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INPUT-OUTPUT SYSTEMS, THEIR TYPES AND APPLICATIONS FOR THE AUTOMATA THEORY

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The paper presents several illustrations of concept of subsystem when applying various types of mappings. There are specifications of notions as input-output system in a certain period of time, stage of system at a given moment, characterization of deterministic, nondeterministic and stochastic system and reasoning over their properties. In the end there are shown some applications of presented terms for the automata theory.

In the article "Some fundamental notions of large variable systems" [7] I have stated several principle terms from the system theory including definition of system. In this paper I like to mention a few more illustrations of system and further specifications of terms frequently used in the automata theory.

Partition of a given system by a choice of a class of mappings of the type $\{Z_1, Z_2, \ldots, Z_n\}$ into subsystems enables obviously certain relativisation of notions of "subsystem" and "element of a system".

Let $\mathscr{S} = \langle U, \mathscr{R} \rangle$ be a system in interval Δt

$$U = \{a_1, \ldots a_{i_1}, a_{i_1+1}, \ldots a_{i_2}, a_{i_2+1}, \ldots a_{i_n}\}$$
$$\mathscr{R} = \{R_1^{(1)}, \ldots, R_{i_n}^{(n)}\},$$

let further be chosen two-argument relation of equivalence $\doteq$ defined in U and let mapping Z be so that:

1) to every set $R_K^{(1)} \in \mathscr{R}$ assigns a set $R_l'^{(1)} \subset R_l^{(1)}$ as its image where for every $a_i \in U$ holds $a_i \in R_l'^{(1)}$ just only,
   if holds: a) $a_i \in R_l^{(1)}$
   b) for no $a_k \in R_l^{(1)}$, $a_i \neq a_k$ holds $a_i \doteq a_k$
   or: if there is $a_k \in R_l^{(1)}$, $a_i \neq a_k$ so that $a_i \doteq a_k$ then a_i is the only (however chosen) element from the set of all elements which are mutually in relation $\doteq$.

*) $a_i \in \mathrm{Cls}\, R_l^{(j)}$ is symbolic denotation of sentence: "a_i is an element from the field of relation $R_l^{(j)}$".

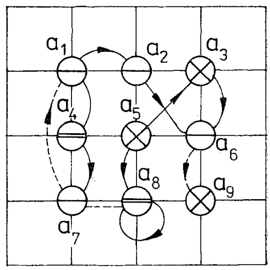
2) to every set $R_l^{(j)} \in \mathscr{R}, j > 1$ assigns a set $R_l'^{(j)} \subset R_l^{(j)}$ as its image where for every $a_k \in U$ holds $a_i \in \text{Cls } R_l'^{(j)*}$) just only

if holds: a) $a_i \in \text{Cls } R_l^{(j)}$

b) for no $a_k \in \text{Cls } R_l^{(j)}$, $a_i \neq a_k$ holds $a_i \doteq a_k$

or: if there is $a_k \in \text{Cls } R_l^{(j)}$, $a_i \neq a_k$ so that $a_i \doteq a_k$ then a_i is the only (however chosen) element from the set of all elements which are mutually in relation $\doteq$.

As a simple illustration, let us consider following system $\mathscr{S}_G$ given graphically (time factor omitted):



Obviously:

$$\mathscr{S}_G = \langle\{a_1, a_2, ..f, a_9\}, \{\ominus, \otimes, \circleddash, \to\to\to, - - \to\}\rangle,$$

$$\ominus = \{a_1, a_2, a_6, a_7\}, \quad \otimes = \{a_3, a_5, a_9\}, \quad \circleddash = \{a_4, a_8\},$$

$$\to\to\to = \{\langle a_1, a_4, a_7\rangle, \langle a_1, a_2, a_6\rangle, \langle a_5, a_3, a_6\rangle, \langle a_5, a_8, a_8\rangle\}$$

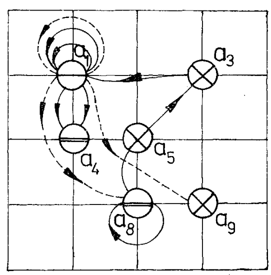
$$- - \to = \{\langle a_7, a_1\rangle, \langle a_7, a_8\rangle, \langle a_6, a_9\rangle\}.$$

Let us define equivalence relation $\doteq_1$ in this way:

$$\doteq \underset{df}{=} \{\langle a_1, a_2\rangle, \langle a_2, a_6\rangle, \langle a_2, a_7\rangle\}$$

(i.e. all white circles are in relation).

From the set $\{a_1, a_2 a_6, a_7\}$ we choose element a_1, for example. By means of transformation Z_1 defined on this base, we form a subsystem $\mathscr{S}_G'$ of the system $\mathscr{S}_G$:



Obviously subsystem $\mathscr{S}'_G = \langle\{a_1, a_3, a_4, a_5, a_8, a_9\}, \{\ominus, \otimes. \ominus, \rightarrow\rightarrow\rightarrow, ---\rightarrow\}\rangle$ where

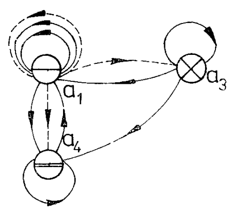
$$\ominus \quad = \{a_1\}, \quad \otimes = \{a_3, a_5, a_9\}, \quad \ominus = \{a_4, a_8\}$$

$$\rightarrow\rightarrow\rightarrow = \{\langle a_1, a_4, a_1\rangle, \langle a_1, a_1, a_1\rangle, \langle a_5, a_3, a_1\rangle, \langle a_5, a_8, a_8\rangle\}$$

$$---\rightarrow = \{\langle a_1, a_1\rangle, \langle a_1, a_8\rangle, \langle a_1, a_9\rangle\}.$$

Let us further define relation of equivalence $\doteq_2$ as the union of relations $\{\langle a_1, a_2\rangle, \langle a_2, a_7\rangle, \langle a_2, a_6\rangle\} \cup \{\langle a_3, a_5\rangle, \langle a_2, a_9\rangle\} \cup \{\langle a_4, a_8\rangle\}$ which is equivalence in sets $\ominus$, $\otimes$, $\ominus$, resp., but does not hold among elements of different sets.

With respect to transformation Z_2, defined on this base, we can create subsystem $\mathscr{S}''_G$ of the system $\mathscr{S}_G$:



Obviously subsystem $\mathscr{S}''_G = \langle\{a_1, a_3, a_4\}, \{\ominus, \otimes, \ominus, \rightarrow\rightarrow\rightarrow\ ---\rightarrow\}\rangle$ where

$$\ominus \quad = \{a_1\}, \quad \otimes = \{a_3\}, \quad \ominus = \{a_4\}$$

$$\rightarrow\rightarrow\rightarrow = \{\langle a_4, a_1, a_4\rangle, \langle a_1, a_1, a_1\rangle, \langle a_3, a_3, a_1\rangle, \langle a_3, a_4, a_4\rangle\}$$

$$---\rightarrow = \{\langle a_1, a_1\rangle, \langle a_1, a_4\rangle, \langle a_1, a_3\rangle\}.$$

Let us introduce one more transformation Z_0, which

to set $\ominus$ assigns itself

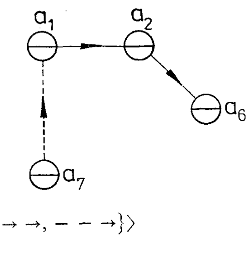
to set $\otimes$ assigns empty set

to set $\ominus$ assigns empty set

to relation $\rightarrow\rightarrow\rightarrow$ assigns set $\{\langle a_1, a_2, a_6\rangle\}$

to relation $---\rightarrow$ assigns set $\{\langle a_7, a_1\rangle\}$.

Thus we define system $\mathscr{S}_{G_0}$, subsystem of the system $\mathscr{S}_G$:



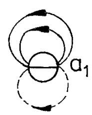
$$\mathscr{S}_{G_0} = \langle\{a_1, a_2, a_6, a_7\}, \{\ominus, \rightarrow\rightarrow\rightarrow, ---\rightarrow\}\rangle$$

Further transformation Z'_0 let assign to the set $\ominus$ itself,

to set $\otimes$ and $\oplus$ empty sets,

to relation $\to \to \to$ assigns set $\{\langle a_1, a_1, a_1 \rangle\}$

to relation $- - \to$ assigns set $\{\langle a_1, a_1 \rangle\}$:



$$\mathscr{S}'_{G_0} = \langle \{a_1\}, \{\ominus, \to \to \to, - - \to\} \rangle$$

Consider now relation between $\mathscr{S}_{G_0}$ and $\mathscr{S}'_{G_0}$. Both are subsystems of the system $\mathscr{S}_G$, but $\mathscr{S}'_{G_0}$ is at the same time subsystem of $\mathscr{S}_{G_0}$. Universe of $\mathscr{S}'_{G_0}$ has only one element a_1, which is somehow "restriction" of $\mathscr{S}_{G_0}$ universe. Similar relations exist among systems $\mathscr{S}_G$, $\mathscr{S}'_G$, $\mathscr{S}''_G$, $\mathscr{S}'_G$ is subsystem of $\mathscr{S}_G$, $\mathscr{S}''_G$ is subsystem of $\mathscr{S}'_G$, but we can define that also as subsystem of $\mathscr{S}_G$. $\mathscr{S}'''_G$ is "restriction" of $\mathscr{S}'_G$. On the contrary the system $\mathscr{S}_{G_0}$ can be considered as a certain "extension" of $\mathscr{S}'_{G_0}$. The element a_1 in given relations of $\mathscr{S}'_{G_0}$ is "foundation" of total system $\mathscr{S}_{G_0}$.

All just mentioned relations are obviously connected with a convenient choice of transformations. Suppose, given "large "system $\mathscr{S}_G$ exists as an ontic system, but not yet known as a whole. Instead of that we know merely system $\mathscr{S}''_G$. Elements of $\mathscr{S}''_G$: a_1, a_3, a_4 we meet at the first approach as "undivisable once" in corresponding relations. At the second attempt we can already "divide" "former elements", but we are not able yet to part the element a_1. The result is formation of "extended system $\mathscr{S}'_G$". At the further approach involving another "partition" of the element a_1 finally we shall identify complete system $\mathscr{S}_G$.

Proposed definitions D 1 – D 12 in [7] may yield interesting description of a certain ontic procedure beginning with poor and primitive information. Its result is synthesis of systems representing considerable simplification of real and complicated systems. Gradually we can synthetize more and more complicated systems because of "elements partition" of former simpler systems and complications of relations. This procedure is motivated by practical needs and demands and enabled by advanced measure and experimental technics.

Requirements of simplified description of large systems lead often to contrary approach. Large systems are divided into subsystems having respective elements from given sets mutually identified and the total system is thus simplified (due to conveniently chosen equivalence relation – denoted by " $\doteq$ "). This procedure if often called "formation of compartments". So obtained "united elements" characterized by a number of these "elements units" and quantitative measures of their characteristics are named "compartments". Corresponding large system then "multicompartmental system".

In modern science is frequently used an approach taking preferably care of only relations between inputs and outputs of a system regardless of their internal structure. Method characterized by using so is called "black box".

In definitions D 5 – D 8 in [7] I tried to specify fundamental notions associated with concepts of "input" and "output" of a system. Now I like to characterize more accurately this approach to systems considering only relations between "inputs" and "outputs" irrespective of internal structure.

I call it temporarily "input-output" approach.

When applying that we shall part properties and relations of elements from universe of a system using following way and so we can specify notion of "*input-output system*".

D 13. A system $\mathscr{S}$ is said to be *input-output system* in a period Δt (symbolically: $\langle \mathscr{S}, \Delta t \rangle \in \mathscr{I} - \mathscr{O}\ \mathscr{S}yst$) iff:

a) $\langle \mathscr{S}, \Delta t \rangle \in \mathscr{S}yst$

b) there exist sets U, V, Z, W so that $\mathscr{S} = \langle U, (V \cup Z \cup W) \rangle$ where

$$V = \{V_K^{t_i} \text{ there is an object } \mathbf{x}_k \text{ and a moment } t_i \in \Delta t \text{ such that } \langle \mathbf{x}_k, \mathscr{S}, t_i \rangle \in \mathscr{I}nprel \text{ and } \langle \mathbf{x}_k, t_i \rangle \in V_k^{t_i}\}$$

($1 \leqq k \leqq n$, n is a number of all input elements of $\mathscr{S}$ in a moment t_i),

$$W = \{W_l^{t_i}; \text{ there is an object } \mathbf{y}_1 \text{ and a moment } t_i \in \Delta t \text{ such that } \langle y_l, \mathscr{S}, t_i \rangle \in \mathscr{O}utel \text{ and } \langle \mathbf{y}_l, t_i \rangle \in W_l^{t_i}\}$$

($1 \leqq l \leqq m$, m is a number of all output elements of $\mathscr{S}$ in a moment t_i), there exist sets $Z_1^{t_{j_1}}, Z_2^{t_{j_2}}, \ldots, Z_m^{t_{j_m}}$ so that

$$Z = \{Z_1^{t_{j_1}}, Z_2^{t_{j_2}}, \ldots, Z_m^{t_{j_m}}\}, \quad t_{j_1}, t_{j_2}, \ldots, t_{j_m} \in \Delta t$$

(Further I shall denote:

$$\langle V \rangle^{t_i} = \langle V_1^{t_i}, V_2^{t_i}, \ldots, V_n^{t_i} \rangle, \quad \langle W \rangle^{t_i} = \langle W_1^{t_i}, W_2^{t_i}, \ldots, W_m^{t_i} \rangle$$

$$\langle Z \rangle^{t_i} = \langle Z_1^{t_i}, Z_2^{t_i}, \ldots, Z_m^{t_i} \rangle$$

there exist relations $\mathscr{R}_\delta, \mathscr{R}_\lambda$ so that

$\mathscr{R}_\delta$ is two-argument relation with the domain given by set of pairs of the type $\langle \langle V \rangle^{t_i}, \langle Z \rangle^{t_i} \rangle$ and with the branch given by set of elements of the type $\langle Z \rangle^{t_j}$,

$\mathscr{R}_\lambda$ is two-argument relation with the domain given by set of pairs of the type $\langle \langle V \rangle^{t_i}, \langle Z \rangle^{t_i} \rangle$ and with the branch given by set of elements of the type $\langle W \rangle^{t_i + |\Delta t'|}$, $t_i < t_i + |\Delta t'|$, $t_i + |\Delta t'| \in \Delta t$.

The "input-output" approach to a real system means a limiting simplification of this system where, abstracting from all elements, their properties and relations and internal structure of that, we admit only input and output elements with their properties and relations. But we introduce "auxiliary" variables (in the latter definition represented by ordered pairs of elements from the set Z, sometimes called "values of stages") and traditionally others named "stages of system".

These stages yield formulation of relations $\mathscr{R}_\lambda$ among ordered

n-tuples of input values

r-tuples of stages values and

m-tuples of output values.

Values of stages are subject to time changes and these changes are characterized by relation $\mathscr{R}_\delta$.

(The names of $\mathscr{R}_\delta$, $\mathscr{R}_\lambda$ relations represent predicates of higher degrees).

The notion given in D 13 is obviously specification of term "automaton", as was specified in [7] in D 1 and D 10 respectively .Given formulation I have chosen to satisfy current, already traditional, specification of "automaton" concept. This concept I shall introduce later by further specification of notion "input-output system".

Now let me form notion "*stage* of system":

D 14. A set Z is called *stage* of system $\mathscr{S}$ at a moment t_i (symbolically: $\langle Z, \mathscr{S}, t_i\rangle \in \in \mathscr{S}t$ or $\langle\langle Z\rangle^{t_i}, \mathscr{S}\rangle \in \mathscr{S}t$), if coincidentally holds:

a) there is time interval Δt so that $t_i \in \Delta t$, $\langle \mathscr{S}, \Delta t\rangle \in \mathscr{I} - \mathscr{O}\,\mathscr{S}yst$

b) there are sets U, V, W so that $\mathscr{S} = \langle U, (V \cup Z \cup W)\rangle$,

$$V = \{V_k^{t_i} \text{ there exists } \mathbf{x}_k \text{ so that } \langle \mathbf{x}_k, \mathscr{S}, t_i\rangle \in \mathscr{I}npel, \langle \mathbf{x}_k, t_i\rangle \in V_k^{t_i}\},$$

$1 \leqq k \leqq n$.

$$W = \{W_l^{t_i} \text{ there exists } \mathbf{y}_l \text{ so that } \langle \mathbf{y}_l, \mathscr{S}, t_i\rangle \in \mathscr{O}utpel, \langle \mathbf{y}_l, t_i\rangle \in W_l^{t_i}\}$$

$1 \leqq l \leqq m$

there exist ordered sets

$$\langle V\rangle^{t_i} = \langle V_1^{t_i}, V_2^{t_i}, \ldots, V_n^{t_i}\rangle$$

$$\langle W\rangle^{t_i} = \langle W_1^{t_i}, W_2^{t_i}, \ldots, W_m^{t_i}\rangle$$

there exist sets $\langle Z\rangle^{t_i}, Z_1^{t_i}, \ldots, Z_m^{t_i}$ so that

$$\langle Z\rangle^{t_i} = \langle Z_1^{t_i}, \ldots, Z_m^{t_i}\rangle$$

there is time interval $\Delta t'$ and $\Delta t''$ so that

$$t_i + |\Delta t'| \in \Delta t,\ t_i + |\Delta t''| \in \Delta t\,,\quad \Delta t'' \subset \Delta t' \subset \Delta t$$

there is relation $\mathscr{R}_\delta$ so that $\langle\langle\langle V\rangle^{t_i}, \langle Z\rangle^{t_i}\rangle, \langle Z\rangle^{t_i+|\Delta t''|}\rangle \in \mathscr{R}_\delta$ there is relation $\mathscr{R}_\lambda$ so that $\langle\langle\langle V\rangle^{t_i}, \langle Z\rangle^{t_i}\rangle, \langle W\rangle^{t_i+|\Delta t'|}\rangle \in \mathscr{R}_\lambda$.

Elements of ordered r-tuple of the type $\langle Z\rangle^{t_i}$ are called values of stage of the system $\mathscr{S}$ at a moment t_i, elements of ordered m-tuple of the type $\langle V\rangle^{t_i}$ are called values of input of the system $\mathscr{S}$ at a moment t_i, elements of ordered m-tuple of the type $\langle W\rangle^{t_i}$ are called values of output of the system $\mathscr{S}$ at a moment t_i.

Let there be given a system $\mathscr{S}$ with given input V and output W respectively and with corresponding values at a moment t_i:

$$\begin{aligned} \langle V\rangle^{t_i} &= \langle V_1^{t_i}, V_2^{t_i}, \ldots, V_n^{t_i}\rangle \\ X &\quad \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\} \\ Y &= \{\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_m\} \\ \langle W\rangle^{t_i} &= \langle W_1^{t_i}, W_2^{t_i}, \ldots, W_m^{t_i}\rangle \end{aligned}$$

Stage Z of the system $\mathscr{S}$ can be considered as "capability" of that at "initial moment" t_i to react on n-tuple of input values by a certain m-tuple of output values after time interval Δt is over. The stage itself of the system is not constant, but varies accordingly to his own past and values of input of the system.

We can assume that from the complete system we know only a certain relation between n-tuple of input values and m-tuple of those of output. Internal structure of the system is unknown for us, as yet. In order to search properties of this relation, we form further entities "stages of the system", "values of the system stages" and look for their convenient time sequences. Mentioned procedure is typical one for modern theory of systems and automata, respectively .

Stage of a system has been generally specified as a set of some elements, $Z_1^{t_{j,1}}, \ldots$ $\ldots, Z_m^{t_{j,m}}$ at some moments t_i. Stage of a given system $\mathscr{S}$ at a given moment t_i I have stated as ordered set of elements $Z_1^{t_i}, \ldots, Z_m^{t_i}$ at these moments. Elements of these sets can be understood as properties of some "artificially constructed" elements from the system universe, $z_1, \ldots, z_m$:

$$\langle R_1, t_{j,1}\rangle \in Z_1^{t_{j,1}}, \ldots, \langle R_m, t_{j,m}\rangle \in Z_m^{t_{jm}}$$

Input-output system stated in this way can be considered as a subsystem of incompletely known large system. From some other viewpoint, such an input-output system can be understood as subsystem created from large and completely known system by means of a mapping which associates:

set of all input elements with itself,

set of all output elements with itself,

set of all internal elements of system with empty set, individual other properties with empty sets.

From the viewpoint of philosophical determinism, this mapping has following capacity:

to individual elements of set $\mathscr{R}$ our large system $\mathscr{S} = \langle U, \mathscr{R}\rangle$ assigns such subsystems, which we shall name "elements of system stage at given moments" ($Z_1^{t_{j,1}}, \ldots$

…, $Z_m^{t_j,m}$). Those elements from our system universe, which are elements of these subsets $Z_1^{t_j,1}, \ldots, Z_m^{t_j,m}$ we shall identify with those "artificially constructed" ones of the type $z_1, \ldots, z_m$. Roughly speaking, stages of system are considered to be its internal characteristics, which are cooperating for regular appearance of certain input values after taking on given input values of the system.

Further I introduce three notions more, which I have taken over from the theory of automata and I try to generalize them for needs of system theory.

D 15. Input-output system $\mathscr{R}$ is in time period Δt deterministic system (symbolically: $\langle \mathscr{S}, \Delta t\rangle \in \mathscr{D}et\mathscr{s}y\mathscr{s}t$) iff:

a) $\langle \mathscr{S}, \Delta t\rangle \in \mathscr{I} - \mathscr{O}\,\mathscr{S}y\mathscr{s}t$

b) relations $\mathscr{R}_\delta$, $\mathscr{R}_\lambda$ from D 13 are uniquely determined ones and by means of them functions δ, λ can be defined so that

$$\langle Z\rangle^{t_i+|\Delta t''|} = \delta(\langle V\rangle^{t_i}, \langle Z\rangle^{t_i}) \underset{\text{def}}{\Leftrightarrow} \langle\langle\langle V\rangle^{t_i}, \langle Z\rangle^{t_i}\rangle, \langle Z\rangle^{t_i+|\Delta t''|}\rangle \in \mathscr{R}_\delta$$

$$\langle W\rangle^{t_i+|\Delta t'|} = \lambda(\langle V\rangle^{t_i}, \langle Z\rangle^{t_i}) \underset{\text{def}}{\Leftrightarrow} \langle\langle\langle V\rangle^{t_i}, \langle Z\rangle^{t_i}\rangle, \langle W\rangle^{t_i+|\Delta t'|}\rangle \in \mathscr{R}_\lambda$$

Remark. At deterministic system, values of system output are obviously uniquely determined by values of input and those of stage of that, respectively.

D 16. Input-output system $\mathscr{S}$ is in time period Δt *nondeterministic* system (symbolically: $\langle \mathscr{S}, \Delta t\rangle \in \mathscr{N}oncl\,et\mathscr{s}y\mathscr{s}t$), iff:

a) $\langle \mathscr{S}, \Delta t\rangle \in \mathscr{I} - \mathscr{O}\,\mathscr{S}y\mathscr{s}t$

b) relations $\mathscr{R}_\delta$, $\mathscr{R}_\lambda$ from D 13 are not uniquely determined, but by means of them it is possible to define functions δ, λ so that

$$\delta(\langle V\rangle^{t_i}, \langle Z\rangle^{t_i}) = \langle Z_1\rangle^{t_i+|\Delta t''|}, \ldots, \langle Z_j\rangle^{t_i+|\Delta t''|}\}$$

iff

$$[\langle\langle\langle V\rangle^{t_i}, \langle Z\rangle^{t_i}\rangle, \langle Z_1\rangle^{t_i+|\Delta t''|}\rangle \in \mathscr{R}_\delta]$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$[\langle\langle\langle V\rangle^{t_i}, \langle Z\rangle^{t_i}\rangle, \langle Z_j\rangle^{t_i+|\Delta t''|}\rangle \in \mathscr{R}_\delta]$$

$$\lambda(\langle V\rangle^{t_i}, \langle Z\rangle^{t_i}\rangle) = \{\langle W_1\rangle^{t_i+|\Delta t'|}, \ldots, \langle W_k\rangle^{t_i+|\Delta t'|}\}$$

iff

$$[\langle\langle\langle V\rangle^{t_i}, \langle Z\rangle^{t_i}\rangle, \langle W_1\rangle^{t_i+|\Delta t'|}\rangle \in \mathscr{R}_\lambda] \ldots [\langle\langle\langle V\rangle^{t_i}, \langle Z\rangle^{t_i}, \langle W_k\rangle^{t_i+|\Delta t'|}\rangle\ \mathscr{R}_\lambda]$$

Remark. At nondeterministic system, output values are not uniquely defined – determined by given input and stage values respectively. By these values there is defined a set of output values (from which later one is being realized).

D 17. Input-output system $\mathscr{S}$ is in time period Δt *stochastic* system, (symbolically: $\langle \mathscr{S}, \Delta t \rangle \in \mathscr{S}\mathit{tochsyst}$) iff:

a) $\langle \mathscr{S}, \Delta t \rangle \in \mathscr{I} - \mathscr{O}\,\mathscr{S}\mathit{yst}$

b) relations $\mathscr{R}_\delta$, $\mathscr{R}_\lambda$ from D 13 are not uniquely determined, but by means of them functions δ, λ can be defined so that

$$\delta(\langle V \rangle^{t_i}, \langle Z \rangle^{t_i}) = \{\langle\langle Z_1 \rangle^{t_i + |\Delta t''|}, p_1 \rangle, \ldots, \langle\langle Z_j \rangle^{t_i + |\Delta t''|}, p_j \rangle\} \; ;$$

$$0 \leqq p_1, \ldots, p_j \leqq 1 \, , \quad \sum_1^j p_i = 1 \, ,$$

iff

$$[\langle\langle\langle V \rangle^{t_i}, \langle Z \rangle^{t_i} \rangle, \langle\langle Z_1 \rangle^{t_i + |\Delta t''|}, p_1 \rangle\rangle \in \mathscr{R}_\delta]$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$[\langle\langle\langle V \rangle^{t_i}, \langle Z \rangle^{t_i} \rangle, \langle\langle Z_j \rangle^{t_i + |\Delta t''|}, p_j \rangle\rangle \in \mathscr{R}_\delta]$$

$$\lambda(\langle V \rangle^{t_i}, \langle Z \rangle^{t_i}) = \{\langle\langle W_1 \rangle^{t_i + |\Delta t_i'}, p_1 \rangle, \ldots, \langle\langle W_k \rangle^{t_i + |\Delta t'|}, p_k \rangle\} \; ;$$

$$0 \leqq p_1, \ldots, p_k \leqq 1 \, , \quad \sum_1^k p_i = 1$$

iff

$$[\langle\langle\langle V \rangle^{t_i}, \langle Z \rangle\rangle^{t_i}, \langle\langle W_1 \rangle^{t_i + |\Delta t'|}, p_1 \rangle\rangle \in \mathscr{R}_\lambda]$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$[\langle\langle\langle V \rangle^{t_i}, \langle Z \rangle^{t_i} \rangle, \langle\langle W_k \rangle^{t_i + |\Delta t'|}, p_k \rangle\rangle \in \mathscr{R}_\lambda$$

At stochastic system obviously by given values of input and stages there is defined a set of possible output values and coincidentally to each value from this set is associated a number from interval $\langle 0,1 \rangle$ expressing degree of probability, so that output value will be realized.

Division of input-output systems into deterministic, nondeterministic and stochastic ones has some interesting gnoseological aspects. It is known, that large systems with various types of elements, properties and relations, studied by biological and social sciences, are mostly stochastic ones. It is necessary to stress that also technical systems functioning on physical bases are stochastic ones.

This fact is given by their large content. For this reason minor differences from strictly deterministic behaviour of individual parts of the system lead often to severe nondeterministic fluctuation of the system, as a whole. Stochastic character have also large systems consisting of not only physical components, but also antropological ones. Scientist and engineer meet often them, describe them by precise language and within this language realize engineering and scientific predictions.

Effort towards uniqueness and exactness of predictions leads frequently to elimination of a subsystem with deterministic behaviour from large stochastic input-output system by means of conveniently chosen transformation. So it is necessary to select from the set of properties and relations of stochastic system such a set or their subset, on the base of which we shall define deterministic subsystem of stochastic system.

Mentioned selection can be formed on the base of series of experiments or theoretically. The first way is realizable for example by applying search for "deterministic" correlations among values of particular subsets of output elements and coincidentally we look for necessary sets of system stage values. The second method is applicable, if we are able to explain stochastic behaviour of system by means of general laws of a theory and at the same time we can find by the use of the same theory deterministic components of the system (we know, for example, why all system of transport devices behaves stochasticaly and when using some of technological sciences, we know which of its components behave deterministicaly). Hence we can say that deterministic subsystems can often be defined on stochastic systems ($\langle \mathscr{S}, \Delta t\rangle \in \in \mathscr{S}tochsyst$ and coincidentally $\langle \mathscr{S}', \Delta t\rangle \in \mathscr{D}etsyst$ and coincidentally $\langle \mathscr{S}', \mathscr{S}, t_i\rangle \in \in \mathscr{S}ubsyst$ for every $t_i \in \Delta t$).

I have omitted so far a question, whether properties and relations involved in systems have qualitative of quantitative character. Stated specifications have been formed to contain systems notions of both, quantitative and qualitative properties and relations.

In offered specifications discussed properties and relations are considered to be in the set denoted by $\mathscr{R}$. If all sets involved in $\mathscr{R}$ are qualitative properties and relations, then also system description has entirely qualitative character and is being realized by means of qualitative predicates.

If some sets from $\mathscr{R}$ are quantitative properties or relations, then they can be considered as subsets of a set which we identify with fundamental qualitative property representing "qualitative background" of quantitative characteristic.

For example, let sets $R_{i,1}^{(1)}, \ldots, R_{i,k}^{(1)}$ of given system $\mathscr{S}$ be quantitative properties of some elements from the universe $\boldsymbol{U}$ of the system $\mathscr{S}$. These properties are considered to be subsets of a set-qualitative property $R_i^{(1)}$. Partition of set $R_i^{(1)}$ into its subsets $R_{i,1}^{(1)}, R_{i,2}^{(1)}, \ldots, R_{i,k}^{(1)}$ is being done by means of a suitable equivalence relation defined in $R_i^{(1)}$. Subsets $R_{i,1}^{(1)}, \ldots, R_{i,k}^{(1)}$ are result of this partition. At exact description of the system, a conveniently chosen metric function associates them with numbers expressing numerically value of property $R_i^{(1)}$ measure.

Obviously, we can consider system $\mathscr{S} = \langle \boldsymbol{U}, \mathscr{R}\rangle$ as qualitatively invariant when identifying the system S in time period Δt, although particular elements from sets of the set $\mathscr{R}$ can posses corresponding properties in distinct measure.

Let an element $\mathbf{a} \in \boldsymbol{U}$ have at every moment t_i from interval Δt property $R_i^{(1)} \in \mathscr{R}$ (system $\mathscr{S}$ is determined by relation: $\langle \mathbf{a}, t_i\rangle \in R_i^{(1)} \times \Delta t$ for every $t_i \in \Delta t$). The element $\mathbf{a}$ can vary as to the measure of possession of property $R_i^{(1)}$ when appearing at respective moments $t_{i1}, t_{12}, \ldots, t_{ik} \in \Delta t$ belonging to subintervals $\Delta t_1, \Delta t_4, \ldots, \Delta t_k$.

Hence

$$\langle \mathbf{a}, t_{i1}\rangle \in R_{i1}^{(1)} \times \Delta t_1$$
$$\vdots$$
$$\langle \mathbf{a}, t_{ik}\rangle \in R_{ik}^{(1)} \times \Delta t_k$$
$$R_i^{(1)} = R_{i1}^{(1)} \cup R_{i2}^{(1)} \cup \ldots \cup R_{ik}^{(1)}$$
$$\Delta t_1 \cap \Delta t_2 \cup \ldots \cup \Delta t_k = \Delta t$$
$$R_{i1}^{(1)} \cap R_{i2}^{(1)} \cap \ldots \cap R_{ik}^{(1)} = \varnothing$$

Given system, formed on the base of qualitative properties and relations among its elements, can go through various partial (quantitative) changes during its existence period. If we find a suitable further equivalence relation enabling repeated partition of some subsets of the type $R_{ik}^{(1)} \in \mathscr{R}$ we can define "extended" system.

For example, let the same specification of system $\mathscr{S}$ hold, but let the element $\mathbf{a}$ have property $R_{ik}^{(1)}$ on interval $\Delta t_l \subset \Delta t$. Thus we form new system $\mathscr{S}'$ for interval Δt_l, $\mathscr{S}' = \langle \boldsymbol{U}, \mathscr{R}'\rangle$ where $R_{ik}^{(1)}$ is now element of $\mathscr{R}'$ at every moment t_{il} of interval Δt_i:

$$\langle \mathbf{a}, t_{il}\rangle \in R_{il}^{(1)} \times \Delta t_i$$

As to the property $R_{il}^{(1)}$ (created by partition $R_i^{(1)}$): we can further divide the set associated with it by means of a convenient equivalence relation into subsets:

$$R_{il,1}^{(1)}, R_{il,2}^{(1)}, \ldots, R_{il,k}^{(1)}$$

The element $\mathbf{a}$ of system $\mathscr{S}'$ (and of course, of system $\mathscr{S}$) can go again through partial changes at particular moments of interval Δt_l concerning the property $R_{il}^{(1)}$:

$$\langle \mathbf{a}, t_{il,1}\rangle \in R_{il,1}^{(1)} \times \Delta t_{l,1} \quad \Delta t_{l,1}, \ldots, \Delta t_{l,k} \subset \Delta t_l \subset \Delta t$$
$$\vdots$$
$$\langle \mathbf{a}, t_{il,k}\rangle \in R_{il,k}^{(1)} \times \Delta t_{l,k} \quad R_{il,1}^{(1)}, \ldots, R_{il,k}^{(1)} \subset R_{il}^{(1)} \subset R_i^{(1)}$$

Given system can be successively extended to other systems, in which we follow time changes running within limits of original system. Sequence of these extended systems represents thus a series of corresponding alternations occurring within original system.

Let a system $\mathscr{S}''$ be obtained from system $\mathscr{S}$ as follows: all remain as in original system $\mathscr{S}$, but in interval $\Delta t_0 \subset \Delta t$ (where Δt_0 follows interval Δt_l of the system $\mathscr{S}'$) let the element $\mathbf{a}$ have property $R_{i0}^{(1)}$ at every moment of interval

$$\Delta t_0 : \langle \mathbf{a}, t_{i0}\rangle \in R_{i0}^{(1)} \times \Delta t_0$$

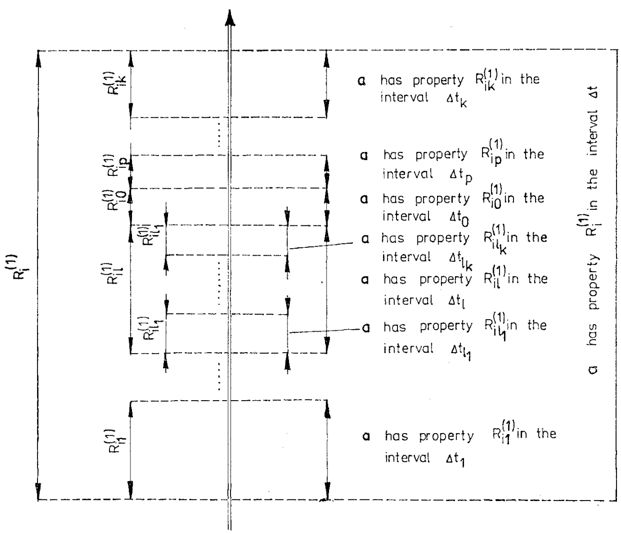
Analogously let us select system $\mathscr{S}'''$: all again remain as in $\mathscr{S}$, but in interval $\Delta t_p \subset \Delta t$ (where Δt_p follows interval Δt_0 of system $\mathscr{S}''$) let the element $\mathbf{a}$ have property $R_{ip}^{(1)}$ at every moment of interval $\Delta t_p : \langle \mathbf{a}, t_{ip}\rangle \in R_{ip}^{(1)} \times \Delta t_p$.

Sequence $\mathcal{S}'$, $\mathcal{S}''$, $\mathcal{S}'''$ represents alternation within the former system $\mathcal{S}$.

System $\mathcal{S}$ can be classified as subsystem of system $\mathcal{S}'$ and $\mathcal{S}''$ and $\mathcal{S}'''$ respectively: it is sufficient to define mapping which assigns to every property or relation resp. of system $\mathcal{S}'$ or $\mathcal{S}''$ or $\mathcal{S}'''$ resp. itself with exception of property $R_{il}^{(1)}$ or $R_{io}^{(1)}$ or $R_{ip}^{(1)}$ resp., to which assigns property $R_i^{(1)}$ so that:

$$R_{il}^{(1)} \subset R_i^{(1)}\,, \quad R_{io}^{(1)} \subset R_i^{(1)}\,, \quad R_{ip}^{(1)} \subset R_i^{(1)}$$

I do hope that given concept of large dynamic systems variable in time well satisfies tendency of modern science and technical practice to study along static problems and systems also dynamic ones, for example systems successively accommodating to enviroment. Proposed concept enables exact specification of time variability of very distinct aspects of real systems, for example that of systems border, sets of input and output elements, output functions and systems stages.



At the end let me form a review of (well known) principle terms from the theory of automata:

D 18. System $\mathcal{S}$ is in time period Δt ***automaton*** (symbolically: $\langle \mathcal{S}, \Delta t\rangle \in \mathcal{A}ut$), if coincidentally holds:

a) $\langle \mathscr{S}, \Delta t\rangle \in \mathscr{I} - \mathscr{O}\,\mathscr{S}yst$
b) there exist sets U, V, W, Z so that $\mathscr{S} = \langle U(V \cup Z \cup W)\rangle$ where V, Z, W are finite sets
c) $\Delta t \subset T$, Δt and T are denumerable sets of time intervals.

D 19. System $\mathscr{S}$ is in time period Δt *abstract automaton* (symbolically: $\langle \mathscr{S}, \Delta t\rangle \in \in \mathscr{A}bstaut$), if coincidentally holds:

a) there exist sets V, W, Z and function γ so that

$$\mathscr{S} = \langle V, W, Z, \gamma\rangle$$

b) there exist system $\mathscr{S}'$ and set U so that

$$\mathscr{S}' = \langle U, (V \cup Z \cup W)\rangle\,, \quad \langle \mathscr{S}', \Delta t\rangle \in \mathscr{A}ut$$

c) there are relations $\mathscr{R}_\delta$, $\mathscr{R}_\lambda$ so that

$$\gamma(\langle\langle V\rangle^{t_i}, \langle Z\rangle^{t_i}) = \langle\langle W\rangle^{t_i+|\Delta t'|}, \langle Z\rangle^{t_i+|\Delta t''|}\rangle$$

where

$$\langle V\rangle^{t_i}, \langle Z\rangle^{t_i}, \langle W\rangle^{t_i+|\Delta t'|}, \langle Z\rangle^{t_i+|\Delta t''|}, \mathscr{R}_\delta, \mathscr{R}_\lambda$$

having the same meaning as in D 13.

D 20. System $\mathscr{S}$ is in time period Δt *deterministic automaton* (symbolically: $\langle \mathscr{S}, \Delta t\rangle \in \mathscr{D}etaut$), if coincidentally holds:

a) $\langle \mathscr{S}, \Delta t\rangle \in \mathscr{D}etsyst$
b) $\langle \mathscr{S}, \Delta t\rangle \in \mathscr{A}ut$

D 21. System $\mathscr{S}$ is in time period Δt *nondeterministic automaton* (symbolically: $\langle \mathscr{S}, \Delta t\rangle \in \mathscr{N}detaut$) if coincidentally holds:

a) $\langle \mathscr{S}, \Delta t\rangle \in \mathscr{N}detsyst$
b) $\langle \mathscr{S}, \Delta t\rangle \in \mathscr{A}ut$

D 22. System $\mathscr{S}$ is in time period Δt *stochastic automaton* (symbolically: $\langle \mathscr{S}, \Delta t\rangle \in \in \mathscr{S}tochaut$), if coincidentally holds:

a) $\langle \mathscr{S}, \Delta t\rangle \in \mathscr{S}tochsyst$
b) $\langle \mathscr{S}, \Delta t\rangle \in \mathscr{A}ut$

Remark. Automata are considered to be systems, whose output and input values and those of stages have finite character and operate in discrete time.

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