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ON THE DECENTRALIZED STABILIZATION OF INTERCONNECTED DISCRETE TIME SYSTEMS

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This paper presents a computationally simpler algorithm for stability analysis of large-scale time invariant discrete time systems described in the state-space model. An attempt is made to develop a decentralized stabilization algorithm based on decentralized observer utilizing the canonical model of Anderson and Luenberger [1]. The results of this paper are illustrated by considering a power system model as an example.

1. INTRODUCTION

The problem of decentralized stabilization of linear interconnected systems has been investigated by a number of workers in recent years. Most of these studies have been concerned with continuous time systems for which conditions for decentralized stabilizability have been obtained. In addition, methods have been proposed for computing the decentralized controllers. The first notable result in these directions seems to be that of Davison [2] who has shown that if the interconnected system have the phase variable canonical models then it is always possible to stabilize the composite system using decentralized state feedback. Özgüner and Perkins [3] have established the decentralized stabilizability of a slightly broader class of composite systems corresponds to lower-block triangular state matrix. Sezer and Huseyin [4]—[5], have further extended these results by establishing the decentralized stabilizability of a general composite system whose subsystems are all completely controllable and observable. Also notable are the results of Šiljak and Vukčevic [6] and Mahalanabis and Singh [7] who have used Liapunov function based aggregation and decomposition techniques for solving the decentralized stabilization problem. More recently, Ikeda and Šiljak [8] have utilized the same approach for establishing decentralized stabilizability of system with a broader class of state matrices.

The aim of the present study has been to examine the utility of the canonical transformation of Anderson and Luenberger [1] in order to obtain a convenient solution of the decentralized stabilization problem of completely controllable and observable interconnected systems. It may be noted that this problem has been earlier studied by Sezer and Huseyin [4], but their solution requires several trial and error runs before the desired decentralized state-feedback controller can be obtained. In this respect, the method of Sezer and Huseyin seems to have no advantage over the method of Wang and Davison [9]. On the other hand, the solutions proposed by Šiljak and Vukčevic [6] and Mahalanabis and Singh [7] appear to be somewhat conservative since these are based on approximation of the interaction effects.

The utility of canonical transformation is first established in Section 2 by considering the problem of decentralized stability analysis of the interconnections of observable discrete time subsystems. This is followed in Section 3 by a study of the decentralized stabilization problem of the interconnections of completely controllable and observable discrete-time systems. The results are illustrated by considering the discretized version of a power system control problem.

2. DECENTRALIZED STABILITY ANALYSIS

Consider a linear time invariant discrete time system which is obtained by interconnecting N subsystems each modelled by the following pair of equations:

$$X_i(k+1) = A_{ii} X_i(k) + \sum_{\substack{i=1 \\ j \neq i}}^{N} A_{ij} X_j(k) + b_{ii} u_i(k) + \sum_{\substack{j=1 \\ j \neq i}}^{N} b_{ij} u_j(k) \tag{1}$$

$$Y_i(k) = C_{ii}^{\mathrm{T}} X_i(k) + \sum_{\substack{j=1 \\ j \neq i}}^{N} C_{ij}^{\mathrm{T}} X_j(k) = C_i^{\mathrm{T}} X(k) \quad \text{for} \quad i = 1, 2, \ldots, N \tag{2}$$

where $X_i(k)$ is the n_i vector state of the ith subsystem and $u_j(k)$ and $Y_i(k)$ are respectively the scalar input and output variables. The system described by equations (1) and (2) will act as input decentralized form when the sampling period is very small compared to the system time constant. In other words, the effect of input interaction term

$$\sum_{\substack{j=1 \\ j \neq i}}^{N} b_{ij} u_j(k)$$

can be neglected under such situation. It is assumed that the eigenvalues of A_{ii} are inside the unit circle and $(A_{ii}, b_i, C_{ii}^{\mathrm{T}})$ constitutes a completely controllable and observable triple.

The problem is to determine the effects of interaction terms in equation (1) on the stability of each subsystem. A straightforward method for doing this is a consider the matrix A, having A_{ij}, $i, j = 1, 2, \ldots, N$, as its submatrices and to find the eigenvalues of this matrix. Unfortunately, this may involve large computations for cases where N is large. The question is whether there exists an alternative, computationally simpler, procedure for ascertaining the stability of the composite system. It appears that the observability properties of the subsystems assumed earlier permits one to resolve this question in a relatively straight forward manner.

To check this, consider the set of subsystems (1)−(2) with $U_i(k) = 0$ for all $i = 1, 2, \ldots, N$. The resultant autonomous system can be described by the equations:

$$X(k+1) = AX(k) \tag{3}$$

$$Y(k) = CX(k) \tag{4}$$

where $X(k)$ is the

$$n = \sum_{i=1}^{N} n_i$$

dimensional state vector of the composite system and $Y(k)$ is the N dimensional output vector of the same system. The matrices A and C have the following partitioned forms:

$$A = \begin{bmatrix} A_{11} & A_{12} & \ldots & A_{1N} \\ A_{21} & A_{22} & \ldots & A_{2N} \\ \vdots & \vdots & & \vdots \\ A_{N1} & A_{N2} & \ldots & A_{NN} \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} C_1^T \\ C_2^T \\ \vdots \\ C_N^T \end{bmatrix}$$

In view of the observability assumption of the subsystems, it follows that the following $n \times n$ matrix P will be nonsingular.

$$P = [C_1 A^T C_1 \ldots (A^T)^{n_1 - 1} C_1, C_2 \ldots (A^T)^{n_N - 1} C_N]^T$$

As shown by Mayne [10], the matrix P can be used to obtain a transformed system having the state $\bar{X}(k) = PX(k)$ for which the system matrices have certain canonical forms. More specifically one gets the transformed state variable model [11]

$$\bar{X}(k+1) = \bar{A}\,\bar{X}(k) \tag{5}$$

$$Y(k) = \bar{C}\,\bar{X}(k) \tag{6}$$

with the matrices $\bar{A}$ and $\bar{C}$ given by

$$\bar{A} = \begin{bmatrix} \bar{A}_{11} & 0 & 0 & \ldots & 0 \\ A_{21} & \bar{A}_{22} & 0 & \ldots & 0 \\ \vdots & \vdots & & & \\ \bar{A}_{N1} & \bar{A}_{N2} & . & \ldots & \bar{A}_{NN} \end{bmatrix}$$

and

$$\bar{C} = \begin{bmatrix} \bar{C}_1^T & 0 & \ldots & 0 \\ 0 & \bar{C}_2^T & \ldots & 0 \\ \vdots & & & \\ 0 & 0 & \ldots & \bar{C}_N^T \end{bmatrix}$$

Further, the blocks of the transformed matrices have the following explicit forms.

$$\bar{A}_{ii} = \left[\begin{array}{c|cccc} 0 & & I_{(n_i-1)\times(n_i-1)} & & \\ \hline \times & \times & \times & \ldots & \times \end{array}\right]_{n_i \times n_i}$$

$$\bar{A}_{ij} = \left[\begin{array}{cccc} & 0 & & \\ \hline \times & \times & \ldots & \times \end{array}\right]_{n_i \times n_j} \qquad i > j$$

and

$$\bar{C}_i^{\mathrm{T}} = [0\ 0\ \dots\ 0\ 1\ 0\ \dots\ 0]$$

where the 1 in $\bar{C}_i^{\mathrm{T}}$ in columns $(1 + n_1 + n_2 + \dots n_i - 1)$ and x's indicate possible non-zero elements.

Once the given system (3)–(4) is transformed to the canonical form (5)–(6) it is easy to see that the stability of the composite system depends on the eigenvalues of the diagonal blocks $\bar{A}_{ii}$ only. Thus, a necessary and sufficient condition for the composite system to be stable is that the eigenvalues of the N matrices $\bar{A}_{11}, \bar{A}_{22} \dots \bar{A}_{NN}$ are all located inside the unit circle. It is thus only necessary to find out the eigenvalues of the transformed subsystems, in order to determine the stability of the composite system.

3. DECENTRALIZED STABILIZATION

Consider now the set of subsystems (1)–(2) and introduce the composite system model

(7) $$X(k + 1) = A\,X(k) + B\,U(k)$$

(8) $$Y(k) = C\,X(k)$$

where A and C have the same forms as in the preceding section and B is an $(n \times N)$-dimensional input matrix. The problem is to obtain a decentralized feedback control law that would place all the poles of the closed loop system inside the unit circle.

The transformation used in the last section, while convenient for stability analysis does not help to obtain the desired solution of the control problem. One can, however, utilize the reorder form of transformation matrix of Anderson and Luenberger [1] in order to obtain the following transformed state-variable model.

(9) $$\tilde{X}(k + 1) = \tilde{A}\,\tilde{X}(k) + \tilde{B}\,U(k)$$

(10) $$Y(k) = \tilde{C}\,\tilde{X}(k)$$

where $\tilde{A} = S^{-1}AS$, $\tilde{B} = S^{-1}B$, $\tilde{C} = CS$ and S is the transformation matrix with $X(k) = S\,\tilde{X}(k)$. The matrices $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ have the following structures:

$$\tilde{A} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} & \dots & \tilde{A}_{1N} \\ 0 & \tilde{A}_{22} & \dots & \tilde{A}_{2N} \\ \vdots & & & \vdots \\ 0 & 0 & & \tilde{A}_{NN} \end{bmatrix} \qquad \tilde{B} = \begin{bmatrix} \tilde{b}_1 & 0 & \dots & 0 \\ 0 & \tilde{b}_2 & \dots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \dots & \tilde{b}_N \end{bmatrix}$$

and

$$\tilde{C} = \begin{bmatrix} \tilde{C}_{11}^{\mathrm{T}} & \tilde{C}_{12}^{\mathrm{T}} & \dots & \tilde{C}_{1N}^{\mathrm{T}} \\ \tilde{C}_{21}^{\mathrm{T}} & \tilde{C}_{22}^{\mathrm{T}} & \dots & \tilde{C}_{2N}^{\mathrm{T}} \\ \vdots & \vdots & & \vdots \\ \tilde{C}_{N1}^{\mathrm{T}} & \tilde{C}_{N2}^{\mathrm{T}} & \dots & \tilde{C}_{NN}^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} \tilde{C}_1^{\mathrm{T}} \\ \tilde{C}_2^{\mathrm{T}} \\ \vdots \\ \tilde{C}_N^{\mathrm{T}} \end{bmatrix}$$

Further, the blocks of $\tilde{A}$ and $\tilde{B}$ have the following explicit forms:

$$\tilde{A}_{ii} = \left[\begin{array}{c|c} 0 & I \\ \hline a_{ii,0} & a_{ii,1},\ a_{ii,2}\ \ldots\ a_{ii,n_i-1} \end{array}\right]_{n_i \times n_i}$$

$$\tilde{A}_{ij} = \left[\begin{array}{c|c} a_{ij,1} & \\ a_{ij,2} & \\ \vdots & 0 \\ a_{ij,n_i} & \end{array}\right]_{n_i \times n_j} \quad (\text{for } i < j) \quad \text{and} \quad \tilde{b}_i = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}_{n_i \times 1}$$

The advantage of this canonical form lies in the obvious solution of the decentralized state feedback control problem starting from the specified closed loop locations. This proceeds by assuming $U_i(k) = = -m_i \tilde{X}_i(k)$ with the result that the closed loop system matrix $\tilde{A}_c = \tilde{A} - \tilde{B}M$ has its diagonal block only affected by the feedback. Once the n closed loop poles are properly assigned to the N-diagonal blocks of $\tilde{A}_c$ the required elements of the vectors m_i can be easily calculated.

Note, however, that is not truly a decentralized control since $\tilde{X}_i$, $i = 1, 2, \ldots, N$, are not available for physical implementation. This difficulty can be circumvented by constructing a decentralized estimator for $\tilde{X}_i(k)$.

Let $\hat{\tilde{X}}_i(k)$ be the estimate of $\tilde{X}_i(k)$ and the dynamics of the estimator (based on one-step delay measurement) be given by the following equation; Willems [12] and Singh [13]

$$(11) \qquad \hat{\tilde{X}}_i(k+1) = \tilde{A}_{ii} \hat{\tilde{X}}_i(k) + \tilde{b}_i U_i(k) + l_i[Y_i(k) - \tilde{C}^{\mathrm{T}}_{ii} \hat{\tilde{X}}_i(k)]$$

where l_i is the observer gain vector of dimensional $n_i \times 1$ and $\tilde{C}^{\mathrm{T}}_{ii}$ is the ith partition of $\tilde{C}^{\mathrm{T}}_i$. The pair $(\tilde{A}_{ii}, \tilde{C}^{\mathrm{T}}_{ii})$ is assumed to be observable. It follows from equations (9)–(11), that the error $e_i(k)$ of the state estimation satisfies the following equations:

$$(12) \qquad e_i(k+1) = (\tilde{A}_{ii} - l_i\tilde{C}^{\mathrm{T}}_{ii})\, e_i(k) + \sum_{j=i+1}^{N} (\tilde{A}_{ij} - l_i\tilde{C}^{\mathrm{T}}_{ij})\, e_j(k) - \\ - \sum_{j=1}^{i-1} l_i\tilde{C}^{\mathrm{T}}_{ij}\, e_j(k) + \sum_{j=i+1}^{N} (\tilde{A}_{ij} - l_i\tilde{C}^{\mathrm{T}}_{ij})\, \hat{\tilde{X}}_j(k) - \sum_{j=1}^{i-1} l_i\tilde{C}^{\mathrm{T}}_{ij}\, \hat{\tilde{X}}_j(k)$$

Because of the upper block triangular structure of $\tilde{A}$ it follows that equation (12) represents a stable system with l_i selected to have the eigenvalues of $(\tilde{A}_{ii} - l_i\tilde{C}^{\mathrm{T}}_{ii})$ inside the unit circle while the contribution of the term

$$\left(-\sum_{j=1}^{i-1} l_i\tilde{C}^{\mathrm{T}}_{ij}\, e_j(k)\right)$$

involve in equation (12) is neglected compared to the total effect of all other terms. Equation (11) would then represent an asymptotically stable decentralized estimator for $\tilde{X}_i(k)$. This leads to the following decentralized controller

$$(13) \qquad U_i(k) = -m_i \hat{\tilde{X}}_i(k)$$

From equations (9)–(13), the following representation is obtained for the controlled system

$$(14) \qquad \bar{X}_i(k+1) = \bar{A}_{ii}\,\bar{X}_i(k) + \sum_{j=i+1}^{N} \bar{A}_{ij}\,\bar{X}_j(k) - \bar{b}_i m_i\,\hat{\bar{X}}_i(k)$$

$$(15) \qquad \hat{\bar{X}}_i(k+1) = (\bar{A}_{ii} - l_i \bar{C}_{ii}^{\mathrm{T}} - \bar{b}_i m_i)\,\hat{\bar{X}}_i(k) + l_i\,Y_i(k)$$

Equation (15) indicates the decentralized estimation scheme for the composite system, for $i = 1, 2, \ldots, N$.

4. NUMERICAL EXAMPLE

In order to illustrate the techniques discussed earlier in the paper, consider the first the equation of decentralized stability analysis of the following system:

$$X(k+1) = A\,X(k) + B\,U(k)\,; \quad Y(k) = C\,X(k)$$

where,

$$A = \begin{bmatrix} 0{\cdot}9993 & 0{\cdot}0589 & 0{\cdot}0009 & -0{\cdot}0599 & 0{\cdot}0002 & 0{\cdot}0 & 0{\cdot}0 \\ -{\cdot}0008 & 0{\cdot}9672 & 0{\cdot}0308 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ -{\cdot}0489 & -0{\cdot}0015 & 0{\cdot}8825 & 0{\cdot}0015 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0054 & 0{\cdot}0002 & 0{\cdot}0 & 0{\cdot}9997 & -0{\cdot}0054 & -0{\cdot}0002 & 0{\cdot}0 \\ 0{\cdot}0002 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0599 & 0{\cdot}9993 & 0{\cdot}0589 & 0{\cdot}0009 \\ 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & -0{\cdot}0008 & 0{\cdot}9672 & 0{\cdot}0308 \\ 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & -0{\cdot}0015 & -0{\cdot}0489 & -0{\cdot}0015 & 0{\cdot}8825 \end{bmatrix}$$

$$B = \begin{bmatrix} 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0019 & 0{\cdot}0 \\ 0{\cdot}1175 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0019 \\ 0{\cdot}0 & 0{\cdot}1175 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 1{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & -1{\cdot}0 & 1{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \end{bmatrix}$$

These represent the discrete version (using a uniform sampling period of 0·01 sec.) of the model for the two-area load-frequency control problem studied by Elgerd [14]. For the decentralized stability analysis, set $U(k) = 0$ and use the transformation matrix P (in Section 2) in order to obtain the following transformed model:

$$\bar{X}(k+1) = \left[\begin{array}{cccc|ccc} 0{\cdot}0 & 1{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 & 1{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 1{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ -0{\cdot}8531 & 3{\cdot}555 & -5{\cdot}550 & 3{\cdot}8485 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ \hline 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 1{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 1{\cdot}0 \\ 0{\cdot}8531 & -2{\cdot}702 & 2{\cdot}849 & -1{\cdot}0 & 0{\cdot}8531 & -2{\cdot}702 & 2{\cdot}849 \end{array}\right] \bar{X}(k)$$

It may be noted that in this particular example, the two transformed subsystems have dimensions 4 and 3 respectively. The eigenvalues of the diagonal blocks are found to be as follows:

For $\tilde{A}_{11}$: $0{\cdot}9942 \pm j\,0{\cdot}03504$; $0{\cdot}9839$; $0{\cdot}87577$

$\tilde{A}_{22}$: $0{\cdot}98681 \pm j\,0{\cdot}0248$; $0{\cdot}87555$

Consider now the eigenvalues of the blocks A_{11} and A_{22} of the original matrix A which are indicative of the system stability in the absence of the interaction terms

For A_{11}: $0{\cdot}99188 \pm j\,0{\cdot}02951$; $0{\cdot}98926$ $0{\cdot}87569$

A_{22}: $0{\cdot}98672 \pm j\,0{\cdot}02484$; $0{\cdot}87557$

It is apparent that $0{\cdot}99188 \pm j\,0{\cdot}02951$; $0{\cdot}87569$; $0{\cdot}98672 \pm j\,0{\cdot}02484$ of the eigenvalues have shifted towards the unit circle which implies for this particular example, a destabilizing action of the interaction terms. This result agrees with the recent result of Mahalanabis and Singh [7] who have utilized a Liapunov function based aggregation technique for analysing the effects of interaction on stability.

Consider now the problem of finding a decentralized feedback. Control for improving the system stability by shifting the closed loop eigenvalues to the following locations:

$$\tilde{A}_{C_{11}}: 0{\cdot}94 \pm j\,0{\cdot}01,\ 0{\cdot}9,\ 0{\cdot}86$$

$$\tilde{A}_{C_{22}}: 0{\cdot}9 \ \pm j\,0{\cdot}04,\ 0{\cdot}8$$

The required transformed model (see Section 3) is first obtained as given below:

$$\tilde{X}(k+1) = \left[\begin{array}{cccc|ccc} 0{\cdot}0 & 1{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & -0{\cdot}998 & 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 & 1{\cdot}0 & 0{\cdot}0 & -1{\cdot}004 & 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 1{\cdot}0 & -1{\cdot}01 & 0{\cdot}0 & 0{\cdot}0 \\ -0{\cdot}8531 & 3{\cdot}555 & -5{\cdot}55 & 3{\cdot}848 & -1{\cdot}015 & 0{\cdot}0 & 0{\cdot}0 \\ \hline 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 1{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 1{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}85314 & -2{\cdot}702 & 2{\cdot}849 \end{array}\right] \tilde{X}(k) + \left[\begin{array}{cc} 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 \\ 1{\cdot}0 & 0{\cdot}0 \\ \hline 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 1{\cdot}0 \end{array}\right] U(k)$$

$$Y(k) = \left[\begin{array}{cccc|ccc} 0{\cdot}0 & -0{\cdot}0002 & 0{\cdot}0002 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ \hline 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0002 & 0{\cdot}0 \end{array}\right] \tilde{X}(k)$$

Assuming,

$$U(k) = -\left[\begin{array}{cccc|ccc} m_{11} & m_{12} & m_{13} & m_{14} & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ \hline 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & m_{25} & m_{26} & m_{27} \end{array}\right] \hat{\tilde{X}}(k)$$

The following constant controller gains are obtained from the specified closed loop poles.

$$M = \left[\begin{array}{cccc|ccc} 0{\cdot}20915 & -0{\cdot}5445 & 0{\cdot}5835 & -0{\cdot}208 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ \hline 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & -0{\cdot}2039 & 0{\cdot}451 & -0{\cdot}249 \end{array}\right]$$

The observer for system under consideration can be designed using the decentralized approach proposed in the last section. For calculating the observer gains, the desired pole locations have been chosen as:

$$0{\cdot}94 \pm \mathrm{j}\,0{\cdot}1,\ 0{\cdot}9,\ 0{\cdot}86: \text{ Sub-observer 1.}$$

$$0{\cdot}9 \pm \mathrm{j}\,0{\cdot}4,\ 0{\cdot}8, \quad : \text{ Sub-observer 2.}$$

The gain vectors l_1 and l_2 are then found to be

$$l_1 = [-1167 \quad -338{\cdot}2 \quad 570{\cdot}1 \quad 1575]^{\mathrm{T}}$$

$$l_2 = [1009 \quad 1052 \quad 1094]^{\mathrm{T}}$$

5. CONCLUSIONS

It has been shown that the composite system stability can be found out with minimum effort by simple calculating the eigenvalues of each diagonal blocks of the transformed model (observable canonical model). It has been noticed that the state interaction has an effect on the stability of the composite system. It can be noted that the Anderson and Luenberger canonical model helps to reduce the computational burden in order to implement decentralized stabilization algorithm based on decentralized observer.

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