INFERENCE IN EXPERT SYSTEMS BASED ON COMPLETE MULTIVALUED LOGIC

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The essence of presumed approach towards reasoning in cases of uncertainty consists in assuming the knowledge base as a fuzzy axiomatic theory, i.e. a set of formulae in which each formula is equipped with a weight specifying the degree of membership to the fuzzy set of axioms of the theory. The task of the inference mechanism in such a case is to determine the degree to which each goal logically follows from this theory, and also other presumptions (the user's answers during the consultation). As a result of these reflections a logical inference mechanism has been designed which was implemented and tested in the System of Automatic Consultations (SAK). One of the advantages of this approach is the possibility of a natural insertion of contexts into knowledge base which has been used for improvement of the work of the SAK- OPTIMALI expert system.

1. THE ESSENCE OF LOGICAL INFERENCE MECHANISM

The task of an inference mechanism in an MYCIN-like expert system is to determine weights of goals both from the weights of user's answers to system's questions and the weights of rules in a knowledge base. For weights the interval of reals $\langle -1, 1 \rangle$ is usually used. Best known methods of evaluation of the weights are based on the Bayesian probabilities (PROSPECTOR, see [4]) or on considerations about measures of belief and disbelief (MYCIN, see [8]).

The essence of our approach to the choice of an inference mechanism consists in understanding the knowledge base as a *fuzzy axiomatic theory*, which is determined by a set of formulae, each of them provided with a weight specifying the degree of its membership to the fuzzy set of axioms of the theory. In that case the task of an *inference mechanism* is to determine the degree to which each goal *logically follows* from this theory and other premises (i.e. user's answers in consultation).

The degree of logical consequence is determined by the semantic of the used multivalued logic (in propositional calculus by the truth functions for calculation truth values of composed propositions).

The activity of any inference mechanism consists in *syntactical deduction* by means of a sequence of elementary inference steps. Thus the inference mechanism is able only (in a better case – being an automated proving procedure) to determine the degree in which the goal is *provable* from the theory and the disposition of consultation. If the used multivalued logic is complete, then for any formula φ and axiomatic theory X it holds generally

$$\text{the degree to which } \varphi \text{ follows from } X \text{ (semantically)} =$$
$$= \text{the degree to which } \varphi \text{ is provable from } X \text{ (syntactically)}\,.$$

For knowledge bases generally represented as arbitrary fuzzy axiomatic theories it is then necessary to construct an inference mechanism as an automated proving procedure in complete multivalued logic.

J. Pavelka in [7] proved that among many isomorphic variants of semantics (which can be reasonably defined as the so called residuated lattices) only the multivalued logic with Lukasiewicz semantics (and its isomorphic variants) has the property of completeness. For the truth values from interval $\langle 0, 1 \rangle$ the Lukasiewicz semantics of logical connectives is given by the following truth functions:

$$
\begin{array}{lll}
\text{implication} & \Rightarrow: & x \rightarrow y = \min(1, 1 - x + y) \\
\text{negation} & \neg: & \neg_* a = 1 - a \\
\text{conjunction} & \wedge: & a \wedge_* b = \min(a, b) \\
\text{disjunction} & \vee: & a \vee_* b = \max(a, b) \\
\text{context} & \&: & a \,\&_*\, b = \max(0, a + b - 1) \\
\text{composition} & \curlyvee: & a \curlyvee_* b = \min(1, a + b)
\end{array}
$$

Each logical consequence of a theory is in the Lukasiewicz logic deducible by using axioms of the theory (provided with degrees) and the inference rules (provided with an instruction for calculation of a degree, in which is deduced the goal of the rule, from degrees, in which premises of rules were deduced).

The basic inference rule is modus ponens

$$
\frac{\varphi, \varphi \Rightarrow \psi}{\psi} \qquad \frac{x, y}{\max(0, x + y - 1)}
$$

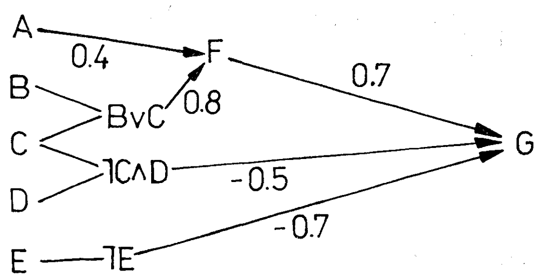
(If formulae φ, $\varphi \Rightarrow \psi$ are proved in degrees x, y respectively then formula ψ may be proved in degree $\max(0, x + y - 1)$.)

2. ILLUSTRATING EXAMPLE

The logical inference mechanism was designed in 1983 and ever since tested for the case of usual propositional rule based knowledge bases.

We shall illustrate on a simple example the method of formal representation of such knowledge bases and the results of deduction in the Lukasiewicz logic.

Knowledge base:



Corresponding fuzzy axiomatic theory:

axiom		degree
Ap	$\Rightarrow F1p$	(0·4)
$Bp \vee Cp$	$\Rightarrow F2p$	(0·8)
$F1p \curlyvee F2p$	$\Rightarrow Fp$	(1)
Fp	$\Rightarrow G1p$	(0·7)
$Cm \wedge Dp$	$\Rightarrow G2m$	(0·5)
Em	$\Rightarrow G3m$	(0·7)
$G1p \,\&\, \neg(G2m \curlyvee G3m)$	$\Rightarrow Gp$	(1)
$\neg G1p \,\&\, (G2m \curlyvee G3m)$	$\Rightarrow Gm$	(1)

Using of weights from the interval $\langle -1, +1\rangle$ is here formally substituted by the "decomposition" of every proposition X in the knowledge base to its "positive part" Xp and "negative part" Xm with truth degrees from $\langle 0, 1\rangle$. (We can notice an analogy with the measures of belief and disbelief in the MYCIN expert system.) Further "decomposition" of propositions (indicated by digits) serves for expressing a composition of rule's contributions by a composed proposition.

A consultation is given by the questionnaire collecting user's weighted answers which form additional axioms to our fuzzy axiomatic theory, e.g.

question	weight	axiom	degree
A	1	Ap	(1)
B	0·4	Bp	(0·4)
C	−0·6	Cm	(0·6)
D	0·8	Dp	(0·8)
E	−1	Em	(1)

The degrees of provability (= the degrees of logical consequence) from the whole fuzzy axiomatic theory representing the obtained uncertain knowledge are as follows:

proposition	degree
$F1p$	(0·4)
$F2p$	(0·2)
Fp	(0·6)
Fm	(0)
$G1p$	(0·3)
$G2m$	(0·1)
$G3m$	(0·7)
Gp	(0)
Gm	(0·5)

Thus the resulting weights of propositions F and G are 0·6 and −0·5 respectively.

We were comfortably surprised that the fuzzy axiomatic theory representing the obtained uncertain information and inference rules of the used logic can be easy implemented as a "program" in PROLOG so that the axioms are "fact-clauses" and the inference rules are "rule-clauses"; besides that it was necessary in the used micro-Prolog version to define only max and min operators.

```
Cp tr 0
Ap tr 1
Bp tr 0·4
Cm tr 0·6
Dp tr 0·8
Em tr 1
(Ap impl F1p) tr 0·4
((Bp vel Cp) impl F2p) tr 0·8
((F1p komp F2p) impl Fp) tr 1
((Fp impl G1p) tr 0·7
((Cm et Dp impl G2m) tr 0·5
(Em impl G3m) tr 0·7
((G1p kont (neg (G2m komp G3m))) impl Gp) tr 1
(((neg G1p) kont (G2m komp G3m)) impl Gm) tr 1
(neg X) tr Y if X tr Z and SUM (Z x 0) and SUM (1 x Y)
(X et Y) tr Z if X tr x and Y tr y and Z min (x y)
(X vel Y) tr Z if X tr x and Y tr y and Z max (x y)
(X kont Y) tr Z if X tr x and Y tr y and SUM (y x z) and SUM (z -1 X1) and Z max (0 X1)
(X komp Y) tr Z if X tr x and Y tr y and SUM (x y z) and Z min (1 z)
X tr Y if (Z impl X) tr x and Z tr y and SUM (x y z) and SUM (z -1 X1) and Y max (0 X1)
X max (X Y) if Y LESS X
X max (Y X) if Y LESS X
X max (X X)
X min (Y X) if X LESS Y
X min (X Y) if X LESS Y
X min (X X)
```

As a matter of interest we mention a shortened trace of the deduction of the *Gm* goal's degree after starting this program with the goal all-trace (*z*: *Gm* tr *z*). The trace is a backward chaining analogy of the logical proof of proposition *Gm* in the fuzzy axiomatic theory corresponding to the knowledge base.

```
all-trace (z : Gm tr z)
(1) : Gm tr X trace? y
(1 1) solved : ((neg G1p) kont (G2m komp G3m)) impl Gm) tr 1
(1 1 1 2 1) solved : (Fp impl G1p) tr 0·7
(1 2 1 1 2 1) solved : ((F1p komp F2p) impl Fp) tr 1
(1 1 2 2 1 1 2 1) solved : (Ap impl F1p) tr 0·4
(2 1 2 2 1 1 2 1) solved : Ap tr 1
(1 2 2 1 1 2 1) solved : F1p tr 0·4
(1 2 2 2 1 1 2 1) solved : ((Bp vel Cp) impl F2p) tr 0·8
(1 2 2 2 2 1 1 2 1) solved : Bp tr 0·4
(2 2 2 2 2 1 1 2 1) solved : Cp tr 0
(2 2 2 2 1 1 2 1) solved : (Bp vel Cp) tr 0·4
(2 2 2 1 1 2 1) solved : F2p tr 0·2
(2 2 1 1 2 1) solved : (F1p komp F2p) tr 0·6
(2 1 1 2 1) solved : Fp tr 0·6
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(1 1 2 1) solved : *G1p* tr 0·3
(1 2 1) solved : (neg *G1p*) tr 0·7
(1 1 2 2 1) solved : ((*Cm* et *Dp*) impl *G2m*) tr 0·5
(1 2 1 2 2 1) solved : *Cm* tr 0·6
(2 2 1 2 2 1) solved : *Dp* tr 0·8
(2 1 2 2 1) solved: (*Cm* et *Dp*) tr 0·6
(1 2 2 1) solved : *G2m* tr 0·1
(1 2 2 1) solved : (*Em* impl *G3m*) tr 0·8
(2 2 2 2 1) solved : *Em* tr 1
(2 2 2 1) solved : *G3m* tr 0·7
(2 2 1) solved : (*G2m* komp *G3m*) tr 0·8
(2 1) solved : ((neg *G1p*) kont (*G*2m komp *G3m*)) tr 0·5
(1) solved : *Gm* tr 0·5

3. THE VERIFICATION OF THE LOGICAL INFERENCE MECHANISM IN THE SAK SYSTEM

In [3] P. Hájek presented an algebraical discussion on several expert systems inference mechanism and exposed that their essence consists in the following combining functions (defined on interval $\langle -1, 1 \rangle$) necessary for a successful calculation of the weights.

NEG (x)	to assess the weight of negation of proposition the weight of which is x
CONJ (x, y)	to assess the weight of conjunction of two propositions with weights x, y
CTR (a, w)	to assess contribution of the rule with weight w, the antecedent of which has weight a
GLOB $(w_1, \ldots, w_k)$	to assess the weight of proposition which is the succedent of rules with contributions $w_1, \ldots, w_k$

Each inference mechanism for handling uncertain information in consulting expert systems is then determined by appropriate combining functions. Therefore the empty expert systems EQUANT [3] and SAK [2] offer a choice of combining functions suitable for the application from the repertoire containing e.g. EMYCIN or PROSPECTOR functions.

The above described logical inference mechanism for the case of usual rule-based knowledge bases is determined by using the following set of combining functions:

$$
\begin{aligned}
\mathrm{NEG}(x) &= -x \\
\mathrm{CONJ}(x, y) &= \min(x, y) \quad \text{for } x, y > 0 \\
&= 0 \qquad\qquad\quad \text{otherwise} \\
\mathrm{CTR}(a, w) &= \max(0, a + |w| - 1) \,.\, \mathrm{sign}(w) \quad \text{for } a > 0 \\
&= 0 \qquad\qquad\qquad\qquad\qquad\qquad \text{otherwise} \\
\mathrm{GLOB}(w_1, \ldots, w_k) &= \min\Big(1, \sum_{w_i > 0} (w_i)\Big) - \min\Big(1, \sum_{w_i < 0} (-w_i)\Big)
\end{aligned}
$$

This fact enabled us to implement logical inference mechanism in 1983 in the empty expert system SAK.

Experiments with the same design as in [3] were performed. The results of consultations under logical inference mechanism were the same or better than the results of these consultations under standard inference mechanism.

4. CONTEXTS IN LOGICAL INFERENCE MECHANISM

The advantage of logical inference mechanism is the possibility to introduce contexts naturally into the knowledge base.

The starting point is that the formula

$$\mathscr{X} \Rightarrow (\varphi \Rightarrow \psi)$$

(expressing "metaknowledge" that the rule $\varphi \Rightarrow \psi$ is conditioned by validity of the context $\mathscr{X}$) is logically equivalent (in the used logic) to the formula

$$(\mathscr{X} \mathbin{\&} \varphi) \Rightarrow \psi$$

where & is the mentioned connective *context*.

Then we can formulate a knowledge base as a loop-free set of rules of the following form:

CONTEXT:	$\mathscr{X}$ (combination of propositions)
IF:	φ (combination of propositions)
THEN:	ψ (proposition)
WITH WEIGHT:	$w \in \langle -1, +1 \rangle$

Let's note that this conceiving of context in the form of contextually conditioned rules is different from its conceiving in other expert systems, e.g. PROSPECTOR's type where a set of control rules with different character from knowledge base rules is used.

A *rule evaluation* according to the semantics of the complete Lukasiewicz logic is realized as follows:

First of all the context part of the rule is evaluated. After that a weight of the inner part of the rule is updated by using the CTR function. Only rules, the modified weight of which is positive, are examined further, which enables to *reduce search space* after the context evaluation.

A contribution of the rule is on the whole equal to

$$\mathrm{CTR}\,(a, \mathrm{CTR}\,(c, w))$$

where c is the weight of the context part $\mathscr{X}$, a is the weight of the antecedent φ and w is the given initial weight of the rule.

The above described approach was exploited for *improving a performance* of the SAK-OPTIMALI expert system which recommends mathematical decision methods adequate to a given decision situation (see [5]).

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