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SUBOPTIMAL PERFORMANCE CRITERION SENSITIVITY OF LARGE-SCALE DECENTRALIZED CONTROL SYSTEMS*

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The purpose of this paper is to consider sensitivity of suboptimal performance criterion of large-scale decentralized control systems. The suggested approach is not crucially dependent on any particular method for decentralized control system design. By calculation the gradient matrices the numerous computations for each parameter of the system are successfully avoided. Moreover, once the suboptimal control problem is solved no further computations are needed to carry out the sensitivity analysis.

Two numerical examples are presented to illustrate the proposed methodology. In the second example the theory is applied to characterize the sensitivity of a power system example which employ five DC terminals to damp out inter-area oscillations due to the AC power system dynamics.

1. INTRODUCTION

In classical control theory it is assumed that control actions are undertaken by a single controller that has all the available information about the system. While there are obvious theoretical advantages control centralization may be difficult for a number of economic and technical reasons. On the contrary when large-scale systems are considered, information processing and control decisions are delegated to a set of agents. Therefore, in recent years, there has been a revival interest in the development of satisfactory control design methods implemented in a decentralized way. In addition the increasing implementation of these methods has been enabled by the availability of increasingly cheap computers and the consequent increase in all branches of engineering of interactive computing facilities to assists in design and analysis. Hence, the study of decentralized control systems is critical when one attempts to design controllers for large-scale systems. One of the most basic issues that arise in this class of problems is the sensitivity of the decentralised design.

The preservation of various system theoretic properties in the face of variations

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in the system model is an important theme in system theory. Growing attention is being paid to the problem of sensitivity of dynamic systems to variations in parameters [1 – 7]. From the sensitivity theory point of view, an important issue is to establish relationship between an infinitesimal variation in a nominal system parameter and the corresponding change in some system properties. Bode [1] was the first who established the significance of sensitivity in the design of feedback control systems. Horowitz [2] has developed the methods of frequency domain to a high extent and has applied them to the design of low sensitivity conventional feedback control systems (see also [3]). In 1963 Dorato [8] called attention to the problem of parameter sensitivity of the performance index of optimal control systems. Great number of publications dealing with the sensitivity problem in optimal and suboptimal control systems systems have appeared in recent years [9 – 22].

It should be pointed out that while the sensitivity theory is primarily concerned with a relationship between infinitesimal variations in a nominal system parameters and the corresponding system property, the robustness theory [23] requires the explicit delination of finite regions of models about the nominal model for which the given property is preserved. The robustness properties of large-scale decentralized control systems have been studied in [24 – 29].

In this paper we consider the problem of sensitivity of the suboptimal performance criterion of decentralized multivariable control systems to small parametes changes, and suboptimal cost sensitivity matrices are derived. The suggested approach is not crucially dependent on any particular method for decentralized control system design. By calculation the gradient matrices the numerous computations for each parameter of the system are successfully avoided. Moreover, once the suboptimal control problem is solved no further computations are needed to carry out the sensitivity analysis. The cost sensitivity matrices can be used to indicate which process parameters most affect the performance index. Thus, the designer can look for information regarding which parameters most affect the performance index in order to decide when a design modification should be made. It is also shown that the problem of selecting the fixed dimensional outputs which will lead to the best decentralized output feedback can be included in the proposed formulation.

The plan of the paper is as follows.

In Section 2 we summarize briefly two approaches for generating decentralized feedback laws for large-scale systems. The main result, i.e., sensitivity analysis of large-scale control systems is given in Section 3, where the problem of selecting the fixed dimensional outputs which lead to the best decentralized feedback is also include. In Section 4 the sensitivity results are illustrated through two numerical examples. In this section we examine the sensitivity of the suboptimal decentralized control design of a power system which employ five DC terminals to damp out inter-area oscillations due to the AC power system dynamics. Finally, in Section 5 we draw some conclusions.

2. DECENTRALIZED CONTROL SYSTEM DESIGN

In this section a brief discussion on two approaches for decentralized feedback designs is given. The approaches enable us to control the system by a set of controllers – each having different information and control variables. For more detailed discussion see [29].

Problem Formulation

Consider a large-scale system

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \sum_{i=1}^{k} \boldsymbol{B}_i \boldsymbol{u}_i(t), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \tag{1}$$

where $i = 1, \ldots, k$ index the control inputs, $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the state of the system, and $\boldsymbol{u}_i(t) \in \mathbb{R}^{m_i}$ is the control input $\sum_{i=1}^{k} m_i = m$. The information available to the local controller is assumed to be

$$\boldsymbol{y}_i(t) = \boldsymbol{C}_i \, \boldsymbol{x}(t) \tag{2}$$

where $\boldsymbol{y}_i(t) \in R^{r_i}$ is a local output vector, $\sum_{i=1}^{k} r_i = r$. The local control $\boldsymbol{u}_i(t)$ is assumed to be a direct feedback from the local output $\boldsymbol{y}_i(t)$, namely

$$\boldsymbol{u}_i(t) = \boldsymbol{E}_i \, \boldsymbol{y}_i(t), \quad i = 1, 2, \ldots, k \tag{3}$$

where $\boldsymbol{E}_i$ is a time-invariant gain matrix.

Design Procedure

Approach I

The first approach is based on computation of a complete state feedback and reduction to a specified control with a decentralized structure. In this case we introduce the performance index

$$J = \frac{1}{2} \int_0^\infty (\boldsymbol{x}^{\mathrm{T}}(t) \, \boldsymbol{Q} \, \boldsymbol{x}(t) + \boldsymbol{u}^{\mathrm{T}}(t) \, \boldsymbol{R} \, \boldsymbol{u}(t)) \, \mathrm{d}t \tag{4}$$

where $\boldsymbol{Q} = \boldsymbol{Q}^{\mathrm{T}} \geqq \boldsymbol{0}$, $\boldsymbol{R} = \operatorname{diag}(\boldsymbol{R}_i) > \boldsymbol{0}$, $i = 1, 2, \ldots, k$, and we seek to determine the optimal control law which minimizes (4) subject to dynamic constraints

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A} \, \boldsymbol{x}(t) + \boldsymbol{B} \, \boldsymbol{u}(t) \tag{5}$$

where $\boldsymbol{B} = [\boldsymbol{B}_1, \boldsymbol{B}_2, \ldots, \boldsymbol{B}_k]$, $\boldsymbol{u}^{\mathrm{T}} = [\boldsymbol{u}_1^{\mathrm{T}}, \boldsymbol{u}_2^{\mathrm{T}}, \ldots, \boldsymbol{u}_k^{T}]$. The solution is given by [30],

$$\boldsymbol{u}(t) = \boldsymbol{F} \, \boldsymbol{x}(t), \quad \boldsymbol{F} = -\boldsymbol{R}^{-1} \, \boldsymbol{B}^{\mathrm{T}} \boldsymbol{K} \tag{6}$$

where $\boldsymbol{K}$ is the positive definite solution of the algebraic Riccati equation

$$\boldsymbol{A}^{\mathrm{T}}\boldsymbol{K} + \boldsymbol{K}\boldsymbol{A} - \boldsymbol{K}\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{K} + \boldsymbol{Q} = \boldsymbol{0} \tag{7}$$

Having the full state feedback the next step is to reduce this to a specified decentralized structure, that is a control given by (3). In [29] different methods have been proposed for this selection (see Appendix I).

Approach II

The second approach is based on minimization of the decentralized quadratic performance index

$$J = \frac{1}{2}\int_0^\infty (\boldsymbol{x}^{\mathrm{T}}(t)\,\boldsymbol{Q}\,\boldsymbol{x}(t) + \sum_{i=1}^{k} \boldsymbol{u}_i^{\mathrm{T}}(t)\,\boldsymbol{R}_i\,\boldsymbol{u}_i(t))\,\mathrm{d}t \tag{8}$$

$$\boldsymbol{Q} = \boldsymbol{Q}^{\mathrm{T}} \geqq \boldsymbol{0}\,, \quad \boldsymbol{R}_i = \boldsymbol{R}_i^{\mathrm{T}} > \boldsymbol{0}\,, \quad i = 1, 2, \ldots, k$$

Both approaches lead to the control law of the form

$$\boldsymbol{u}_i(t) = \boldsymbol{E}_i\boldsymbol{C}_i\,\boldsymbol{x}(t) \tag{9}$$

where the values of the matrices $\boldsymbol{E}_i$, $i = 1, 2, \ldots, k$, depend on specified approach chosen (see Appendix I).

The implementation of any of the decentralized control laws proposed in this section leads to the closed loop system of the form

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A} + \sum_{i=1}^{k} \boldsymbol{B}_i\boldsymbol{E}_i\boldsymbol{C}_i)\,\boldsymbol{x}(t) \tag{10}$$

To reduce the dependence of the decentralized gains on the initial state $\boldsymbol{x}_0$ we suppose that $\boldsymbol{x}_0$ is a random variable uniformly distributed on the unit sphere with $\mathsf{E}\{\boldsymbol{x}_0\} = 0$, and $\mathsf{E}\{\boldsymbol{x}_0\boldsymbol{x}_0^{\mathrm{T}}\} = \boldsymbol{I}$, and we minimize the expected value of the performance index [31]. In this case, when the decentralized control is applied to the system (1) the value of the performance index is

$$J = \operatorname{tr} \boldsymbol{P} \tag{11}$$

where $\boldsymbol{P}$ satisfies the matrix equation

$$(\boldsymbol{A} + \sum_{i=1}^{k} \boldsymbol{B}_i\boldsymbol{E}_i\boldsymbol{C}_i)^{\mathrm{T}}\,\boldsymbol{P} + \boldsymbol{P}(\boldsymbol{A} + \sum_{i=1}^{k} \boldsymbol{B}_i\boldsymbol{E}_i\boldsymbol{C}_i) + \sum_{i=1}^{k} \boldsymbol{C}_i^{\mathrm{T}}\boldsymbol{E}_i^{\mathrm{T}}\boldsymbol{R}_i\boldsymbol{E}_i\boldsymbol{C}_i + \boldsymbol{Q} = \boldsymbol{0} \tag{12}$$

3. SENSITIVITY ANALYSIS

In this section we consider the problem of sensitivity of the suboptimal performance criterion of large-scale decentralized control systems to small parameter variations, and suboptimal cost sensitivity matrices are derived. The problem of selecting the fixed dimensional outputs which will lead to the best decentralized output-feedback is also included in this section.

Suppose that originally the system parameter values are given and that a decentralized control law based on these values is computed. However, in practice the parameter values may change. Three basic types of variations exist:

(a) Mathematical modeling and data errors in defining the nominal system and plant.

(b) Variations in dynamic characteristics caused by changes in environmental conditions, manufacturing tolerances, aging, wear, noncritical material failures, and off nominal power supplies.

(c) Maintenance induced errors in calibration, installation and adjustment.

Hence, the designer may look for information regarding which parameters most affect the decentralized control system. Dynamic systems, especially control systems, are often designed such that certain performance index takes either optimal or pre-assigned value. In these cases the quality of the system is characterized by the performance index, and it is quite logical that the sensitivity measure of interest is the performance index sensitivity. Thus, the designer can look for information regarding which parameters most affect the performance criterion in order to decide when a design modification should be made.

Suppose that the actual system matrices $\boldsymbol{A}$, $\boldsymbol{B}_i$ and $\boldsymbol{C}_i$, $i = 1, 2, \ldots, k$, are given by

$$\bar{\boldsymbol{A}} = \boldsymbol{A} + \Delta\boldsymbol{A}\,, \quad \bar{\boldsymbol{B}}_i = \boldsymbol{B}_i + \Delta\boldsymbol{B}_i\,, \quad \bar{\boldsymbol{C}}_i = \boldsymbol{C}_i + \Delta\boldsymbol{C}_i \tag{13}$$

In a similar way, the weighting matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$, i.e., $\boldsymbol{R}_i$, $i = 1, 2, \ldots, k$, in the performance indices (4) and (8) can be represented as

$$\bar{\boldsymbol{Q}} = \boldsymbol{Q} + \Delta\boldsymbol{Q}\,, \quad \bar{\boldsymbol{R}} = \boldsymbol{R} + \Delta\boldsymbol{R}\,, \quad \bar{\boldsymbol{R}}_i = \boldsymbol{R}_i + \Delta\boldsymbol{R}_i \tag{14}$$

where matrices $\Delta\boldsymbol{A}$, $\Delta\boldsymbol{B}_i$, $\Delta\boldsymbol{C}_i$, $\Delta\boldsymbol{Q}$, $\Delta\boldsymbol{R}$, i.e., $\Delta\boldsymbol{R}_i$ are small variations in the nominal values of the corresponding matrices, respectively.

The variation of the suboptimal criterion when the nominal value of the matrices change from $\boldsymbol{X}$ to $\boldsymbol{X} + \Delta\boldsymbol{X}$, where $\boldsymbol{X}$ is any of the matrices given in (13) and (14), can be defined as

$$\frac{\partial \boldsymbol{J}}{\partial \boldsymbol{X}} = \lim_{\Delta \boldsymbol{X} \to 0} \frac{\Delta \boldsymbol{J}}{\Delta \boldsymbol{X}} \tag{15}$$

The results of this section are based on the following theorem.

Theorem. The variations of the suboptimal criterion when the process parameters change are given by:

(a)

$$\frac{\partial \boldsymbol{J}}{\partial \boldsymbol{A}} = 2\boldsymbol{P}\boldsymbol{V} \tag{16}$$

(b)

$$\frac{\partial \boldsymbol{J}}{\partial \boldsymbol{B}_i} = 2\boldsymbol{P}\boldsymbol{V}\boldsymbol{C}_i^{\mathrm{T}}\boldsymbol{E}_i^{\mathrm{T}} \tag{17}$$

(c)

$$\frac{\partial \boldsymbol{J}}{\partial \boldsymbol{C}_i} = -2(\boldsymbol{E}_i\boldsymbol{B}_i^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{E}_i^{\mathrm{T}}\boldsymbol{R}_i\boldsymbol{E}_i\boldsymbol{C}_i)\,\boldsymbol{V} \tag{18}$$

(d)

$$\frac{\partial J}{\partial Q} = V \tag{19}$$

(e)

$$\frac{\partial J}{\partial R_i} = E_i C_i P C_i^T E_i^T \tag{20}$$

where E_i is defined by (9), P is the positive definite solution of (12) and V is the solution of

$$V\left(A + \sum_{i=1}^{k} B_i E_i C_i\right)^T + \left(A + \sum_{i=1}^{k} B_i E_i C_i\right) V + I = 0 \tag{21}$$

Proof. Because the proof of the conditions (a)–(e) are based on similar arguments we shall merely show to obtain (a) and (c). The proofs employ matrix differentiation techniques based on [32–34].

(a) For $\bar{A} = A + \Delta A$ and using (15) it follows that

$$\frac{\partial J}{\partial A} = \lim_{\Delta A \to 0} \frac{\Delta J}{\Delta A} \tag{22}$$

where

$$\Delta J = \mathrm{tr}\, \Delta P \tag{23}$$

Therefore, implicit in what has been assumed is that the initial state is a random variable uniformly distributed on the unit sphere.

For $\bar{A} = A + \Delta A$ equation (12) becomes,

$$\left(A + \Delta A + \sum_{i=1}^{k} B_i E_i C_i\right)^T (P + \Delta P) + (P + \Delta P)\left(A + \Delta A + \sum_{i=1}^{k} B_i E_i C_i\right) + \\ + \sum_{i=1}^{k} C_i^T E_i^T R_i E_i C_i + Q = 0 \tag{24}$$

Subtracting (12) from (24), and neglecting all the second-order terms, it follows that

$$\Delta A^T P + \left(A + \sum_{i=1}^{k} B_i E_i C_i\right)^T \Delta P + P\,\Delta A + \Delta P\left(A + \sum_{i=1}^{k} B_i E_i C_i\right) = 0 \tag{25}$$

that is,

$$\Delta P = \int_{\infty}^{0} \exp\left[\left(A + \sum_{i=1}^{k} B_i E_i C_i\right) t\right] \left[\Delta A^T P + P\,\Delta A\right] \exp\left[\left(A + \sum_{i=1}^{k} B_i E_i C_i\right) t\right] \mathrm{d}t \tag{26}$$

Now, it can be easily proved that

$$\Delta J = \mathrm{tr}\left[V\,\Delta A^T P + V P\,\Delta A\right] \tag{27}$$

where the matrix V is the solution of (21). Applying the matrix differential calculation rules [32] on eqn. (22) the expression

$$\frac{\partial J}{\partial A} = 2PV \tag{28}$$

follows immediately.

(c) For $\bar{C}_i = C_i + \Delta C_i$ we have

$$\frac{\partial J}{\partial C_i} = \lim_{\Delta C_i \to 0} \frac{\Delta J}{\Delta C_i}, \quad i = 1, 2, \ldots, k \tag{29}$$

Using the same techniques as in the previous case, we obtain

$$\Delta P = \int_0^\infty \exp\left[\left(A + \sum_{i=1}^{k} B_i E_i C_i\right) t\right] \left[(B_i E_i \, \Delta C_i)^{\mathrm{T}} P + \Delta C_i^{\mathrm{T}} E_i^{\mathrm{T}} R_i E_i C_i + \right.$$
$$\left. + C_i^{\mathrm{T}} E_i^{\mathrm{T}} R_i E_i \, \Delta C_i + P(B_i E_i \, \Delta C_i)\right] \exp\left[\left(A + \sum_{i=1}^{k} B_i E_i C_i\right) t\right] \mathrm{d}t \tag{30}$$

that is

$$\Delta J = \mathrm{tr}\left[V \Delta C_i^{\mathrm{T}} (E_i^{\mathrm{T}} B_i^{\mathrm{T}} P + E_i^{\mathrm{T}} R_i E_i C_i) + V(P B_i E_i + C_i^{\mathrm{T}} E_i^{\mathrm{T}} R_i E_i) \Delta C\right] \tag{31}$$

Applying the matrix differential rules [32] on eqn. (29) the expression

$$\frac{\partial J}{\partial C_i} = -2(E_i^{\mathrm{T}} B_i^{\mathrm{T}} P + E_i^{\mathrm{T}} R_i E_i C_i) V \tag{32}$$

is obtained, where the matrix V is the solution of (21).

Notice that the proposed approach is not strictly connected with any particular method for decentralized control system design. Once the design problem is solved, no extra computations are required to obtain the sensitivity matrices. Moreover, the expressions are based on the process parameters of the initial design (i.e., around which sensitivity is examined). For example, the approximated variation of the performance criterion due to a small variation ΔB_i, of the B_i, $i = 1, 2, \ldots, k$, is given by:

$$\Delta J = \mathrm{tr}\left[2VP \, \Delta B_i E_i C_i\right] \tag{33}$$

Notice that by calculation the gradient matrices we avoid numerous computations for each parameter of the system. As mentioned, in practice, due to environmental effects, aging, perturbations, etc., the parameter values may change. This point out to the fundamental problem of obtaining the characterization of the uncertainties (modeling errors or parameter variations) associated with a given model. The results presented in this section show that this knowledge can be acquired not only by experience with a real applications but also using the proposed sensitivity analysis of the suboptimal performance criterion. This information can help a designer to answer the following question: which parameter changes are so essential that they require particular attention, not only in the modeling process but also in the physical realization of the control system. The answer to this question is of particular interest in situations where physical intuition is of little or no help in pinning down the critical elements.

Another important case is the following. Sometimes in the control systems due to economical or technical reasons it may be desirable to change the parameter values, during the operation of the system. In these cases the explicit expressions of the perturbation matrices, ΔA, ΔB_i etc., are usually available. If not, the information about the less sensitive variations is of interest, especially if there is some freedom in choosing the matrices ΔA, ΔB_i etc.

In what follows we consider the problem of selecting fixed dimensional outputs which will lead to the best decentralized output feedback design. More precisely we consider the following problem. Given the system (1)–(3), where the dimension of matrices C_i, $i = 1, 2, \ldots, k$, is fixed, how to choose the parameters of C_i in order to obtain the best achievable decentralized output feedback design. Notice that the answer to this problem can still be useful, even in the case where a designer does have the advantage of choosing the output matrices C_i, $i = 1, 2, \ldots, k$, because it provides a lower bound for performance index for fixed dimensions of matrices C_i. The solution of this problem is based on the following corollary.

Corollary. The optimal matrix C_i is defined by

$$C_i = -(E_i^{\mathrm{T}} R_i E_i)^{-1} E_i^{\mathrm{T}} B_i^{\mathrm{T}} P \tag{34}$$

where the matrix E_i is defined by (9).

Proof. Follows trivially from the Theorem.

The computational algorithm to solve this problem can be easily developed on the basis of existing algorithms for decentralized output feedback design, e.g. those proposed in Section 2.

4. NUMERICAL EXAMPLES

To illustrate some of the results presented in the previous sections let us consider two examples.

Example 1.

Consider a second order system with

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \tag{35}$$

$$C_1 = [0 \;\; 1], \quad C_2 = [1 \;\; 0] \tag{36}$$

The design of the decentralized control system is based on the calculation of complete state feedback (by linear quadratic methodology) with

$$Q = \begin{bmatrix} 14 & 8 \\ 8 & 6 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{37}$$

The decentralized control law is obtained by simply setting the gains in the centralized gain matrix which correspond to a state not available for feedback, to zero.

It can be easily shown that the solution of the corresponding Riccati equation is given with

(38) $$K = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$$

The decentralized control is defined with

(39) $$E_1 C_1 = [0 \quad -2], \quad E_2 C_2 = [-2 \quad 0]$$

The corresponding value of the matrix P, eqn. (12) is

(40) $$P = \frac{1}{7}\begin{bmatrix} 53 & -5 \\ -5 & 15 \end{bmatrix}$$

First, we shall examine the variation in system actuator matrices B_i, $i = 1, 2$. Define the actual value of these matrices as

(41) $$\bar{B}_i = B_i + \Delta B_i$$

where

(42) $$B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

and

(43) $$\Delta B_1 = \begin{bmatrix} b \\ b \end{bmatrix}, \quad \Delta B_2 = \begin{bmatrix} b \\ b \end{bmatrix}$$

where b is a small scalar perturbation parameter. From (33) it can be easily shown that the approximated variation of the suboptimal performance criterion is

(44) $$\Delta J_1 = \frac{2b}{7 \cdot 76} \,.\, 488 = 1 \cdot 6842 b$$

(45) $$\Delta J_2 = \frac{2b}{7 \cdot 76} \,.\, 1212 = 4 \cdot 5564 b$$

for B_1 and B_2, respectively. The corresponding value of the matrix V is

(46) $$V = \frac{1}{76}\begin{bmatrix} 12 & 3 \\ 3 & 8 \end{bmatrix}$$

Notice that the suboptimal performance criterion is much more sensitive to variations in the matrix B_2 than to those in the matrix B_1, that is

(47) $$\Delta J_1 / \Delta J_2 = 0 \cdot 3696$$

In a similar way we can calculate the sensitivity of the performance criterion due to the small parameter variations in the output matrices C_i, $i = 1, 2$,

(48) $$C_1 = [0 \quad 1], \quad C_2 = [1 \quad 0]$$

Notice that in this care

(49) $$\Delta J = -2 \operatorname{tr} [V(PB_i E_i + C_i^{\mathrm{T}} E_i^{\mathrm{T}} R_i E_i) \Delta C_i]$$

Define the variations in the output matrices C_i, $i = 1, 2$, by

$$\bar{C}_i = C_i + \Delta C_i\,, \quad i = 1, 2\,. \tag{50}$$

where

$$\Delta C_i = \begin{bmatrix} c & c \end{bmatrix}, \quad i = 1, 2 \tag{51}$$

and c a small scalar perturbation parameter. From (49) it follows that

$$\Delta J_3 = -\frac{2c}{7 \cdot 76} \,.\, 224 = -0 \cdot 8421 c \tag{52}$$

$$\Delta J_4 = -\frac{2c}{7 \cdot 76} \,.\, 936 = -3 \cdot 5188 c \tag{53}$$

and

$$\Delta J_3 / \Delta J_4 = 0 \cdot 2393 \tag{54}$$

Therefore, the suboptimal performance criterion is much more sensitive to the variations in the matrix C_2. In this way a designer has some information regarding which parameters most affect the value of the performance criterion.

Example 2. Five Terminal MTDC System.

A multiterminal DC system embedded in a conventional AC power system can be used as a control for damping inter-area oscillations. The design of the multiterminal DC control system involves a large scale, multivariable system with the sensors and actuators geographically distributed. Besides its huge size and information constraints, one of the basic features of this system is the presence of different type of perturbations i.e., parameter variations in the actual plant. Thus, one of the most basic issues that arises in this class of problems, is the sensitivity analysis of the decentralized design.

A power system example which employ five DC terminals to damp out inter-area oscillations due to the AC power system dynamics is used to illustrate the theory developed in the previous sections. A physical interpretation of the dynamics associated with this system is given in [29, 35]. An one-line diagram of this system is shown in Figure 1. The dimensions of the matrices A and B_i, $i = 1, 2, 3, 4$ are 9×9, 9×1, 9×1, 9×1, 9×1, respectively (see Appendix II).

As known, the angle of an area cannot be measured easily, thus only the frequency information is available to the controllers. Therefore, we consider the following information pattern: feedback using local frequency and frequency of the terminal 1. This decentralized information pattern corresponds to

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{55}$$

$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{56}$$

(57)
$$C_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

(58)
$$C_4 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

For decentralized output feedback design the two approaches proposed in Section 2 have been used. Based on the modal-decomposition concept, the state weighting matrix $\boldsymbol{Q}$ was selected to penalize the complex modes associated with inter-area

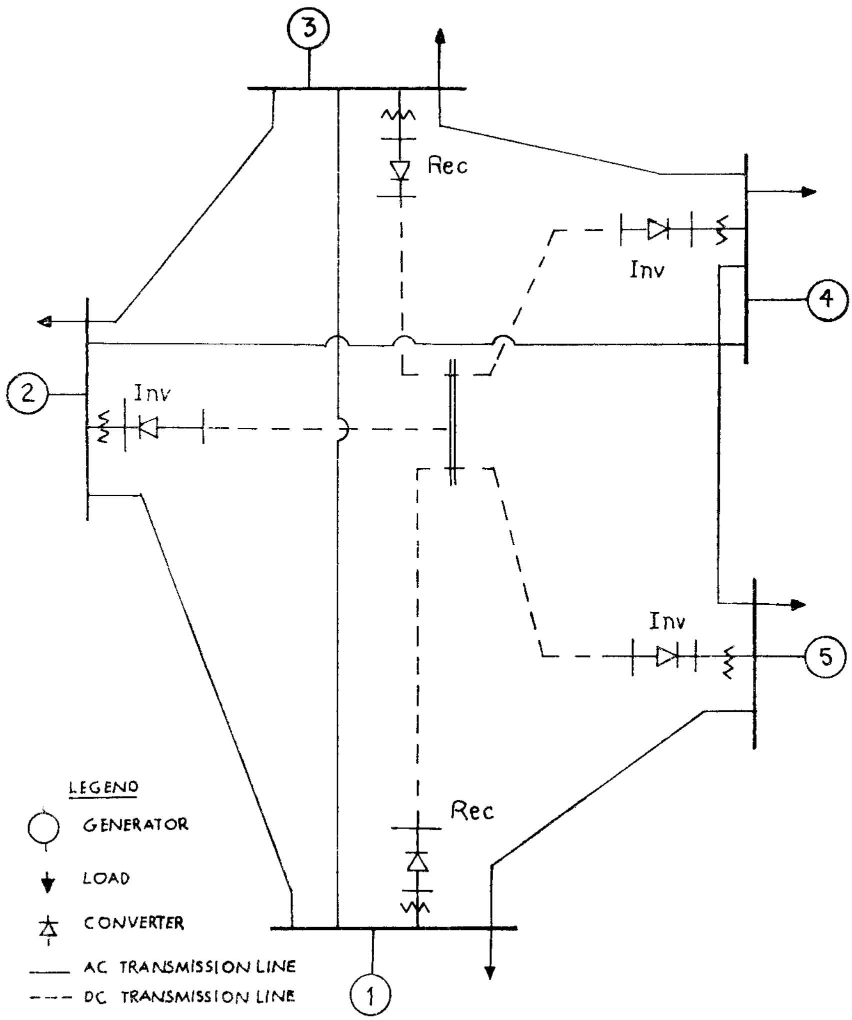

Fig. 1. A power system with 5 dc terminals and 5 ac nodes, each representing an area or a group of coherent generators.

oscillations and the control penalty matrix $\boldsymbol{R}$ is selected as an identity matrix weighted by a scalar [35].

First, we shall examine the variations in the system matrix $\boldsymbol{A}$. Define the actual value of this matrix as

$$\bar{A} = A + \Delta A \tag{59}$$

As mentioned, by calculation the gradient matrices we avoid numerous computations for each parameter of the system. Therefore, although the matrix A has 81 elements we can easily search for information regarding which element variations most affect the suboptimal performance criterion. The sensitivity of the suboptimal performance index due to small variations in some elements of the matrix A is given in Table 1.

Table 1. Sensitivity of J due to variations in some elements of the matrix A.

Element	ΔJ		
	Method 1	Method 2	Method 3
A (1, 1)	$-0{\cdot}31484 \,.\, 10^{5}$	$-0{\cdot}11026 \,.\, 10^{6}$	$-0{\cdot}10365 \,.\, 10^{6}$
A (2, 1)	$-0{\cdot}50834 \,.\, 10^{-6}$	$-0{\cdot}47236 \,.\, 10^{-6}$	$-0{\cdot}45014 \,.\, 10^{-6}$
A (4, 1)	$-0{\cdot}24823 \,.\, 10^{-6}$	$-0{\cdot}22904 \,.\, 10^{-6}$	$-0{\cdot}22741 \,.\, 10^{-6}$
A (6, 1)	$0{\cdot}15020 \,.\, 10^{-6}$	$0{\cdot}14902 \,.\, 10^{-6}$	$0{\cdot}14376 \,.\, 10^{-6}$
A (8, 1)	$-0{\cdot}19758 \,.\, 10^{-6}$	$-0{\cdot}17498 \,.\, 10^{-6}$	$-0{\cdot}17157 \,.\, 10^{-6}$
A (3, 3)	$-0{\cdot}61047 \,.\, 10^{4}$	$-0{\cdot}66877 \,.\, 10^{4}$	$-0{\cdot}62678 \,.\, 10^{4}$
A (4, 4)	$-0{\cdot}89587 \,.\, 10^{8}$	$-0{\cdot}30516 \,.\, 10^{8}$	$-0{\cdot}25742 \,.\, 10^{8}$
A (8, 9)	$-0{\cdot}20175 \,.\, 10^{-7}$	$0{\cdot}17101 \,.\, 10^{-7}$	$0{\cdot}18942 \,.\, 10^{-7}$
A (9, 9)	$-0{\cdot}12279 \,.\, 10^{5}$	$-0{\cdot}10812 \,.\, 10^{5}$	$-0{\cdot}10490 \,.\, 10^{5}$

The results indicate that the suboptimal performance criterion is far more sensitive to the variations in the elements $A(1, 1)$, $A(3, 3)$, $A(4, 4)$ and $A(9, 9)$ than in the other elements.

In a similar way, we can calculate the effect of the variations in the system actuator matrices B_i, $i = 1, 2, 3, 4$, and the output matrices C_i, $i = 1, 2, 3, 4$. Table 2 and Table 3 give the sensitivity of the suboptimal performance index due to small perturbations in the matrices B_i and C_i, $i = 1, 2, 3, 4$.

Therefore, the calculation of the gradient matrices allows us to easily provide information regarding which elements of the matrices A, B_i and C_i most affect the suboptimal performance criterion.

Table 2. Sensitivity of J due to variations in B_i, $i = 1, 2, 3, 4$, $\Delta B_i = 0{\cdot}02 B_i$, $i = 1, 2, 3, 4$.

Matrix	ΔJ		
	Method 1	Method 2	Method 3
B_1	$0{\cdot}573787 \,.\, 10^{6}$	$0{\cdot}657111 \,.\, 10^{6}$	$0{\cdot}683157 \,.\, 10^{6}$
B_2	$-0{\cdot}825454 \,.\, 10^{6}$	$-0{\cdot}1141077 \,.\, 10^{7}$	$-0{\cdot}114413 \,.\, 10^{7}$
B_3	$-0{\cdot}101736 \,.\, 10^{5}$	$0{\cdot}1416788 \,.\, 10^{5}$	$0{\cdot}176724 \,.\, 10^{5}$
B_4	$0{\cdot}239774 \,.\, 10^{2}$	$0{\cdot}665754 \,.\, 10^{5}$	$0{\cdot}735508 \,.\, 10^{5}$

Table 3. Sensitivity of J due to variations in C_i, $i = 1, 2, 3, 4$, $\Delta C_i = 0{\cdot}01 C$, $i = 1, 2, 3, 4$.

Matrix		ΔJ		
		Method 1	Method 2	Method 3
C_1	C_1 (1, 1)	$-0{\cdot}23736 \,.\, 10^7$	$-0{\cdot}66018 \,.\, 10^7$	$-0{\cdot}69789 \,.\, 10^7$
	C_1 (2, 3)	$-0{\cdot}13229 \,.\, 10^8$	$-0{\cdot}13027 \,.\, 10^8$	$-0{\cdot}14000 \,.\, 10^8$
C_2	C_2 (1, 1)	$0{\cdot}98375 \,.\, 10^7$	$0{\cdot}10967 \,.\, 10^8$	$0{\cdot}11422 \,.\, 10^8$
	C_2 (2, 5)	$0{\cdot}97476 \,.\, 10^7$	$0{\cdot}10907 \,.\, 10^8$	$0{\cdot}11356 \,.\, 10^8$
C_3	C_3 (1, 1)	$-0{\cdot}50508 \,.\, 10^4$	$0{\cdot}22925 \,.\, 10^3$	$0{\cdot}40771 \,.\, 10^3$
	C_3 (2, 7)	$-0{\cdot}50508 \,.\, 10^4$	$0{\cdot}22925 \,.\, 10^3$	$0{\cdot}40771 \,.\, 10^3$
C_4	C_4 (1, 1)	$0{\cdot}63631 \,.\, 10^3$	$-0{\cdot}58052 \,.\, 10^3$	$-0{\cdot}33816 \,.\, 10^3$
	C_4 (2, 9)	$0{\cdot}63631 \,.\, 10^3$	$-0{\cdot}58052 \,.\, 10^3$	$-0{\cdot}33816 \,.\, 10^3$

5. CONCLUSIONS

A computationally efficient method for suboptimal performance criterion sensitivity in large-scale decentralized control systems has been proposed. The suggested approach is not crucially dependent on any particular method for decentralized control systems design. The problem of selecting the fixed dimensional outputs which lead to the best decentralized feedback also been considered. The sensitivity results were illustrated through two numerical examples.

APPENDIX I

First Approach

When there are some states which are not feedback into the control system, it is impossible to obtain the optimal feedback control which agrees with the optimal one for state feedback system. Then the trajectory of the decentralized feedback system becomes different from that of the state feedback one. Thus, the objective is to determine the control law (3) so that the system (1) and (5) are "near" each other. There are a variety of quantities one can choose to minimize in such a case. Two obvious possibilities are:

Method 1:

$$\min_{E_i} \|B_i(E_i C_i - F_i)\| \tag{60}$$

where F_i is the row of matrix F which is the optimal state feedback gain minimizing (4). In this case

$$E_i = F_i C_i^{\mathrm{T}}(C_i C_i^{\mathrm{T}})^{-1} \tag{61}$$

Method 2:

$$\min_{E_i} \int_0^{\infty} \sum_{i=1}^{k} x^{\mathrm{T}}(t)\,(E_i C_i - F_i)^{\mathrm{T}}\, R_i(E_i C_i - F_i)\, x(t)\, \mathrm{d}t \tag{62}$$

In this case

$$E_i = F_i V C_i^{\mathrm{T}} (C_i C_i^{\mathrm{T}})^{-1} \tag{63}$$

where V is a solution of the Lyapunov matrix equation

$$V(A + BF)^{\mathrm{T}} + (A + BF)V + I = 0 \tag{64}$$

Second Approach

$$E_i = -R_i^{-1} B_i K V C_i^{\mathrm{T}} (C_i C_i^{\mathrm{T}})^{-1} \tag{65}$$

where matrices K and V are the solutions of

$$\Big(A + \sum_{i=1}^{k} B_i E_i C_i\Big)^{\mathrm{T}} K + K\Big(A + \sum_{i=1}^{k} B_i E_i C_i\Big) + \sum_{i=1}^{k} C_i^{\mathrm{T}} E_i^{\mathrm{T}} R_i E_i C_i + Q = 0 \tag{66}$$

$$\Big(A + \sum_{i=1}^{k} B_i E_i C_i\Big) V + V\Big(A + \sum_{i=1}^{k} B_i E_i C_i\Big)^{\mathrm{T}} + I = 0 \tag{67}$$

APPENDIX II

$$A = \left[\begin{matrix}
-0{\cdot}1892\mathrm{D}+00 & 0{\cdot}5864\mathrm{D}-02 & 0{\cdot}2136\mathrm{D}-10 & 0{\cdot}2315\mathrm{D}-02 & 0{\cdot}1497\mathrm{D}-09 \\
-0{\cdot}3770\mathrm{D}+03 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}3770\mathrm{D}+03 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 \\
0{\cdot}8253\mathrm{D}-11 & -0{\cdot}3376\mathrm{D}-01 & -0{\cdot}1892\mathrm{D}+00 & 0{\cdot}9396\mathrm{D}-02 & 0{\cdot}1309\mathrm{D}-09 \\
-0{\cdot}3770\mathrm{D}+03 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}3770\mathrm{D}+03 \\
0{\cdot}9585\mathrm{D}-14 & 0{\cdot}4958\mathrm{D}-02 & 0{\cdot}8591\mathrm{D}-11 & -0{\cdot}1214\mathrm{D}-01 & -0{\cdot}1691\mathrm{D}+00 \\
-0{\cdot}3770\mathrm{D}+03 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 \\
-0{\cdot}1048\mathrm{D}-11 & 0{\cdot}8276\mathrm{D}-02 & 0{\cdot}7498\mathrm{D}-11 & 0{\cdot}6369\mathrm{D}-02 & 0{\cdot}5038\mathrm{D}-10 \\
-0{\cdot}3770\mathrm{D}+03 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 \\
0{\cdot}1078\mathrm{D}-10 & 0{\cdot}3652\mathrm{D}-02 & 0{\cdot}1498\mathrm{D}-10 & 0{\cdot}1550\mathrm{D}-02 & 0{\cdot}1052\mathrm{D}-09
\end{matrix}\right.$$

$$\left.\begin{matrix}
0{\cdot}2090\mathrm{D}-02 & 0{\cdot}2281\mathrm{D}-10 & 0{\cdot}6905\mathrm{D}-02 & 0{\cdot}1060\mathrm{D}-10 \\
0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 \\
0{\cdot}7056\mathrm{D}-02 & 0{\cdot}2020\mathrm{D}-10 & 0{\cdot}2783\mathrm{D}-02 & 0{\cdot}9622\mathrm{D}-11 \\
0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 \\
0{\cdot}3682\mathrm{D}-02 & 0{\cdot}9114\mathrm{D}-11 & 0{\cdot}9376\mathrm{D}-03 & 0{\cdot}4589\mathrm{D}-11 \\
0{\cdot}0000\mathrm{D}+00 & 0{\cdot}3770\mathrm{D}+03 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 \\
-0{\cdot}2419\mathrm{D}-01 & -0{\cdot}1823\mathrm{D}+00 & 0{\cdot}3313\mathrm{D}-02 & 0{\cdot}4088\mathrm{D}-11 \\
0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}3770\mathrm{D}+03 \\
0{\cdot}3994\mathrm{D}-02 & 0{\cdot}1600\mathrm{D}-10 & -0{\cdot}3349\mathrm{D}-01 & -0{\cdot}1892\mathrm{D}+00
\end{matrix}\right]$$

$$B = \begin{bmatrix}
0{\cdot}1199\mathrm{D}-02 & 0{\cdot}2028\mathrm{D}-02 & 0{\cdot}1471\mathrm{D}-02 & 0{\cdot}6536\mathrm{D}-03 \\
0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 \\
-0{\cdot}3516\mathrm{D}-02 & 0{\cdot}2477\mathrm{D}-03 & -0{\cdot}9898\mathrm{D}-03 & -0{\cdot}1231\mathrm{D}-04 \\
0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 \\
-0{\cdot}4005\mathrm{D}-03 & -0{\cdot}1124\mathrm{D}-02 & -0{\cdot}3533\mathrm{D}-03 & -0{\cdot}1286\mathrm{D}-04 \\
0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 \\
-0{\cdot}8851\mathrm{D}-03 & 0{\cdot}2233\mathrm{D}-04 & -0{\cdot}4752\mathrm{D}-02 & -0{\cdot}4695\mathrm{D}-03 \\
0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 & 0{\cdot}0000\mathrm{D}+00 \\
0{\cdot}3781\mathrm{D}-03 & 0{\cdot}9191\mathrm{D}-03 & -0{\cdot}3256\mathrm{D}-04 & -0{\cdot}5189\mathrm{D}-02
\end{bmatrix}$$

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