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EXTENSION PRINCIPLE AND FUZZY-MATHEMATICAL PROGRAMMING

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A concept of fuzzy-mathematical programming problem is introduced in this paper. It is a problem of optimizing an objective function subject to a constraint set which is a fuzzy set. Solution of a FMP problem is defined as a fuzzy set by means of the "Extension principle" applied to a set-to-set mapping. Our philosophy is applied to the linear programming problem, where the set of all feasible solutions is subjected to fuzzyfication. The fuzzy-optimal solution can be obtained by an algorithmic way as demonstrated by a simple example.

1. INTRODUCTION

A general problem of mathematical programming has the following form

$$\text{(1)} \qquad \begin{aligned} &\text{maximize } g(x)\,, \\ &\text{subject to } x \in Z\,, \end{aligned}$$

where $g : X \to E_1$, $Z \subset X$, X is a set. The constraint set Z in (1) is understood to be a "deterministic" subset of X. It is usual that Z is given by a system of inequalities, i.e.

$$Z = \{x \in X\, ;\ g_i(x) \leqq 0,\ i = 1, \dots, m\}\,,$$

where $g_i : X \to E_1$. Generally, when we speak about constraints we suppose them to be known. Unfortunately, the real problems do not always benefit from this favour. Very often the constraints are vaguely formulated, the set Z is not given strictly by its elements, the elements of X belong to Z with lower or higher relative strength of competence. There immediately arises a question how the optimal solution of (1) should be understood in case of fuzzy constraint set Z. Such a problem and relative ones form the scope of this paper.

2. PRELIMINARIES

In what follows the reader is supposed to be familiar with the notion of a fuzzy set and also with the endeavours to use this notion in modeling the decision-making process. Acquitance with [3] published in this journal is sufficient for the following arguments, however, some basic notions will be recalled in this section.

A fuzzy set Z in X is fully determined by its membership function $\mu : X \to [0, 1]$. It is obvious to express Z symbolically

$$Z = \int_X \mu(x)/x\,,$$

where the symbol of integral is understood as a symbol of the union for all $x \in X$.

For $\beta \in [0, 1]$ we define a *β-level set of Z* as follows:

$$Z^\beta = \{x \in X;\ \mu(x) \geqq \beta\}\,. \tag{2}$$

We can see that Z^β is a deterministic set. Note that a deterministic set A, $A \subset X$ may be taken as a special case of a fuzzy set with the membership function which equals to the characteristic function of A. From now on, when we speak about a "set" or "subset", we always mean a "deterministic set" or "deterministic subset".

By symbol $\exp X$ we denote the family of all subsets of the set X, by $\operatorname{fexp} X$ the family of all fuzzy sets on X is meant.

The following definition has the principal importance in this paper.

Definition. Let G be a set-to-set mapping, such that

$$G : \exp X \to \exp Y\,,$$

where X, Y are arbitrary sets. Define the mapping G^f,

$$G^f : \operatorname{fexp} X \to \operatorname{fexp} Y$$

by the formula

$$G^f(Z) = \int_Y \vartheta(y)/y \tag{3}$$

where

$$Z = \int_X \mu(x)/x\,, \tag{4}$$

$$\mu : X \to [0, 1]\,.$$

$$\vartheta(y) = \max\{0, \sup\{\beta \in [0, 1];\ y \in G(Z^\beta)\}\}\,, \tag{5}$$

The mapping G^f is said to be *the fuzzy extension of the mapping G.*

Remark. A point-to-point mapping G, $G : X \to Y$, can be understood as a set-to-set mapping, using the formula

$$G(V) = \bigcup_{v \in V} \{G(v)\} \tag{6}$$

for any $V \in \exp X$. When we speak about the fuzzy extension of the mapping G, G being a point-to-point mapping, we always mean the set-to-set interpretation of the mapping G given by (6).

Lemma. Let $G : X \to Y$ be a point-to-point mapping, $Z = \int_X \mu(x)/x$, then for any $y \in Y$

$$(7) \qquad = \sup \{\mu(x) \in [0, 1];\ y = G(x),\ x \in X\} = \sup \{\beta \in [0, 1];\ y \in G(Z^\beta)\} .$$

Proof. For a given $y \in Y$ denote

$$L = \{\mu(x) \in [0, 1];\ x \in X,\ y = G(x)\} ,$$
$$P = \{\beta \in [0, 1];\ y \in G(Z^\beta)\} .$$

(a) Take $\alpha \in L$, then there is $x \in X$, such that $\alpha = \mu(x)$ and $G(x) = y$. By (2) we obtain $x \in Z^\alpha$ which implies $G(x) \in G(Z^\alpha)$, however, $y = G(x)$, leaving us with $\alpha \in P$, such that $\sup L \leqq \sup P$.

(b) Choose $\beta \in P$, then $y \in G(Z^\beta)$ and there is $x \in Z^\beta$ such that $G(x) = y$. Since $x \in Z^\beta$, we have $\mu(x) \geqq \beta$. We have just proven that for any $\beta \in P$ there is $\mu(x) \in L$ such that $\mu(x) \geqq \beta$. Thus, $\sup L \geqq \sup P$. $\square$

Remark. The following definition of the extension principle is known from the original Zadeh's paper [7]:

Let $G : X \to Y$ be a point-to-point mapping, $Z = \int_X \mu(x)/x$. Then the extension mapping $\bar{G}^f$ of G is defined by this formula

$$(8) \qquad \bar{G}^f(Z) = \int_Y \bar{\vartheta}(y)/y ,$$

where

$$(9) \qquad \bar{\vartheta}(y) = \max \{0, \sup\{\mu(x) \in [0, 1];\ x \in X,\ y = G(x)\}\} .$$

By means of the Lemma given above and the previous Remark, it is evident that definition of fuzzy extension (3), (4), (5) and definition (8), (9) introduced by L. A. Zadeh give the same fuzzy extension of the point-to-point mapping G, i.e.

$$G^f(Z) = \bar{G}^f(Z) .$$

Our definition is more general then the one introduced by L. A. Zadeh.

3. RESULTS

In the present section we shall investigate some properties of the fuzzy extension of the set-to-set mapping G,

$$G : \exp X \to \exp X .$$

Let $Z = \int_X \mu(x)/x$, $\mu : X \to [0, 1]$ and let ϑ be defined by formula (5).

For the purpose of the present and the next sections we shall consider the following two properties of the mapping G

(V 1) $\qquad \emptyset \neq U \subset X \quad$ implies $\quad G(U) \subset U$,

(V 2) $\qquad \emptyset \neq V \subset W \subset X \quad$ implies $\quad V \cap G(W) \subset G(V)$.

Further, denote for $x \in X$

$$Q_x = \{\alpha \in [0, 1];\ x \in G(Z^\alpha)\}. \tag{10}$$

Proposition. 1. Let $G : \exp X \to \exp X$ have property (V 1). Then for any $x \in X$:

$$\vartheta(x) \leqq \mu(x). \tag{11}$$

Proof. Consider arbitrary $x \in X$. If $\vartheta(x) = 0$, then $\vartheta(x) \leqq \mu(x)$. Assume that $\vartheta(x) > 0$, by definition (5) of function ϑ the set $\{\alpha \in [0, 1];\ x \in G(Z^\alpha)\}$ is nonempty. Choose $\gamma \in [0, 1]$ such that $x \in G(Z^\gamma)$. Using property (V 1) we obtain $x \in Z^\gamma$, thus $\mu(x) \geqq \gamma$ which implies $\mu(x) \geqq \sup\{\alpha \in [0, 1];\ x \in G(Z^\alpha)\} = \vartheta(x)$. □

Proposition 2. Let G have properties (V 1), (V 2), and let $x \in X$. Then

$$\begin{aligned} \vartheta(x) &= \mu(x) \quad \text{for} \quad x \in \bigcup_{\alpha\in[0,1]} G(Z^\alpha), \\ \vartheta(x) &= 0 \quad \text{for} \quad x \notin \bigcup_{\alpha\in[0,1]} G(Z^\alpha). \end{aligned} \tag{12}$$

Proof.

(a) Choose $x \in \bigcup_{\alpha\in[0,1]} G(Z^\alpha)$, then there is $\alpha \in [0, 1]$ such that $x \in G(Z^\alpha)$ and by (V 1), $G(Z^\alpha) \subset Z^\alpha$, thus $\mu(x) \geqq \alpha$. Evidently, $Z^{\mu(x)} \subset Z^\alpha$ and $x \in Z^{\mu(x)} \cap G(Z^\alpha)$, then by (V 2) we obtain $x \in G(Z^{\mu(x)})$, consequently, $\mu(x) \leqq \sup Q_x = \vartheta(x)$. The opposite inequality follows directly from Proposition 1.

(b) Assume that $x \notin \bigcup_{\alpha\in[0,1]} G(Z^\alpha)$, then Q_x is empty, thus by (5) we have $\vartheta(x) = 0$. □

Corollary. According to Proposition 2 we can write

$$G^f(Z) = \int_R \mu(x)/x$$

where $R = \bigcup_{\alpha\in[0,1]} G(Z^\alpha)$.

Proposition 3. Let G have properties (V 1) and (V 2). Then for $0 < \beta \leqq 1$ the β-level set $(G^f(Z))^\beta$ of the fuzzy set $G^f(Z)$ satisfies

$$(G^f(Z))^\beta = \bigcup_{\alpha \geqq \beta} G(Z^\alpha). \tag{15}$$

Proof. Choose $x \in (G^f(Z))^\beta$. Using (2) and (5) we have $\vartheta(x) \geqq \beta > 0$, such that $Q_x \neq \emptyset$. Thus, by Proposition 2 $\vartheta(x) = \mu(x)$. The last equation implies the existence of $\gamma \in [0, 1]$ with $x \in G(Z^\gamma)$ and $x \in Z^\beta$. Consider

(i) $\gamma \geqq \beta$, then $G(Z^\gamma) \subset \bigcup_{\alpha \geqq \beta} G(Z^\alpha)$, thus $x \in \bigcup_{\alpha \geqq \beta} G(Z^\alpha)$.

(ii) $\gamma < \beta$, then $Z^\beta \subset Z^\gamma$. As $x \in Z^\beta \cap G(Z^\alpha)$ and using property (V 2) we obtain $x \in G(Z^\beta) \subset \bigcup_{\alpha \geqq \beta} G(Z^\alpha)$.

On the other hand, choose $x \in \bigcup_{\alpha \geqq \beta} G(Z^\alpha)$, then there is $\gamma \in [\beta, 1]$ such that $x \in G(Z^\gamma)$. Since $\beta > 0$, we have $\gamma \in Q_x$, which implies that $\vartheta(x) = \sup Q_x \geqq \gamma \geqq \beta$. Consequently, $x \in (G^f(Z))^\beta$. □

Remark. Proving Proposition 3, we use properties (V 1) and (V 2) only in the first part of the proof, namely proving the inclusion

$$(G^f(Z))^\beta \subset \bigcup_{\alpha \geqq \beta} G(Z^\alpha).$$

The opposite inclusion holds without any additional assumption.

4. FUZZY-MATHEMATICAL PROGRAMMING

Our concept of the problem of fuzzy-mathematical programming differs essentially from the one introduced by Negoita and Sularia in [4] or from other similar concepts, see [1], [6], [8].

Let $g : X \to E_1$ be an objective function defined on a set X, let Z be a fuzzy set on X, i.e.

$$Z = \int_X \mu(x)/x,$$

where $\mu : X \to [0, 1]$, and μ is not identically zero function. The problem of maximizing the objective g on Z, symbolically written as

$$\text{(16)} \qquad \text{maximize } g(x), \quad \text{subject to } Z = \int_X \mu(x)/x,$$

is said to be *the fuzzy-mathematical programming problem* (FMP).

Let a set-to-set mapping $G : \exp X \to \exp X$ be defined by the following formula: for $U \subset X$ set

$$\text{(17)} \qquad G(U) = \{x \in X;\ x \in U,\ g(x) = \sup_{u \in U} g(u)\}.$$

Further, let G^f means the fuzzy extension of the mapping G defined by (17). Then *the fuzzy-optimal solution* Z_{opt} of the FMP problem is defined like this:

$$\text{(18)} \qquad Z_{\text{opt}} = G^f(Z).$$

The fuzzy-optimal value g_{opt} of the solution of the FMP problem is defined as follows:

$$g_{\text{opt}} = g^f(Z_{\text{opt}}).$$

Remark. Notice that both the solution of the FMP problem and the value of the optimal solution are fuzzy sets. The mapping g^f is the fuzzy extension of $g : X \to E_1$, see Remarks in Section 2.

Proposition 4. Let $G : \exp X \to \exp X$ be defined by expression (17). Then G has properties (V 1) and (V 2).

Proof. Property (V 1) follows directly from definition (17). Let $V \subset W \subset X$ and choose $x \in V \cap G(W)$. It follows that

$$g(x) = \sup_{u \in U} g(u) . \tag{19}$$

Since $x \in V \subset W$, we have

$$g(x) \leqq \sup_{u \in V} g(u) \leqq \sup_{u \in W} g(u) . \tag{20}$$

Combining (19) and (20) we get $g(x) = \sup_{u \in V} g(u)$, hence $x \in G(V)$. □

Now, Propositions 2 and 3 can be used to characterize the optimal solution Z_{opt} of the FMP problem (16). The membership function ϑ of the fuzzy-optimal solution of the FMP problem is characterized by Proposition 2. Proposition 3 gives the characterization of the fuzzy-optimal solution of the FMP problem by means of level sets.

Remark. If the constraint set Z is a deterministic set (as a special case of a fuzzy set) then the fuzzy-optimal solution and fuzzy-optimal value are deterministic sets, too, and they coincide with the usual concept of optimal solution and optimal value used in mathematical programming.

5. APPLICATION

In the present section we shall apply the general results derived in the foregoing sections to a special case of fuzzy mathematical programming problem. For this purpose consider the following linear programming problem (LP):

$$\text{maximize} \sum_{j=1}^{n} c_j x_j , \tag{21}$$

$$\text{subject to} \sum_{j=1}^{n} a_{ij} x_j \leqq b_i \quad \text{for} \quad i \in M = \{1, \ldots, m\} . \tag{22}$$

$$x_j \geqq 0 \quad \text{for} \quad j \in N = \{1, \ldots, n\} ,$$

where a_{ij}, c_j, b_i are given real numbers, $i \in M, j \in N$. For $i \in M$ set

$$K_i = \{x = (x_1, \ldots, x_n) \in E_n;\ x \geqq 0,\ \sum_{j=1}^{n} a_{ij} x_j \leqq b_i\} \tag{23}$$

and let $\chi_i : E_n \to E_1$ be the characteristic function of the set K_i, i.e.

$$\chi_i(x) = 1 \quad \text{for} \quad x \in K_i\,,$$
$$\chi_i(x) = 0 \quad \text{for} \quad x \notin K_i\,.$$

Further, to every constraint $\sum_{j=1}^{n} a_{in} x_j \leqq b_i$, a given weight $w_i > 0$ is assigned, with $\sum_{i \in M} w_i = 1$.

Now, we define the constraint set Z, which is a fuzzy set, and is determined by its membership function $\mu : E_n \to [0, 1]$:

$$\mu(x) = \sum_{i \in M} w_i \, \chi_i(x)\,. \tag{24}$$

Hence, we shall deal with a special type of the FMP problem (16):

$$\text{maximize} \ \sum_{j \in N} c_j x_j\,, \tag{25}$$

$$\text{subject to } Z = \int_{E_n} \mu(x)/x\,, \tag{26}$$

where μ is defined by (24), $c = (c_1, \ldots, c_n)$, $x = (x_1, \ldots, x_n)$. The fuzzy-optimal solution and the fuzzy-optimal value of problem (25), (26) are defined by (18) and (19), where for $U \subset E_n$, we have

$$G(U) = \{x \in E_n;\ x \in U,\ c^{\mathrm{T}} x = \sup_{u \in U} c^{\mathrm{T}} u\}\,. \tag{27}$$

For $I \in \exp M$ we define

$$s(I) = \sup \{c^{\mathrm{T}} x;\ x \in \bigcap_{i \in I} K_i\}\,, \tag{28}$$

$$S(I) = \{x \in E_n;\ x \in \bigcap_{i \in I} K_i,\ c^{\mathrm{T}} x = s(I)\}\,, \tag{29}$$

$$w(I) = \sum_{i \in I} w_i\,. \tag{30}$$

For $\alpha \in [0, 1]$ set

$$M_\alpha = \{I \in \exp M;\ w(I) \geqq \alpha\}\,. \tag{31}$$

Proposition 5. Let $\alpha \in [0, 1]$. Then

$$G(Z^\alpha) = \{x \in E_n;\ x \in \bigcup_{I \in M_\alpha} S(I),\ c^{\mathrm{T}} x = \max_{I \in M_\alpha} s(I)\}\,. \tag{32}$$

Proof. It is easy to verify that

$$Z^\alpha = \bigcup_{I \in M_\alpha} \bigcap_{i \in I} K_i\,. \tag{33}$$

For the purpose of this proof denote

$$R = \{x \in E_n,\ x \in \bigcup_{I \in M_\alpha} S(I),\ c^{\mathrm{T}} x = \max_{I \in M_\alpha} s(I)\}\,.$$

By means of (27) and (33) we get

$$G(Z^\alpha) = \{x \in E_n;\; x \in \bigcup_{I \in M_\alpha} \bigcap_{i \in I} K_i,\; c^T x = \sup_{u \in \cup \cap K_i} c^T u\} =$$

$$= \{x \in E_n;\; x \in \bigcup \bigcap K_i,\; c^T x = \max_{I \in M_\alpha} \sup_{u \in \cap K_i} c^T u\} =$$

$$= \{x \in E_n;\; x \in \bigcup \bigcap K_i,\; c^T x = \max_{I \in M_\alpha} s(I)\} .$$

Since $S(I) \subset \bigcap_{i \in I} K_i$, we have $R \subset G(Z^\alpha)$. Now, let $x \in \bigcup \bigcap K_i$ and $c^T x = \max_{I \in M_\alpha} \sup_{u \in \bigcap_{i \in I} K_i} c^T u$, then there is $I_0 \in M_\alpha$ such that $x \in \bigcap_{i \in I_0} K_i$ and

$$\max_{I \in M_\alpha} \sup_{u \in \bigcap_{i \in I} K_i} c^T u = \sup_{u \in \bigcap_{i \in I_0} K_i} c^T u = s(I_0)$$

Hence, $x \in S(I_0)$, consequently, $x \in R$, and $G(Z^\alpha) \subset R$. □

Remark. To characterize the level sets of the fuzzy-optimal solution Z_{opt} and the fuzzy-optimal value g_{opt}, it is sufficient to use Proposition 5 and then Proposition 3. The procedure of the application is demonstrated on a simple example in the next section.

6. EXAMPLE

Consider the following LP problem

$$\begin{aligned} \text{maximize} \quad & 3x_1 + 4x_2 \\ \text{subject to} \quad & -2x_1 + x_2 \leqq 1 , \\ & 3x_1 + x_2 \leqq 6, \\ & x_1 + 2x_2 \leqq 4, \\ & 2x_1 + 2x_2 \leqq 5, \\ & x_1, x_2 \geqq 0. \end{aligned}$$

According to the notation of Section 5, denote:

$$K_1 = \{x \in E_2;\; -2x_1 + x_2 \leqq 1,\; x_1, x_2 \geqq 0\} ,$$

$$K_2 = \{x \in E_3;\; 3x_1 + x_2 \leqq 6,\; x_1, x_2 \geqq 0\} ,$$

$$K_3 = \{x \in E_2;\; x_1 + 2x_2 \leqq 4,\; x_1, x_2 \geqq 0\} ,$$

$$K_4 = \{x \in E_2;\; 2x_1 + 2x_2 \leqq 5,\; x_1, x_2 \geqq 0\} ,$$

with

$$w_1 = 0{\cdot}1 , \quad w_2 = 0{\cdot}2 , \quad w_3 = 0{\cdot}3 , \quad w_4 = 0{\cdot}4 .$$

By means of (28)–(30) we calculate

I	$w(I)$	$s(I)$	$S(I)$
$\{1\}$	0·1	$+\infty$	$\emptyset$
$\{2\}$	0·2	24·0	$x = (0; 6)$
$\{3\}$	0·3	9·0	$x = (3; 0)$
$\{4\}$	0·4	10·0	$x = (0; 2{\cdot}5)$
$\{1, 2\}$	0·3	15·0	$x = (1; 3)$
$\{1, 3\}$	0·4	9·0	$x = 3; 0)$
$\{1, 4\}$	0·5	9·5	$x = (0{\cdot}5; 2)$
$\{2, 3\}$	0·5	9·6	$x = (1{\cdot}6; 1{\cdot}2)$
$\{2, 4\}$	0·6	8·25	$x = (1{\cdot}75; 0{\cdot}75)$
$\{3, 4\}$	0·7	9·0	$x = (1; 1{\cdot}5)$
$\{1, 2, 3\}$	0·6	9·6	$x = (1{\cdot}6; 1{\cdot}2)$
$\{1, 2, 4\}$	0·7	9·5	$x = (0{\cdot}5; 2)$
$\{1, 3, 4\}$	0·8	9·0	$x = (1; 1{\cdot}5)$
$\{2, 3, 4\}$	0·9	9·0	$x = (1; 1{\cdot}5)$
$\{1, 2, 3, 4\}$	1·0	9·0	$x = (1; 1{\cdot}5)$

Using expression (33) we obtain

$$\begin{aligned}
G(Z^{0\cdot1}) &= \emptyset \\
G(Z^{0\cdot2}) &= \{(0{\cdot}0; 6{\cdot}0)\} \\
G(Z^{0\cdot3}) &= \{(1{\cdot}0; 3{\cdot}0)\}\,, \\
G(Z^{0\cdot4}) &= \{(0{\cdot}0; 2{\cdot}5)\}\,, \\
G(Z^{0\cdot5}) &= \{(1{\cdot}6; 1{\cdot}2)\}\,, \\
G(Z^{0\cdot6}) &= \{(0{\cdot}5; 2{\cdot}0)\}\,, \\
G(Z^{0\cdot7}) &= \{(0{\cdot}5; 2{\cdot}0)\}\,, \\
G(Z^{0\cdot8}) &= \{(1{\cdot}0; 1{\cdot}5)\}\,, \\
G(Z^{0\cdot9}) &= \{(1{\cdot}0; 1{\cdot}5)\}\,, \\
G(Z^{1\cdot0}) &= \{(1{\cdot}0; 1{\cdot}5)\}\,.
\end{aligned}$$

Using (15) we obtain the level sets Z^{α}_{opt} of the fuzzy-optimal solution Z_{opt}:

$$\begin{aligned}
Z^{0\cdot1}_{\text{opt}} &= Z^{0\cdot2}_{\text{opt}} = \{(0; 6), (1; 3), (0; 2{\cdot}5), (1{\cdot}6, 1{\cdot}2), (0{\cdot}5; 2), (1; 1{\cdot}5)\}\,, \\
Z^{0\cdot3}_{\text{opt}} &= \{(1; 3), (0; 2{\cdot}5), (1{\cdot}6; 1{\cdot}2), (0{\cdot}5; 2), (1; 1{\cdot}5)\}\,, \\
Z^{0\cdot4}_{\text{opt}} &= \{(0; 2{\cdot}5), (1{\cdot}6; 1{\cdot}2), (0{\cdot}5; 2), (1; 1{\cdot}5)\}\,, \\
Z^{0\cdot5}_{\text{opt}} &= \{(1{\cdot}6; 1{\cdot}2), (0{\cdot}5; 2), (1{\cdot}\ 1{\cdot}5)\}\,, \\
Z^{0\cdot6}_{\text{opt}} &= Z^{0\cdot7}_{\text{opt}} = \{(0{\cdot}5; 2), (1; 1{\cdot}5)\}\,, \\
Z^{0\cdot8}_{\text{opt}} &= Z^{0\cdot9}_{\text{opt}} = Z^{1\cdot0}_{\text{opt}} = \{(1; 1{\cdot}5)\}\,.
\end{aligned}$$

Using the symbolic way of description of a fuzzy set we obtain

$$\begin{aligned}
Z_{\text{opt}} = {} & 0{\cdot}2/(0; 6) + 0{\cdot}3/(1; 3) + 0{\cdot}4/(0; 2{\cdot}5) + 0{\cdot}5/(1{\cdot}6; 1{\cdot}2) + \\
& + 0{\cdot}7/(0{\cdot}5; 2) + 1{\cdot}0/(1; 1{\cdot}5)\,.
\end{aligned}$$

Definition (15) leads to the level sets g^α_{opt} of the fuzzy-optimal value g_{opt} of problem (25), (26):

$$g^{0\cdot1}_{opt} = g^{0\cdot2}_{opt} = \{24\cdot0,\ 15\cdot0,\ 10\cdot0,\ 9\cdot6,\ 9\cdot5,\ 9\cdot0\}\,,$$
$$g^{0\cdot3}_{opt} = \{15\cdot0,\ 10\cdot0,\ 9\cdot6,\ 9\cdot5,\ 9\cdot0\}\,,$$
$$g^{0\cdot4}_{opt} = \{10\cdot0,\ 9\cdot6,\ 9\cdot5,\ 9\cdot0\}\,,$$
$$g^{0\cdot5}_{opt} = \{9\cdot6,\ 9\cdot5,\ 9\cdot0\}\,,$$
$$g^{0\cdot6}_{opt} = g^{0\cdot7}_{opt} = \{9\cdot5,\ 9\cdot0\}\,,$$
$$g^{0\cdot8}_{opt} = g^{0\cdot9}_{opt} = g^{1\cdot0}_{opt} = \{9\cdot0\}\,.$$

Symbolically

$$g_{opt} = 0\cdot2/24\cdot0 + 0\cdot3/15\cdot0 + 0\cdot4/10\cdot0 + 0\cdot5/9\cdot6 + 0\cdot7/9\cdot5 + 1\cdot0/9\cdot0\,.$$

7. CONCLUSION

In the present paper a concept of fuzzy-mathematical programming is introduced. A FMP problem is a problem of optimizing an objective function subject to a constrait set which is a fuzzy set. A solution of FMP problem is also defined as a fuzzy set by means of the "Extension principle" applied on certain set-to-set mapping.

Our philosophy is applied to the LP problem, where the set of all feasible solutions is subjected to "fuzzyfication". The fuzzy-optimal solution can be obtained by an algorithmic way as demonstrated on a simple example. The presented fuzzy model of linear programming enables to incorporate different importance of individual linear constraints or to handle inconsistent system of constraints. From this point of view the usual LP problem is a special case of our fuzzy problem.

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