estimate can be justified by applying the nonparametric Bayesian estimation under poor prior information. This is clearly "cheap" by-product of the evaluations performed. Due to outliers, it is more probable that $S_z(\Theta)$ will be "overestimated" by $\hat{S}_z$ than "underestimated". This situation can be shown safer (both from stability and rate of convergence view points) than the opposite one.

2. Choice of H_y, H_u.

As a rule, user's wishes are expressed in other terms than in the terms of the quadratic loss. The developed tools give us a chance to tune the input and output penalties according to these wishes. The idea si simple:

For fixed H_u, H_y and a sample Θ_i, the full description of the closed loop behaviour is available. Consequently, the stationary distribution of the corresponding state can be evaluated, i.e. the distribution of the stationary state (which is uncertain due to Θ uncertainty) can be constructed and analyzed. If the results of the analysis are satisfactory from the user's view point, the tuning is stopped, otherwise the penalties are changed (either manually or automatically).

Search for a balance between the input and output dispersions serves as an example of the sketched methodology. For each optimal closed loop realization, these dispersions are evaluated and their distributions are constructed in the same way as that of q. Then, it is easy to see whether the ranges of the probable input-output data are acceptable or not. At present, an automatic search for proper input penalty for SISO systems is resolved. It uses regula falsi method resting on monotonic relation of the penalty and input dispersion.

7. ILLUSTRATIVE EXAMPLE

The following example illustrates behaviour and computational demands of the designed tool.

Series of experiments have been performed on IBM PC AT compatible using a package for simulation, identification and control (SIC) [9]. This package contains also the mentioned algorithm for the structure determination of regression models [4]. SIC runs under a special monitor KOS (abbreviation of Czech name for an interactive supporting system) [10]. The intended CAD package will use services of KOS, too.

A typical example of third order single-input single-output system is presented.

Simulated system

$$p(y(t) \mid t-1; P, R) = N_{y(t)}(\hat{y}(t), R)$$

with the conditional expectation of the form

$$\hat{y}(t) = P' z(t) =$$

$$[b_0, a_1, b_1, a_2, b_2, a_3, b_3, k]\,[u(t), y(t-1), \ldots, y(t-3), u(t-3)\,1]' =$$

$$= 2{\cdot}01y(t-1) + 0{\cdot}00793u(t-1) - 1{\cdot}35y(t-2) + 0{\cdot}0236u(t-2) +$$

$$+ 0{\cdot}301y(t-3) + 0{\cdot}00435u(t-3)$$

and the variance

$$R = 1$$

Identification

White gaussian noise with zero mean and unit variance was used for exciting the system through the identification phase. Bayesian identification was used estimating the linear normal model with the correct third order. Recursive least squares producing the needed statistics for Gauss-inverse-Wishart distribution were initialized as follows

$$\hat{P} = 0\,, \quad \hat{R} = 1\,, \quad G = I\,, \quad d_f = 3$$

where I denotes unit matrix.

Five hundred data pairs were processed without forgetting.

Loss function

Quadratic loss (13) predicted was specified by

$$H_y^2 = 1\,, \quad H_u^2 = 0{\cdot}2\,, \quad H_z H_z' = 100I\,, \quad T = 2000$$

Simulation of selftuner behaviour

The simulated system was controlled by the LQ selftuner which assumed the correct model structure and optimized the quadratic criterion specified by the weights given above. It used the enforced separation of identification and control design. The optimization was based on so called iterations-spread-in-time (IST) strategy. IST strategy computes single iteration of Riccati equation per identification step using the preceding results as initial condition (for details see [5]). Prior information was specified by

$$\hat{P} = [1, 0{\cdot}9, 0, \ldots, 0]\,, \quad \hat{R} = 3\,, \quad G = 3I\,, \quad d_f = 3$$

The long-run sample value of the assumed quadratic loss (10 000 simulation steps) was 42·3.

Results of prediction

The result of the proposed parametric prediction based on 2000 samples can be seen in Figure 1. Plausibility of log-normal form assumed can be judged from Figure 2 which shows results of the nonparametric estimation based on 1000 samples. The

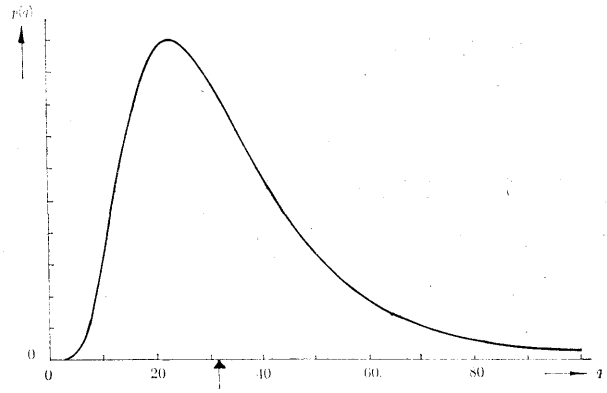

Fig. 1. Shape of the p.d.f. $p(q)$ constructed under the log-normal hypothesis (↑ marks the realization of the sample loss through the adaptive control simulation).

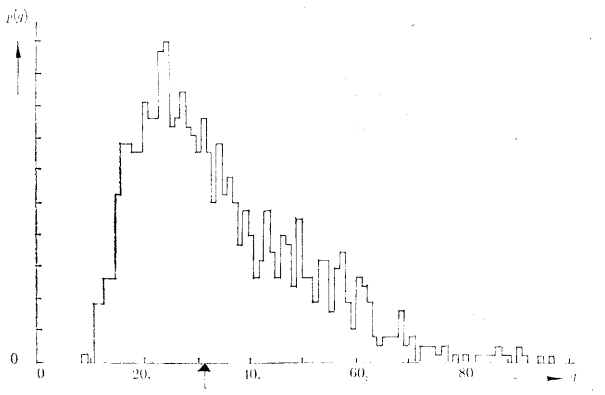

Fig. 2. Nonparametric estimate of the p.d.f. $p(q)$ (↑ marks the realization of the sample loss through the adaptive control simulation).

sample loss found through the simulation of selftuning control is in good agreement with the prediction: its value falls in the range of highly probably loss values.

Computation time was about 3 minutes in the parametric case, about 15 minutes in the nonparametric case. Substantial time saving has been reached by the smaller amount of samples needed for a comparable precision. This fact is of vital importance for the time-consuming optimization planned for the penalty tuning.

8. CONCLUSIONS

A tool for prior evaluation of LQG selftuning controller and a step for its pre-tuning have been described.

The closed loop behaviour has been predicted under incomplete knowledge of the controlled system. The problem formulation reflects the practical aim: to facilitate by a necessary software support the routine use of the selftuners described in [5]. Other sensitivity-type studies are of importance when alternative goals are followed.

Of course, the presented algorithms are not able to bridge a gap between real world and models and they have to fail when, for instance, the controlled system is strongly nonlinear. Our experience, however, has shown that a range of real systems can be well controlled when relying on proper linear normal regression model.

Within class of these models, this contribution completes a sequence which covers all (except experiment design) basic steps of automatic pretuning of SISO LQG selftuners. This fact is illustrated in the table below. Its first column lists the parameters which have to be selected for LQG controller, second column refers to algorithms for their prior tuning and the third gives information which has to be supplied by the user. Naturally, access to representative data measured on the process is necessary prerequisite of the referred tools.

Table. Pre-tuning of key tuning knobs of LQG selftuners.

Pre-tuned parameter	See	User asked for	Remarks
model order, transport delay, significant inputs and external variables	[4]	the structure of the most complex assumed model	
sampling period	[7]	admissible model complexity	
initial conditions for least squares	[11]		forgotten result of least squares gained from available data
forgetting factor		forgetting factor	no ready solution: structure determination [4] applicable
control horizon	[5]		avoided for favourite IST strategy
terminal state and input penalizations	here	required range of inputs	no ready solution for MIMO systems

The real decrease of user's burden when he/she is pre-tuning the selftuner should be seen from the table.

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REFERENCES

[1] G. J. Bierman: Factorization Methods for Discrete Sequential Estimation. Academic Press, New York 1977.

[2] T. S. Fergusson: A Bayesian analysis of some nonparametric problems. Ann. Statist. *1* (1973), 209—230.

[3] T. Jeníček: Prior Analysis of Expected Control Quality when Using Selftuning Controllers. (in Czech). Ph. D. Dissertation, Institute of Information Theory and Automation, Czechoslovak Academy of Sciences, Prague 1988.

[4] M. Kárný and R. Kulhavý: Structure determination of regression-type models for adaptive prediction and control. In: Bayesian Analysis of Time Series and Dynamic Models (J. C. Spall, ed.), Marcel Dekker, Inc., New York and Basel 1988.

[5] M. Kárný, A. Halousková, J. Böhm, R. Kulhavý and P. Nedoma: Design of linear quadratic adaptive control: theory and algorithms for practice. Supplement to Kybernetika *21* (1981), No. 3—6.

[6] M. Kárný: Quantification of prior knowledge about global characteristics of linear models. Kybernetika *20* (1984), 376—385.

[7] M. Kárný: Estimation of sampling period for selftuners. Submitted to 11th IFAC World Congress, Tallinn 1990.

[8] V. Kučera: Discrete Linear Control — The Polynomial Approach. J. Wiley, Chichester 1979.

[9] R. Kulhavý: SIC: User's Guide (version 1.1). Institute of Information Theory and Automation, Czechoslovak Academy of Sciences, Prague 1988.

[10] P. Nedoma and R. Kulhavý: KOS: User's Guide (version 1.1). Institute of Information Theory and Automation, Czechoslovak Academy of Sciences, Prague 1988.

[11] V. Peterka: Bayesian approach to system identification. In: Trends and Progress in System Identification (P. Eykhoff, ed.), Pergamon Press, Oxford 1981.

[12] Preprints of 2nd IFAC Workshop on Adaptive Systems in Control and Signal Processing, 1—3 July, 1986, Lund, Sweden.

[13] Preprints of 4th IFAC Symposium on CAD in Control Systems, CAD'88, 23—25 Aug., 1988, Beijing, P. R. C.

[14] R. C. Rao: Linear Statistical Inference and Its Application. Second Edition. John Wiley, New York 1973.

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