

**Knihy došlé do redakce  
(Books received)**

*Heinz-Erich Erbs, Otto Stolz:* Einführung in die Programmierung mit PASCAL, 2., überarbeitete und erweiterte Auflage (Mikro-Computer-Praxis). B. G. Teubner, Stuttgart 1984. 240 Seiten, mit zahlreichen Abbildungen, Illustrationen, Beispielen und Übungen; DM 24,80.

*Kurt Schmidt, Wilhelm Wildegger, Franz Pätzold, Gottfried Schwarz:* Mikroprogrammierbare Schnittstellen. (Leitfäden der angewandten Informatik.) B. G. Teubner, Stuttgart 1984. 224 Seiten, mit 97 Bildern und 3 Tabellen; DM 32,—.

*Ludovic Lebart, Alain Morineau, Jean-Pierre Fénelon:* Statistische Datenanalyse — Methoden und Programme. (Titel der französischen Originalausgabe: Traitement des données statistique — méthodes et programmes, Bordas, Paris 1982). Akademie-Verlag, Berlin 1984. 188 Seiten, mit 27 Abbildungen und 15 Tabellen; M 19,50.

J. PONSTEIN

**Approaches to the Theory  
of Optimization**

Cambridge University Press, Cambridge 1980.

xii + 205 pages.

The aim of this book is to indicate and describe in rigorous mathematical setting some of the existing approaches to the treatment of optimization problems. The author's choice of included material was governed by two main ideas: generality and simple and full presentation of results. Although these two requirements may seem contradictory at the first glance, the book of Prof. Ponstein proves the possible solution of this dilemma.

General treatment of optimization problems in Banach and locally convex topological vector spaces is presented. However, the style of presentation without any logical gaps or formal references enables also for a non-

professional reader to follow presented results in this promising and evergrowing area of applied mathematics. Clearly, some standard level of knowledge of functional analysis and topology is necessary.

What is really worth mentioning is the ability of the author to enlighten also deep and far reaching results without unnecessary mathematical formalism which can often burden also the smartest ideas lying behind. So the reader has the opportunity to get acquainted with fairly general results and ideas of optimization theory in a convenient way.

On the other hand, the limited extent of the book results necessarily in neglectation of some topics. For example, regularity of optimization problems is omitted as general setting of the respective conditions may be too strong for practical purposes. Also the convex processes and multiobjective optimization problems are only touched. As optimization problems have their origin in practical applications in engineering, economics, the social and mathematical sciences, the stress is laid upon "practical" impact of the presented theory. Also the natural and clear exposition and development of the subject will be certainly appreciated by the wide range of readers.

The book consists of 7 chapters and 5 appendices. In the first chapter the most convenient formulation of a general optimization problem within the framework of mathematical programming theory is given which is based on a thorough analysis of various examples of optimization problems. An intuitive approach to the solution of this problem is explained in Chapter 2 including perturbations, duality, the Lagrangian, etc.

A global approach in locally convex topological vector spaces using bifunctions is followed in Chapter 3 and includes separation and duality theorems, normality, and Lagrangian and Fenchel duality. The following Chapter 4 presents conjugate duality approach to mathematical programming problems. Local approach in Banach spaces is analysed in

Chapter 5. It includes Fréchet differentiability, first-order and second-order conditions, linearizing sets etc.

In Chapter 6 some other approaches are reviewed, e.g. the fixed point approach and modified Lagrangians. Final chapter contains some applications such as Chebyshev approximation, optimal control and multiobjective optimization. Appendices summarize the basic facts in topology and functional analysis. There are no references to the literature in the main text. There are together with comments combined in separate final section.

To conclude it is possible to state that Prof. Ponstein has provided a concise account of optimization which should be readily accessible to anyone with basic understanding of topology and functional analysis. The book can be recommended to advanced students and professionals concerned with operational research, optimal control and mathematical programming.

*Jaroslav Doležal*

M. J. D. POWELL  
**Approximation Theory  
and Methods**

Cambridge University Press, Cambridge  
1981.

339 pages.

This book gives an introduction to the theory of contemporary most often used approximation methods. It is based on a course of lectures to third-year undergraduates in mathematics at Cambridge University. Professor Powell is well known specialist in numerical analysis and the author of very useful numerical methods for unconstrained optimization.

The present book deals with methods of function approximation by polynomials and piecewise polynomials, called spline functions. The book consists of 24 chapters and 2 appendices. The first chapter includes the problem formulation and the idea of best approximation in normed linear spaces. Second chapter introduces the concept of convexity and gives the conditions for uniqueness of the best approximation. Polynomial and piecewise polynomial approximations are introduced in the

third chapter. Contents of the fourth chapter is polynomial interpolation by the Lagrange formula. Fifth chapter deals with divided differences and with interpolation formulae by Newton and Hermite. In the sixth chapter we find the theorems about uniform convergence of polynomial approximations, i.e. the Weierstrass theorem and its proof using Bernstein polynomials. The seventh chapter introduces minimax approximation. We find here the Haar condition and characterization theorem based on it. The eighth chapter presents the exchange algorithm for calculation of the element that minimizes the maximum error of approximation in  $C(a, b)$  by elements satisfying the Haar condition. Convergence conditions of this algorithm are given in the ninth chapter. The tenth chapter applies this algorithm for the case of a rational approximation. Here also an alternative method of rational approximation is introduced based on linear programming. The least squares approximation is presented in the eleventh chapter. In the twelfth chapter the basic properties of orthogonal polynomials are studied and Gaussian quadrature formula is introduced. Thirteenth chapter deals with problems of approximation of periodic functions by trigonometric polynomials. The theory of best  $L_1$  approximation is discussed in chapters fourteen and fifteen. In the sixteenth chapter the order of convergence of polynomial approximation is investigated also for the case of Lipschitz non-differentiable functions. In chapter seventeen the necessary condition of uniform convergence of approximations is derived being formulated as uniform boundedness of norms of linear approximation operators. This concept is applied both to algebraic and trigonometric polynomials. Chapter eighteen introduces the reader to the world of spline-functions as an advantageous tool of approximation. Chapter twenty introduces the  $B$ -splines and their fundamental properties including Schoenberg-Whitney theorem about uniqueness of interpolation by  $B$ -splines. Also the convergence properties of spline approximations are presented in this chapter. Chapter twenty one is concerned with problems of knot positions in context with spline inter-

polation. In chapter twenty two the important Peano kernel theorem is proved, which expresses in integral form the linear bounded functionals on  $C^{k+1}(a, b)$ , that vanish for all polynomials of degree  $k$ . This theorem gives general and useful techniques for expressing the errors of approximations. In the chapter twenty three, in connection with variational problems, the natural and perfect splines are introduced. Finally, chapter twenty four is dedicated to the problem of optimal interpolation, which naturally leads to the concept of spline-functions. Appendix A is devoted to the equivalent formulations and to some consequences

of the Haar condition. In Appendix B remarks concerning related works and references are given.

The book is written on high mathematical level, however, it is readable also to persons without special knowledge and training in approximation theory. Attention is chiefly paid to the theoretical background. All chapters are completed by interesting non-trivial exercises. The book is highly suitable as a very useful textbook for students, scientists, engineers and generally to all who are in need of approximation methods for practical use.

*Antonín Tuzar*