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ON THE PERFORMANCE AND STABILITY OF A SIMPLE GATED CONFLICT RESOLUTION ALGORITHM

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A simple "gated" conflict resolution algorithm is described that is simpler than the well-known tree conflict resolution algorithm due to Capetanakis, Tsybakov and Mikhailov. Here each cycle begins with the estimation of the number of contending users to select the number of slots in a frame. Then each contending user transmits its packet in one of those slots, chosen at random. Performance analysis based on Markov chain theory together with the region of stability of the protocol is provided. It is shown that the capacity of this algorithm is slightly less than that of Capetanakis' static algorithm.

1. INTRODUCTION

Multiple access protocols are used to schedule the transmission of packets over a broadcast communication channel that is shared by a distributed population of stations. We assume the channel to be a resource that can transmit successfully only one packet at a time.

Several factors need to be considered in the selection of a multiple access protocol. While at first it might seem that the most efficient multiple access protocol that yet has been devised must be used, one must keep in mind that achieving maximum capacity is by no means the only goal in choosing a protocol for a network. Simplicity, long term stability, robustness to errors in feedback information and good delay characteristics are also important.

Slotted Aloha is among the simplest multiple access protocols for communication systems, and a viable access technique in certain applications (e.g., a system supporting a large number of lightly loaded users). However, as many authors have pointed out [1 – 3], it is not stable, even for small traffic intensities, unless it is very carefully controlled.

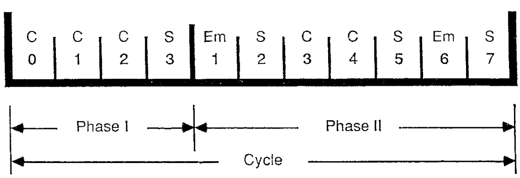
Protocols based on the tree algorithms are more advanced and take advantage of the feedback that a collision occurred sometimes in the past [4–6]. Such protocols are stable, but suffer two major problems from a practical point of view. The first is failure due to incorrect or inconsistent feedback (e.g., deadlock), the second is that the current "state" of the protocol encodes the channel activity for a long term in the past, so that stations may have difficulty entering or leaving the network.

In this paper we analyze a variation of a simple multiple access protocol introduced by Greenberg [7]. Some numerical results are obtained, along with the region of stability of the protocol. In Section 2 we present the model and description of the protocol. Throughput analysis is presented in Section 3. Finally discussion of results and conclusions are presented in Sections 4 and 5 respectively.

2. THE MODEL AND DESCRIPTION OF THE PROTOCOL

We assume the standard slotted Aloha channel model. The channel is divided into slots of length equal to one packet transmission time and all the terminals are synchronized to the channel time. Furthermore at the end of each slot, all users can determine whether zero, one or multiple packet transmission took place in that slot. Multiple packets per slot correspond to a collision and under such circumstances none of the packets is successful. The source model is assumed to be Poisson, i.e., an infinite number of independent users that collectively generate n packets per slot, where n is a Poisson random variable with mean λ.

The operation of the protocol consists of a sequence of "cycles" and each cycle is composed of two phases. New messages are only permitted to join the protocol at the start of a cycle. Phase I is used to estimate k, the number of contending users participating in the protocol during that cycle, and phase II is used for transmission of those packets. In the first slot of phase I (slot 0) all the contending users transmit their packets with probability $1/2^0 = 1$ to signal their participation in the protocol. If zero or one packet is transmitted in this slot, then the cycle is terminated. Other-



C: Collision Slot, Em: Empty Slot and S: Successful Slot

Fig. 1. Example of 4 successful transmissions in a cycle of duration 11 slots.

wise a collision occurred and phase I continues with each of the contending users transmitting with probability $1/2^1$ in the following slot (slot 1) and so on until slot u (where each of the contending users transmitted with probability $1/2^u$) which is collision free for the first time. In phase I we found an estimate of $\log_2 k$, namely u. Thus in phase II, we allocate 2^u – {number of packets successfully transmitted in the uth slot}, and each of the remaining contending users transmitt its packet in any of those slots with equal probability. Those users who due to a collision do not succeed in phase II, will join the protocol at the beginning of the next cycle together with new arrivals in the present cycle. The protocol is examplified for a typical cycle in Figures 1 and 2.

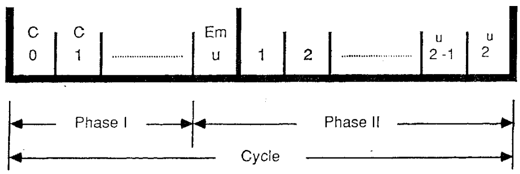


Fig. 2. Structure of a cycle.

3. ANALYSIS OF THE PROTOCOL

In this section we derive an expression for the throughput. First we obtain the conditional throughput, given the number of ready users at the beginning of a cycle. Then we use the Markov property to obtain the average throughput. For this derivations we use the following notations:

- k: Number of contending users at the beginning of a cycle
- $P_k(j)$: $\Pr(u = j \mid k)$, distribution of phase I, given k
- $f_k(i)$: $\Pr(0$ or 1 transmission in slot i of phase I, given $k)$
- $\beta(s \mid n, m)$: Pr (s successful transmission in phase II of given duration m slots, where the number of packets participating in this phase is given to be n)
- γ_k: Conditional throughput, i.e., the ratio of the average number of successful transmission in a cycle to the average duration of a cycle, given k
- $\Pr(i \mid k)$: Transition probability of number of contending users at the beginning of a cycle
- Π_k: Stationary probability of having k contending users at the beginning of a cycle.

Now we have (see Figure 2)

$$(1) \qquad f_k(j) = \left(1 - \frac{1}{2^j}\right)^k + \frac{k}{2^j}\left(1 - \frac{1}{2^j}\right)^{k-1} = \left(1 + \frac{k}{2^j - 1}\right)\left(1 - \frac{1}{2^j}\right)^k$$

In phase I, each user transmits with probability $1/2^j$ in slot j until zero or one transmission takes place. Thus

(2) $$P_k(j+1) = \prod_{l=1}^{j} [1 - f_k(l)] f_k(j+1) \qquad \forall k, \quad j = 1, 2, \ldots$$

which gives the following recursion:

(3) $$P_k(1) = (k+1)\frac{1}{2^k} \qquad \forall k$$

$$P_k(j+1) = P_k(j)\,\frac{1 - f_k(j)}{f_k(j)}\, f_k(j+1) \qquad \forall k, \quad j = 1, 2, \ldots$$

where $f_k(j)$ is given by (1). Then it can be shown that

(4) $\gamma_k =$

$$= \frac{\sum_j \left\{\left(1 - \frac{k}{k+2^j-1}\right) k \left(1 - \frac{1}{2^j}\right)^{k-1} + \frac{k}{k+2^j-1}\left[(k-1)\left(1 - \frac{1}{2^j-1}\right)^{k-2} + 1\right]\right\} P_k(j)}{\sum_j \left[(2^j + j) + \left(1 - \frac{k}{k+2^j-1}\right)\right] P_k(j)} \qquad k \geqq 2$$

$$\gamma_1 = 0$$

$$\gamma_0 = 0$$

The number of contending users at the beginning of a cycle is Markovian. To find the transition probabilities, we note the following occupancy problem [8], take $\beta(s \mid n, m)$ as the probability that exactly s cells, each having exactly 1 object when n unlike objects are distributed randomly into m unlike cells, then we have [8]

(5)

$$\beta(s \mid n, m) = \frac{\binom{m}{s}}{m^n} \sum_{i=0}^{\min(m-s-1,\,n-s)} (-1)^i \binom{m-s}{i} = \frac{n!}{(n-s-i)!}\,(m-s-i)^{n-s-i}$$

Conditioning on u, the conditional transition probability becomes

(6)

$$\Pr(i \mid k, u = j) = \sum_{s=0}^{\min(k,2^j)} \left[\left(1 - \frac{k}{k+2^j-1}\right)\beta(s \mid k, 2^j) + \frac{k}{k+2^j-1}\beta(s-1 \mid k-1, 2^j)\right] \cdot$$

$$\cdot \frac{(\lambda(2^j + j + 1))^{i-k+s}\, e^{-\lambda(2^j+j+1)}}{(i-k+s)!}$$

where

$$\beta(-1 \mid \cdot, \cdot) \triangleq 0$$

and then the transition probability becomes

(7) $$\Pr(i \mid k) = \sum_j \Pr(i \mid k, u = j)\, P_k(j)$$

where $P_k(j)$ is obtained by the recursion (3).

To find the stationary probability distribution of the number of contending users at the beginning of a cycle, we solve the matrix equation $\mathbf{\Pi} = \mathbf{\Pi P}$ when $\mathbf{\Pi} = (\Pi_0, \Pi_1, \Pi_2, \ldots)$ and the ki element of matrix $\mathbf{P}$ is given by $\Pr(i \mid k)$. The stationary probabilities exist, provided that the chain is ergodic [9]. Then the average throughput can be easily found.

In what follows we show that for $\lambda < \lambda^* = \cdot 3123$ the chain is ergodic. To accomplish this we use the following lemma.

Pake's Lemma (cf. [10]). Let $[N_i]_1^\infty$ be an irreducible aperiodic Markov chain whose state space is the set of nonnegative integers. The following conditions are *sufficient* for the chain to be ergodic:

(8a) $$|\mathsf{E}\{N_{i+1} - N_i \mid N_i = k\}| < \infty \qquad \forall k \quad \text{finite}$$

and

(8b) $$\limsup_{k \to \infty} \mathsf{E}\{N_{i+1} - N_i \mid N_i = k\} < 0$$

where $\mathsf{E}\{\cdot \mid \cdot\}$ denotes the conditional expectation.

Denote N_i to be the number of contending users at the beginning of ith cycle, it can be shown that

(9) $$\mathsf{E}\{N_{i+1} - N_i \mid N_i = k\} = \sum_j (\lambda - \gamma_k) \left[(2^j + j) + \left(1 - \frac{1}{k + 2^j - 1}\right)\right] P_k(j)$$
$$k \geqq 2$$

and for (8a) and (8b) to hold it is sufficient that

$$\lambda < \lambda^* = \lim_{k \to \infty} \gamma_k$$

where γ_k is the conditional throughput and is given by (4). Numerical results (see Fig. 5) shows that λ^* up to four digits accuracy is given by $\cdot 3123$.

4. DISCUSSION OF RESULTS

Numerical results show that phase I gives a good estimate of $\log_2 k$ where k is the number of contending users. This can be seen from Figures 3 and 4 where mean and coefficient of variation C ($C = \sqrt{(\text{variance})}/\text{mean}$) of phase II vs. k are plotted respectively. For all range of k, C is less than 1 and it approaches rather quickly to $\cdot 689$. Figure 5 displays the throughput vs. k. Under heavy loading which results in large k with high probability, γ_k approaches to $\cdot 3123$ which as it was shown in Section 3 is the upper bound on the capacity of the protocol. The same figure also shows the performance in the case of perfect estimate with no cost.

Figure 6 depicts the conditional throughput as a function of k when minislots of duration a, $a \leqq 1$, instead of whole slots are used in phase I. It is evident that asymptotically no gain in throughput is obtained. This is to be expected since the

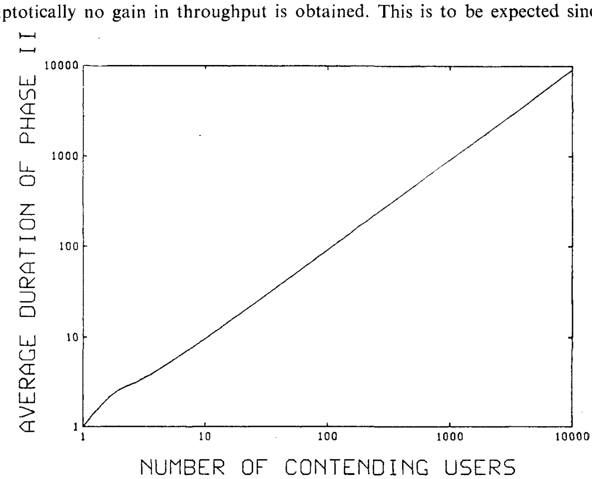


Fig. 3. The estimate vs. the number of contending users.

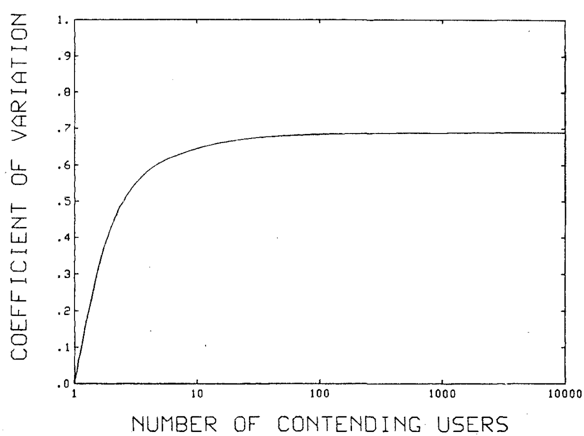


Fig. 4. Coefficient of variation of phase II vs. number of contending users.

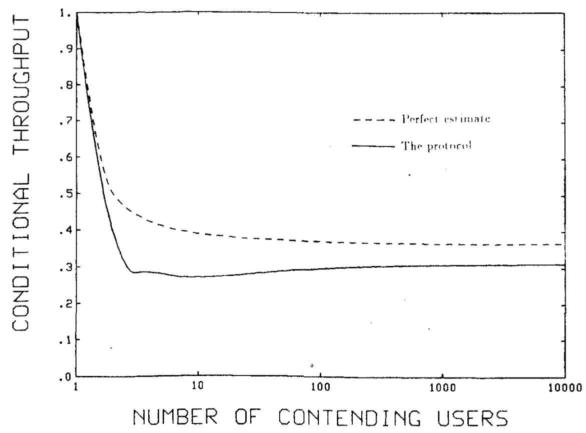


Fig. 5. Conditional throughput of the protocol compare achievable performance with perfect estimate.

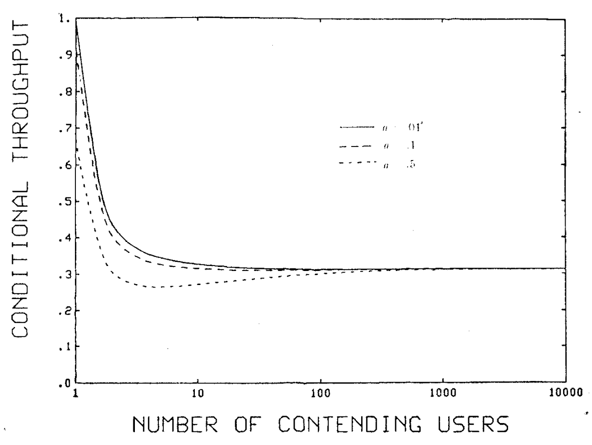


Fig. 6. Conditional throughput of the protocol with estimation phase composed of minislots.

average duration of phase I is of the order of $\log_2^k$, which is asymptotically negligible compared to phase II.

5. CONCLUSIONS

In this paper we analyzed a simple multiple access protocol which allocates the channel capacity by estimating the number of contending users. We showed that stability is achieved with only a slight degradation in the capacity (·3123 compare to $1/e$ of the unstable slotted Aloha).

Finally as a source of improvement we remark that if in phase II the stations were to monitor the proportion of idle, successful and collision slots, the estimate of the number of contending users could be improved: by having too many idle or collision slots, the stations might conclude that their estimate is high or too low respectively and accordingly shorten or expand the remainder of phase II. One algorithm for doing this was given by Hajek and Van Loon [3]. The idea of using an estimation process is also useful in more complicated algorithms. For example, Schoute [11] has been able to achieve a capacity of ·426 using a more complex estimation process and a slightly different cycle structure.

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REFERENCES

[1] M. J. Ferguson: On the control stability and waiting time in slotted Aloha random access system. IEEE Trans. Comm. *COM-23* (1975), 1306—1311.

[2] L. Kleinrock and S. S. Lam: Packet switching in a multiaccess broadcast channel: performance evaluation. IEEE Trans. Comm. *COM-23* (1975), 410—423.

[3] B. Hajek and T. V. Loon: Decentralized dynamic control of a multiaccess broadcast channel. IEEE Trans. Automat. Control *AC-27* (1982), 559—569.

[4] J. F. Hayes: Adaptive technique for local distribution. IEEE Trans. Comm. *COM-26* (1978), 1178—1186.

[5] B. Tsybakov and V. A. Mikhailov: Free synchronous packet access in a broadcast channel with feedback. Problems Inform. Transmission *14* (1979), 259—280.

[6] J. J. Capetanakis: Tree algorithms for packet broadcasting channels. IEEE Trans. Inform. Theory *IT-25* (1979), 505—515.

[7] A. G. Greenberg: On the Time Complexity of Broadcast Communication Schemes. Technical Report 82-04-02, April 1982, Dept. of Computer Science, University of Washington.

[8] V. F. Kolchin, B. A. Sevast'yanov and P. P. Chistyakov: Random Allocations. V. H. Winston and Sons, Washington, D. C. 1978.

[9] S. Karlin and H. M. Taylor: A First Course in Stochastic Processes. Second Edition. Academic Press, New York 1975.

[10] G. Fayolle, E. Gelenbe and J. Labetoulle: Stability and optimal control of the packet switching broadcast channel. J. Assoc. Comput. Mach. *24* (1977), 375—386.

[11] F. C. Schoute: Dynamic frame length ALOHA. IEEE Trans. Comm. *COM-31* (1983), 565—568.

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