

ON VARIOUS DYNAMIC COMPENSATIONS

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The aim of this modest note is to show that certain compensation schemes frequently used in control theory, namely regular output feedback, combined dynamic compensation, and dynamic precompensation, are input-output equivalent.

1. INTRODUCTION

Let $R^{p \times q}$ and $R^{p \times q}\{w\}$ be respectively the sets of scalar and proper rational $p \times q$ matrices in one indeterminate w over the real field R . The units of the rings $R^{n \times n}$ and $R^{n \times n}\{w\}$ are respectively the non-singular and bi-proper matrices. Let us recall that an $H(w)$ is a unit of $R^{n \times n}\{w\}$ if and only if $H(0)$ is a unit of $R^{n \times n}$. That is to say, a rational matrix is bi-proper if it is proper together with its inverse. As usual, I will denote the identity matrix.

Consider a system Σ

$$(1) \quad \begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

described by the quadruple of matrices $A \in R^{n \times n}$, $B \in R^{n \times q}$, $C \in R^{p \times n}$ and $D \in R^{p \times q}$ which gives rise to the transfer matrix

$$T(w) = D + wC(I - wA)^{-1}B \in R^{p \times q}\{w\}.$$

Thus w is the inverse differential operator.

Further let

$$(2) \quad \dot{x}_d = u_d$$

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be a dynamic extension of Σ , a bunch of n_d integrators, and let v denote a command signal, say r -dimensional one.

Consider the problem of modifying $T(w)$ by means of a control law. The following three control laws are frequently used in the literature.

1. Regular Output Feedback

$$(3) \quad \begin{bmatrix} u \\ u_d \end{bmatrix} = F \begin{bmatrix} y \\ x_d \end{bmatrix} + Gv$$

$$(4) \quad \begin{cases} F = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \in R^{(q+n_d) \times (p+n_d)} \\ G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \in R^{(q+n_d) \times r} \end{cases}$$

and

$$(5) \quad I - F_{11}D = \text{unit of } R^{q \times q}.$$

This static control law acting on the extended system (1)–(2) is often used to study the dynamic output feedback for the original system (1), see e.g. Wonham [4]. The regularity condition (5) was introduced by Descusse and Malabre [1] in order to prevent the derivatives of v to appear in x . The class of control laws (3) satisfying (4)–(5) will be denoted by $C_1(\Sigma)$.

2. Combined Dynamic Compensation

$$(6) \quad u = P(w)y + Q(w)v$$

where

$$(7) \quad P(w) \in R^{q \times p}\{w\}, \quad Q(w) \in R^{q \times r}(w)$$

and

$$(8) \quad I - P(w)T(w) = \text{unit of } R^{q \times q}\{w\}.$$

This is a general compensation scheme which makes explicit the presence of a feed-forward and a feedback in the control law. It has been found useful in the polynomial equation approach, see e.g. Kučera [2]. In most cases, the regularity condition (8) is tacitly assumed. The class of control laws (6) satisfying (7)–(8) will be denoted by $C_2(\Sigma)$.

3. Dynamic Precompensation

$$(9) \quad u = K(w)v$$

where

$$(10) \quad K(w) \in R^{q \times r}\{w\}.$$

This is a standard way of introducing a compensator when the transfer-function approach is adopted. The scheme is flexible and can be used to represent a combined compensation or a dynamic feedback alone, see e.g. Wolovich [3]. The class of control laws (9) which satisfy (10) will be denoted by $\mathcal{C}_3(\Sigma)$.

We shall say that two classes are *input-output equivalent* if, for any control law of one class, we can find a control law in the other class such that their application to a given system will result in overall systems having the same transfer matrices.

This kind of equivalence reflects just the ability of two control laws to produce the same input-output behaviour. In particular, this equivalence says nothing about dynamical order, stability, sensitivity or other properties of control systems which depend on a particular realization. Nevertheless, this concept is useful when various feedforward/feedback configurations are studied from the input-output point of view. This is the case, for example, when solving the disturbance rejection, exact model matching or model following problems in dynamical systems.

2. RESULT

The aim of this note is to show that the three control laws are input-output equivalent. This would not be surprising if the control laws were unrestricted by (5), (8) and (10). What is less obvious is that the *regularity* condition (5) or (8) is equivalent to the *properness* condition (10).

Claim. The classes $\mathcal{C}_1(\Sigma)$, $\mathcal{C}_2(\Sigma)$ and $\mathcal{C}_3(\Sigma)$ are input-output equivalent.

Proof. The easiest way is to establish the chain of implications

$$\mathcal{C}_1(\Sigma) \Rightarrow \mathcal{C}_2(\Sigma) \Rightarrow \mathcal{C}_3(\Sigma) \Rightarrow \mathcal{C}_1(\Sigma).$$

a) To show that each element of $\mathcal{C}_1(\Sigma)$ can be realized as an element of $\mathcal{C}_2(\Sigma)$, consider a control law defined by (3)–(5). Using (1)–(3), calculate the transfer matrices from y and v to u . On identifying with (6), we obtain

$$(11) \quad \begin{aligned} P(w) &= F_{11} + wF_{12}(I - wF_{22})^{-1}F_{21} \\ Q(w) &= G_1 + wF_{12}(I - wF_{22})^{-1}G_2. \end{aligned}$$

Since $I - wF_{22}$ is bi-proper, both $P(w)$ and $Q(w)$ are proper. Moreover

$$I - P(0)T(0) = I - F_{11}D$$

is non-singular whence (8) holds. The control law (11) thus belongs to $\mathcal{C}_2(\Sigma)$.

b) To show that each element of $\mathcal{C}_2(\Sigma)$ can be realized as an element of $\mathcal{C}_3(\Sigma)$, consider a control law defined by (6)–(8). Calculate the transfer matrix from v to u and compare it with (9) to obtain

$$(12) \quad K(w) = [I - P(w)T(w)]^{-1}Q(w).$$

The properties (7)–(8) then imply (10); hence the control law defined by (12) belongs to $C_3(\Sigma)$.

c) Finally, let us show that each element of $C_3(\Sigma)$ can be realized as an element of $C_1(\Sigma)$. Given any proper $K(w)$, let

$$K(w) = D_0 + wC_0(I - wA_0)^{-1}B_0$$

for some realization (A_0, B_0, C_0, D_0) . Then

$$(13) \quad \begin{aligned} F_{11} &= 0 & F_{12} &= C_0 & G_1 &= D_0 \\ F_{21} &= 0 & F_{22} &= A_0 & G_2 &= B_0 \end{aligned}$$

defines a control law of the form (3). Moreover, $I - F_{11}D$ is the identity. Hence the control law (13) belongs to $C_1(\Sigma)$. \square

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