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ON THE DETECTION OF A COMPLEX FINITE BINARY SEQUENCE IN THE PRESENCE OF AN INTERFERING COMPLEX BINARY SEQUENCE

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A method of removing an unwanted binary sequence from the additive mixture of this sequence with a wanted one and with a noise sequence is investigated. The case of complex binary sequences — that is the case of quadrature — channel signal reception — is considered.

1. INTRODUCTION

In [1], a method of separation of a wanted binary sequence from an inwanted one has been investigated. Real sequences have been considered. In this article there will be shown that the method can be used with only very slight modifications in the case of complex binary sequences.

2. SOME FUNDAMENTAL RELATIONS

Let us suppose that the actually received signal $\{x_i\}$ is complex:

$$x_i = |x_i| \, e^{i\xi_i} = S \, e^{i\psi} t_i + U \, e^{i\varphi} v_i + n_i \tag{1}$$

$i = 1, 2, \ldots, N$, $\{t_i\}$ is a wanted real binary sequence, normalized to $|t_i| = 1$, $\{v_i\}$ is an unwanted real binary sequence, $|v_i| = 1$, n_i is a bivariate white Gaussian sequence symmetrically distributed about the origin, with independent real and imaginary components, $\sigma_x = \sigma_y = 1$. Further $U > 0$, $S \geqq 0$, and

$$U \gg S \,, \quad U \gg \sigma \tag{2}$$

Now there will be required

$$\sum \big| |x_i| \, e^{i\xi_i} - s \, e^{i\psi} t_i - u \, e^{i\varphi} v_i \big|^2 = \Phi(s, u, \psi, \varphi) = \min \,. \tag{3}$$

The limits 1, N in the sum are omitted in (3) and will be omitted in what follows.

As in [1], we are interested here only in s. Differentiating (3) with respect to s, u

ψ, φ, four equations result and can be further simplified by an assumption based on (2)

$$\mathrm{e}^{i\varphi} v_i = \mathrm{e}^{i\xi_i}, \quad \mathrm{e}^{-i\varphi} v_i = \mathrm{e}^{-i\xi_i}, \quad i = 1, \ldots, N. \tag{4}$$

The following two equations can be derived from which s can be computed.

$$2Ns + Au = B, \tag{5}$$

$$As + 2Nu = 2 \sum |x_i|, \tag{6}$$

with

$$A = \sum t_i (\mathrm{e}^{i(\psi - \xi_i)} + \mathrm{e}^{-i(\psi - \xi_i)}), \tag{7}$$
$$B = \sum t_i |x_i| (\mathrm{e}^{i(\psi - \xi_i)} + \mathrm{e}^{-i(\psi - \xi_i)}).$$

It is seen that A, B are real. From (5), (6)

$$s = \frac{2NB - 2 \sum |x_i| A}{4N^2 - A^2}. \tag{8}$$

3. THE METHOD OF QUADRATURE DETECTION

The way by which (8) has been derived can now be forgotten and only the properties of the numerator of (8) will be investigated.

$$NB - \sum |x_i| A = \mathrm{e}^{i\psi} [N \sum t_i \bar{x}_i - \sum |x_i| \sum t_i \mathrm{e}^{-i\xi_i}] + \tag{9}$$
$$+ \mathrm{e}^{-i\psi} [N \sum t_i x_i - \sum |x_i| \sum t_i \mathrm{e}^{i\xi_i}].$$

With a bar, complex conjugates are denoted.

Now, since $|e^{i\psi}| = 1$, it is clear that the investigation can be restricted on the random vector

$$W = N \sum t_i x_i - \sum |x_i| \sum t_i \mathrm{e}^{i\xi_i} \tag{10}$$

(or, equivalently, $\overline{W}$).

The expression (10) is a direct generalization of (15) of [1]. Remembering that $\{t_i\}$ is supposed to be known, that $\{x_i\}$ are values the components of which are measured in quadrature channel and, finally

$$e^{i\xi_i} = x_i / |x_i| \tag{11}$$

it is seen that W can indeed be computed.

To find the properties of the distribution of W from the respective real and imaginary parts is an elementary but somewhat lengthy procedure. But from (10) with (2) and (4), one can expect that the vectors W will lie substantially on the line containing the origin and the direction given by the angle φ ($-\varphi$ for $\overline{W}$).

The results of computer simulations in Tables 1 and 2 of Section 5 confirm this expectation. The noise being symmetric about the origin, W can thus be more naturally decomposed into two orthogonal components Y, Z, the substantial being Y.

Repeating then the computation from [1], one finds easily that Y is Gaussian and

$$\mathsf{E}(Y) = S(N^2 - C^2)\cos(\varphi - \psi)\,, \tag{12}$$

$$\sigma^2(Y) = \sigma^2 N(N^2 - C^2) \tag{13}$$

with

$$C = \sum t_i v_i \tag{14}$$

$S\cos(\varphi - \psi)$ is the projection of the wanted signal "amplitude" on Y. It is seen that with the conditions (2), only C (and not U) has influence on (12), (13).

The nonsubstantial component Z is also Gaussian. It is not easy to get simple and sufficiently accurate expressions for $\mathsf{E}(Z)$, $\sigma^2(Z)$, since no simple approximations of the second term in (10) exist. But

$$\mathsf{E}(Z) = \mathsf{E}(Y)\,O(U^{-1})\,, \tag{15}$$

$$\sigma^2(Z) = \sigma^2(Y)\,O(U^{-2}) \tag{16}$$

with O having the known meaning. The degree of approximation in (12), (13), (15), (16) can be checked in Tables 1, 2 in 5. paragraph. For practical purposes, the distribution of W can be considered as the one-dimensional distribution of Y, Z being very small.

If, as usual in quadrature reception, the absolute value $|W|$ is used for detection, then (12)–(16) show that all formulas of [1] for the threshold setting can be directly used with the trivial modification that now the one-sided Gaussian and "t" distributions are to be applied and that the term $\cos(\varphi - \psi)$ in (12) must be taken into account, φ and ψ being not estimated. Both angles can be considered as randomly, independently and uniformly distributed in $\langle 0 \cdot 2\pi)$, and this term can be replaced by $2/\pi \doteq \frac{2}{3}$.

4. THE SIGNAL/NOISE RATIO AND TWO TYPES OF UNWANTED SEQUENCES

From (12), (13)

$$\left(\frac{\mathsf{E}(Y)}{\sigma(Y)}\right)^2 = \frac{S^2}{2\sigma^2}\,\frac{N^2 - C^2}{N}\,2\cos^2(\varphi - \psi)\,. \tag{17}$$

The mean value of $\cos^2(\varphi - \psi)$ is $\frac{1}{2}$ under the above assumptions. Thus

$$(s/n)_o = (s/n)_i\,N\,\frac{N^2 - C^2}{N^2}\,. \tag{18}$$

This is the same formula as (23) in [1] and is now somewhat optimistic due to neglection of the nonsubstantial component Z of W. The last term on the right represents the diminution of s/n due to the presence of the unwanted sequence.

Two interesting cases of this binary sequence will now be considered.

Firstly, let all sequences be real and let

$$v_i = 1\,, \quad i = 1, \ldots, N \tag{19}$$

Generally,

$$C = \sum t_i v_i = N - 2m\,, \tag{20}$$

where m is the number of places on which $\{t_i\}$ and $\{v_i\}$ differ by signs. Now with respect to (19)

$$C = \text{the number of } (+1) \text{ in } \{t_i\} - \text{the number of } (-1) \text{ in } \{t_i\}\,. \tag{21}$$

It is well known that there exist binary sequences very good with respect to (21), e.g. all PN sequences, in contrast to other sequence, e.g., Barker- 5 or Barker- 13 which are not so good.

Secondly, let $\{t_i\}$ be a given real binary sequence and let $\{v_i\}$ be real binary sequence chosen at random so that independently on i the probabilities

$$p(v_i = 1) = p(v_i = -1) = p = 1/2\,. \tag{22}$$

Then, the distribution of sign differences of $\{t_i\}$, $\{v_i\}$ is binomial,

$$p(m) = \binom{N}{m} p^N \tag{23}$$

and for last term on the right in (18), there is to be computed

$$\mathsf{E}\left(\frac{N^2 - C^2}{N^2}\right) = \sum_{C=-N}^{N} \frac{N^2 - C^2}{N^2}\, p\left(\frac{N^2 - C^2}{N^2}\right). \tag{24}$$

The step of C in this sum is 2.

From (20)

$$m = (N - C)/2\,, \tag{25}$$

$$(N^2 - C^2)/N^2 = 4m(N - m)/N^2 \tag{26}$$

and obviously for a given C

$$p\left(\frac{N^2 - C^2}{N^2}\right) = p(m) \tag{27}$$

with m from (25) and $p(m)$ from (23). Then

$$\mathsf{E}\left(\frac{N^2 - C^2}{N^2}\right) = \frac{4}{N^2} \sum_{m=0}^{N} m(N - m)\, p(m) = 4p(1 - p)\left(1 - \frac{1}{N}\right) = 1 - \frac{1}{N}\,, \tag{28}$$

due to (22).

Thus, the diminution of s/n is independent of the wanted sequence and is very small for sufficiently great N.

5. SIMULATION RESULTS

The formulas of the preceding paragraphs have been tested by many computer simulations from which some results will be shown here. In whar follows

$$\{t_i\} = + + + - - + - = \text{Barker-7} \tag{29}$$
$$\{v_i\} = + + + - - - -$$

$S = 0, 1$, $U = 4, 16$, $\varphi = 0°$, $\psi = 30°, 60°, 90°$, the noise n is "pseudo-Gaussian" with independent real and imaginary components, $\sigma_x = \sigma_y = 1$.

In the following two tables the results of repeated 25 random experiments are contained. The bivariate distribution of $\overline{W}$ has been observed. Empirical mean values and dispersions are contained in both tables, the values computed from (12), (13) are shown in parentheses.

Table 1.

$U = 4$

	$S = 0$		$S = 1$, $\psi = 30°$		$S = 1$, $\psi = 60°$		$S = 1$, $\psi = 90°$	
$\bar{x}$	0·027	(0)	19·4	(20·9)	10·9	(12·0)	−0·122	(0)
$\bar{y}$	−0·93	(0)	−0·76		−0·79		−1·13	(0)
s_x^2	128	(168)	137	(168)	139	(168)	150	(168)
s_y^2	11·5		15·6		9·2		8·3	

Table 2.

$U = 16$

	$S = 0$		$S = 1$, $\psi = 30°$		$S = 1$, $\psi = 60°$		$S = 1$, $\psi = 90°$	
$\bar{x}$	−0·43	(0)	20·3	(20·9)	11·5	(12·0)	−0·47	(0)
$\bar{y}$	−0·23	(0)	−0·17		−0·20		−0·24	(0)
s_x^2	135	(168)	137	(168)	139	(168)	139	(168)
s_y^2	0·71		0·79		0·69		0·67	

Further, the results of repeated 100 random experiments are contained in the following Table 3 with the sequences from (29), $S = 0, 1$, $U = 4, 16$, $\varphi = 30°$, $\psi = 60°$, $\sigma_x = \sigma_y = 1$. Cumulative absolute frequencies of the distribution of $|W|$ grouped in intervals of the width 10 are shown.

The values in parentheses have been computed as follows. From (12) $\mathsf{E}(W) = 20{\cdot}9$. From (13) $\sigma(W) = 13{\cdot}0$. Thus the standardized mean in the case of $S = 0$ (1) is $\mathsf{E}(W)/\sigma(W) = 0$ (1·61). The standardized threshold is $20/\sigma(W) = 1{\cdot}54$. Now, instead of constructing the table of one-sided Gaussian distribution, one can choose the pro-

Table 3.

Interval	$S = 0$		$S = 1$	
	$U = 4$	$U = 16$	$U = 4$	$U = 16$
⟨ 0, 10)	51	55	23	22
⟨10, 20)	91	88 (87·6)	50	49 (47·2)
⟨20, 30)	98	97	79	76
⟨30, 40)	100	100	98	94
⟨40, 50)			100	99
⟨50, 60)				100

bability that the stândard normal variable is contained in the interval $(-1{\cdot}54, 1{\cdot}54)$ for $S = 0$ and in the interval $(-1{\cdot}54-1{\cdot}61,\ 1{\cdot}54-1{\cdot}61) = (-3{\cdot}15,\ -0{\cdot}07)$ for $S = 1$. Analogously one could use the table of the "t"-distribution.

6. CONCLUDING REMARKS

There remain some interesting questions to be answered, e.g.

1) what would happen if (2) were not valid,

2) what would be the profit from computing all the values s, u, ψ, φ.

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