KYBERNETIKA — VOLUME 16 (1980), NUMBER 5

Observer-Based Deadbeat Controllers: A Polynomial Design

VLADIMÍR KUČERA

A new method of designing observer-based deadbeat controllers for linear multivariable systems is presented. The design procedure consists in solving a linear equation in polynomial matrices and yields the entire controller, including the observer plus state feedback, in a single step. The resulting algorithm is remarkably simple, efficient, and leads to an economical realization of the controller.

INTRODUCTION

Deadbeat control is a typical example of linear control strategies in discrete-time systems. The standard objective is to drive any initial state of the system to zero in a shortest time possible. If the entire state is available for direct measurement, this objective can be accomplished by a linear state-variable feedback. Moreover, this feedback is constant and independent of the initial state. Standard methods for calculating such a control law can be found in Ackermann [1] or Strejc [6]; more general results are reported by Mullis [5].

If, however, some state variables are not accessible, this control law cannot be implemented. A common solution is to introduce an observer which would reconstruct the actual state from measurable data in a shortest time possible, and then apply the state-variable control law. This additional dynamics inevitably prolongs the control process but renders it possible to satisfy the deadbeat requirement for every initial state of the system. Moreover, the observer characteristics are independent of this initial state. Various techniques of designing deadbeat observers are discussed by Ackermann [1] or Kwakernaak and Sivan [4].

In a recent paper [3], the author proposed a new method for calculating state-variable deadbeat control laws. The method is based on solving a simple linear equation in polynomial matrices. The purpose of this paper is to extend the polynomial design technique so as to cope with unmeasurable states. In fact, a straightforward procedure is presented which enables us to design the entire controller, including the observer plus state feedback, in a single step. The procedure is not restricted to reversible systems and remains intact for systems defined over arbitrary fields. The resulting computational algorithm is remarkably simple and efficient.

STATE SPACE DESIGN

It is assumed that the reader is proficient in state space techniques and, in particular, in the theory of observers and deadbeet control. The entire state-space design procedure is just briefly summarized here for later reference.

Consider an n dimensional, discrete-time, completely controllable and constructible system

(1)
$$\begin{aligned} x_{t+1} &= Fx_t + Gu_t \\ y_t &= Hx_t \end{aligned}$$

where G is $n \times m$ and has rank m while H is $l \times n$ and has rank l. The following two notions will be needed later. Let $\{A_k\}$ be a sequence of $n \times m$ matrices and let

$$r_k = \operatorname{rank}[A_1 \ldots A_k] - \operatorname{rank}[A_1 \ldots A_{k-1}], \quad k = 1, 2, \ldots$$

Then a *column selection* for $\{A_k\}$ is a sequence of matrices $\{C_k\}$ with C_k $m \times r_k$ for which

$$\operatorname{image}[A_1C_1 \ldots A_kC_k] = \operatorname{image}[A_1 \ldots A_k]$$

for each k. The choice of C_k need be no more complicated than eliminating any column of $[A_1 \ldots A_k]$ which is linearly dependent on the columns which precede it. Now let $\{A_k\}$ be a sequence of $l \times n$ matrices and let

$$s_k = \operatorname{rank}\begin{bmatrix} A_1 \\ \vdots \\ A_k \end{bmatrix} - \operatorname{rank}\begin{bmatrix} A_1 \\ \vdots \\ A_{k-1} \end{bmatrix}, \quad k = 1, 2, \ldots$$

Then a *row selection* for $\{A_k\}$ is a sequence of matrices $\{R_k\}$ with R_k $s_k \times l$ for which

$$\operatorname{kernel}\begin{bmatrix} R_1A_1 \\ \vdots \\ R_kA_k \end{bmatrix} = \operatorname{kernel}\begin{bmatrix} A_1 \\ \vdots \\ A_k \end{bmatrix}$$

for each k. Again, the choice of R_k may just represent the elimination of any row of $\begin{bmatrix} A_1 \\ \vdots \\ A_k \end{bmatrix}$ which is linearly dependent on the rows which precede it.

The first step in the design is to find the deadbeat control law

(2) $$u_t = -Lx_t$$

as if the state were measurable. Such an L is defined by the requirement that $(F - GL)^k x = 0$ for every state x which can be driven to the origin in time k, and can be calculated via the following recursive procedure. Let $F_0 = F$ and let $C_{k1}, \ldots$ $\ldots, C_{kk}$ be column selection for $[F_{k-1}^{k-1}G \ldots F_{k-1}G \ G]$. Define matrices F_k, L_k, and N_k by

(3) $$\begin{aligned} N_k[F_{k-1}^{k-1}GC_{k1} \ldots F_{k-1}GC_{kk-1} \ GC_{kk}] &= [C_{k1} \ 0 \ldots 0] \\ L_k &= N_k F_{k-1}^k \\ F_k &= F_{k-1} - GL_k \end{aligned}$$

Then

(4) $$L = L_1 + \ldots + L_q$$

where q is the reachability index of the pair (F, G). If F is invertible, the recursive procedure can be shortcut by setting $F_{q-1} = F$.

The next step is to design the deadbeat observer. It is an $n - l$ dimensional system of the form

(5) $$\begin{aligned} z_{t+1} &= Tz_t + U_1 y_t + U_2 u_t \\ \hat{x}_t &= Vz_t + Wy_t \end{aligned}$$

producing an estimate $\hat{x}_t$ of x_t given $y_t, y_{t-1}, \ldots$ and $u_t, u_{t-1} \ldots$. Let H' complement H to a nonsingular matrix and denote

$$\begin{bmatrix} H \\ H' \end{bmatrix}^{-1} = [J \ J']$$

Then the observer matrices are

(6) $$\begin{aligned} T &= (H' - KH)FJ' \\ U_1 &= (H' - KH)F(J + J'K) \\ U_2 &= (H' - KH)G \\ V &= J' \\ W &= J + J'K \end{aligned}$$

where the observer gain matrix K is determined by the requirement that $T^k z = 0$ for every state z which can be driven to the origin in time k. Writing

$$T = \bar{F} - K\bar{H}$$

 for $\bar{F} = H'FJ'$ and $\bar{H} = HFJ'$, a recursive procedure dual to the one given in (3) can be applied. Let $\bar{F}_0 = \bar{F}$ and let $R_{k1}, \ldots, R_{kk}$ be a row selection for

$$\begin{bmatrix} \bar{H}\bar{F}_{k-1}^{k-1} \\ \vdots \\ \bar{H}\bar{F}_{k-1} \\ \bar{H} \end{bmatrix}.$$

Define matrices $\bar{F}_k$, K_k, and M_k by

(7)
$$\begin{bmatrix} R_{k1}\bar{H}\bar{F}_{k-1}^{k-1} \\ \vdots \\ R_{k\,k-1}\bar{H}\bar{F}_{k-1} \\ R_{kk}\bar{H} \end{bmatrix} M_k = \begin{bmatrix} R_{k1} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$K_k = \bar{F}_{k-1}^k M_k$$
$$\bar{F}_k = \bar{F}_{k-1} - K_k\bar{H}$$

Then

(8)
$$K = K_1 + \ldots + K_p$$

where p is the observability index of the pair $(\bar{H}, \bar{F})$. When $\bar{F}$ is invertible, the recursive procedure can be shortcut by setting $\bar{F}_{p-1} = \bar{F}$.

Having obtained the state-variable feedback and the observer, we implement the control law

(9)
$$u_t = -L\hat{x}_t$$

in place of (2). Defining the variable

$$e_t = z_t - (H' - KH)\,x_t$$

the resultant closed-loop system obeys the equation

(10)
$$\begin{bmatrix} x_{t+1} \\ e_{t+1} \end{bmatrix} = \begin{bmatrix} F - GL & -GLJ' \\ 0 & \bar{F} - K\bar{H} \end{bmatrix} \begin{bmatrix} x_t \\ e_t \end{bmatrix}$$

Thus the closed-loop system is the same as if the true state were fed back; however, it is no longer free (i.e., excited by x_0 only) but it is driven by the sequence e generated by a difference between x_0 and z_0. For reversible systems, $e_p = 0$ by construction of K and $x_{p+q} = 0$ by construction of L.

POLYNOMIAL EQUATION APPROACH

We shall now demonstrate how matrix polynomial equations can effectively be used to obtain the observer-based deadbeat controller in a single step. Write $u = \{u_t\}$ and $y = \{y_t\}$ for the input and output sequences, respectively, and introduce the delay operator d by the relation $dx_t = x_{t-1}$ for any sequence $\{x_t\}$.

The system (1) will now be described by the equation

$$A(d)\, y = B(d)\, u + C(d) \tag{11}$$

where A, B, and C is a triple of left coprime polynomial matrices in the delay operator d. The A is $l \times l$ with $A(0)$ invertible, the B is $l \times m$ with $B(0) = 1$, and the $l \times 1$ matrix C represents the effect of the initial state, x_0, on the system output. Passing from (1) to (11), we have the relationships

$$A^{-1}B = dH(I_n - dF)^{-1}\, G$$
$$A^{-1}C = H(I_n - dF)^{-1}\, x_0$$

where I_n is the $n \times n$ identity matrix.

The observer-based controller is another dynamical system of the form

$$P(d)\, u = -Q(d)\, y + R(d) \tag{12}$$

where P, Q, and R is a triple of left coprime polynomial matrices in the delay operator d. The P must be $m \times m$ with $P(0)$ invertible, the Q is $m \times l$, and the $m \times 1$ matrix R represents the effect of the initial state, z_0, on the controller output.

The deadbeat problem consists in transferring every initial state x_0 to the origin in a shortest time possible. The system (1) being completely controllable and constructible, this corresponds to making both input and output sequences finite and as short as possible. Moreover, the state feedback as well as the observer does not depend on x_0. This calls for the overall controller to be independent of C.

To start the derivation, introduce two right coprime polynomial matrices A_1 and B_1, respectively $m \times m$ and $l \times m$, such that

$$A^{-1}B = B_1 A_1^{-1} \tag{13}$$

These matrices serve to define the input-output equation of the system in the form

$$y = B_1(d)\, w\,, \quad u = A_1(d)\, w \tag{14}$$

where w is an internal variable. Further define two right coprime polynomial matrices P_1 and Q_1, respectively $l \times l$ and $m \times l$, by

$$P^{-1}Q = Q_1 P_1^{-1} \tag{15}$$

These matrices can be used to characterize the input-output behaviour of the controller as follows:

$$u = -Q_1(d)\, v\,, \quad y = P_1(d)\, v \tag{16}$$

where v is another internal variable.

For the sake of brevity we shall hereafter suppress the argument d. Substituting (16) into (11) one obtains

$$(AP_1 + BQ_1)\, v = C$$

and inserting (14) into (12) yields

$$(PA_1 + QB_1)\, w = R$$

Since

$$\begin{aligned} y &= P_1 v + B_1 w \\ u &= -Q_1 v + A_1 w \end{aligned}$$

we finally have

$$\begin{aligned} y &= P_1(AP_1 + BQ_1)^{-1}\, C + B_1(PA_1 + QB_1)^{-1}\, R \\ u &= -Q_1(AP_1 + BQ_1)^{-1}\, C + A_1(PA_1 + QB_1)^{-1}\, R \end{aligned} \tag{17}$$

Now, both u and y are to be finite sequences, i.e., polynomials in d, to satisfy the deadbeat requirement. Moreover, this is to hold true independently of C. Complete constructibility of our system implies that there is no factor common to all C's and hence the P_1 and Q_1 must satisfy the equation

$$AP_1 + BQ_1 = I_l \tag{18}$$

To each pair of matrices P_1, Q_1 satisfying (18) there exists a (unique) pair of matrices P, Q satisfying both (15) and the equation

$$PA_1 + QB_1 = I_m$$

because

$$\begin{bmatrix} A & B \\ -Q & P \end{bmatrix}\begin{bmatrix} P_1 & -B_1 \\ Q_1 & A_1 \end{bmatrix} = \begin{bmatrix} I_l & 0 \\ 0 & I_m \end{bmatrix}$$

Thus in view of (17) the equation (18) is necessary and sufficient to guarantee finite sequences u and y independently of C (and R).

Complete controllability of our system implies left coprimeness of A and B. Hence equation (18) has always a solution. However, not all solutions qualify for a deadbeat controller. Observing that

$$\begin{aligned} y &= P_1 C + B_1 R \\ u &= -Q_1 C + A_1 R \end{aligned} \tag{19}$$

and that both u and y are to be of least possible degree for every polynomial vector C, we must take the solution of (18) which minimizes the degree of each column of the matrix $\begin{bmatrix} P_1 \\ Q_1 \end{bmatrix}$.

Incidentally, such a solution may not be unique.

To summarize, the design procedure is seen to be extremely simple. Given a system (1) by polynomial matrices A and B, we have just to find a minimum-degree solution P_1, Q_1 of equation (18). Any minimal realization of (16) then gives the deadbeat controller.

SOLVING THE POLYNOMIAL EQUATION

A simple and efficient procedure for computing the minimum-degree solution of equation (18) will now be given in algorithmic form.

Step 1. Form the polynomial matrix

$$\begin{bmatrix} A & B \\ I_l & 0 \\ 0 & I_m \end{bmatrix} \tag{20}$$

Step 2. By elementary (unimodular) column operations, reduce (20) to the form

$$\begin{bmatrix} I_l & 0 \\ D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}$$

where the polynomial matrix $D = \begin{bmatrix} D_{12} \\ D_{22} \end{bmatrix}$ is column reduced, i.e., the real matrix consisting of the leading coefficients of the highest degree polynomials in each column of D has rank m. Clearly, any solution of (18) can be expressed in the form $D_{11} + D_{12}T$, $D_{21} + D_{22}T$ for some polynomial matrix T.

Step 3. For each column c_{i1} of $\begin{bmatrix} D_{11} \\ D_{21} \end{bmatrix}$ perform a sequence of division algorithms as follows. Denote D_1 the matrix obtained from D by deleting the columns whose degree exceeds the degree of c_{i1}, and calculate a quotient t_{i1} and remainder c_{i2} such that

$$c_{i1} = D_1 t_{i1} + c_{i2}$$

and degree $c_{i2} <$ degree D_1. Denote D_2 the matrix obtained from D_1 by deleting the columns whose degree exceeds the degree of c_{i2}, and calculate a quotient t_{i2} and remainder c_{i3} such that

$$c_{i2} = D_2 t_{i2} + c_{i3}$$

and degree $c_{i3} <$ degree D_2. Define D_3 similarly and continue the process until

$$\operatorname{rank} D_{k_i} < \operatorname{rank} [c_{ik_i} \ D_{k_i}]$$

Step 4. A desired minimum-degree solution P_1, Q_1 of (18) is then given by

$$\begin{bmatrix} P_1 \\ Q_1 \end{bmatrix} = [c_{1k_1} \ c_{2k_2} \ldots c_{lk_l}]$$

To illustrate the preceding theory we consider the deadbeat control problem for the system (1) given by

$$F = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$$

Let us first apply the state space technique. The recursive procedure (3) yields

$$L_1 = \begin{bmatrix} \beta & 1+\beta & 0 \\ \alpha & \alpha & 1 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 1-\beta & 1-\beta & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

for any reals α, β and hence all deadbeat state variable feedback gains are given by (4):

$$L = \begin{bmatrix} 1 & 2 & 0 \\ \alpha & \alpha & 1 \end{bmatrix}$$

To find the deadbeat observer (5), choose

$$H' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and calculate

$$J = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad J' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Applying the recursive procedure (7) to

$$\bar{F} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, \quad \bar{H} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

one obtains

$$K_1 = \begin{bmatrix} -1 + 2\gamma \\ 1 - \gamma \end{bmatrix}, \quad K_2 = \begin{bmatrix} 4 - 2\gamma \\ -2 + \gamma \end{bmatrix}$$

for any real γ. Thus the observer gain is given by (8):

$$K = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

and the observer matrices (6) read

$$T = \begin{bmatrix} -2 & -4 \\ 1 & 2 \end{bmatrix}, \quad U_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad U_2 = \begin{bmatrix} -3 & -3 \\ 1 & 2 \end{bmatrix}$$

$$V = \begin{bmatrix} 0 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad W = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}.$$

Implementing the control law (9), it follows from equation (10) that every state x_0 (and z_0) is transferred to zero in no more than four steps.

Let us now illustrate the polynomial equation approach. We first change the description (1) to (11) by calculating

$$A = 1 - 2d + d^3$$

$$B = [d - 2d^2 + d^3 \quad d - d^2 - d^3]$$

and for completeness,

$$C = (\xi_1 + \xi_3) - (2\xi_1 - \xi_2 + \xi_3)\, d + (\xi_1 - \xi_2 - \xi_3)\, d^2$$

where

$$x_0 = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix}$$

Next we solve the equation (18). Step 1 of the algorithm gives the matrix

$$\begin{bmatrix} 1 - 2d + d^3 & d - 2d^2 + d^3 & d - d^2 - d^3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and after performing Step 2 we have

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 - 15d & -2d + 3d^2 & d \\ 8 + 5d & 1 - d - d^2 & 0 \\ 9 - 10d & 1 - 3d + 2d^2 & -1 + d \end{bmatrix}$$

Thus any solution of (18) can be expressed in the form

$$P_1 = 1 - 15d + [-2d + 3d^2 \quad d] \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} \tag{21}$$

$$Q_1 = \begin{bmatrix} 8 + 5d \\ 9 - 10d \end{bmatrix} + \begin{bmatrix} 1 - d - d^2 & 0 \\ 1 - 3d + 2d^2 & -1 + d \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

where t_1 and t_2 are polynomials. Step 3 tells us that $1 - 15d$, $\begin{bmatrix} 8 + 5d \\ 9 - 10d \end{bmatrix}$ is already a minimum-degree solution; all such solutions are

$$(22) \qquad P_1 = 1 - (15 - \tau)\,d$$

$$Q_1 = \begin{bmatrix} 8 + 5d \\ (9 - \tau) - (10 - \tau)\,d \end{bmatrix}$$

on putting $t_1 = 0$, $t_2 = \tau$, an arbitrary real number. Any minimal realization of (22) then gives the desired deadbeat controller. The transient due to x_0 can be calculated from (19):

$$\begin{aligned} y = (\xi_1 + \xi_3) &+ [(\tau - 17)\,\xi_1 + \xi_2 + (\tau - 16)\,\xi_3]\,d - \\ &- [(2\tau - 31)\,\xi_1 - (\tau - 16)\,\xi_2 + (\tau - 14)\,\xi_3]\,d^2 + \\ &+ [(\tau - 15)\,\xi_1 - (\tau - 15)\,\xi_2 - (\tau - 15)\,\xi_3]\,d^3 \end{aligned}$$

and

$$\begin{aligned} u = &\begin{bmatrix} 8\xi_1 + 8\xi_3 \\ (9 - \tau)\,\xi_1 + (9 - \tau)\,\xi_3 \end{bmatrix} + \begin{bmatrix} -11\xi_1 + 8\xi_2 - 3\xi_3 \\ (3\tau - 18)\,\xi_1 - (\tau - 9)\,\xi_2 + (2\tau - 19)\,\xi_3 \end{bmatrix} d - \\ &- \begin{bmatrix} 2\xi_1 + 3\xi_2 + 13\xi_3 \\ (3\tau - 29)\,\xi_1 + (2\tau - 19)\,\xi_2 + \xi_3 \end{bmatrix} d^2 + \\ &+ \begin{bmatrix} 5\xi_1 - 5\xi_2 - 5\xi_3 \\ (\tau - 10)\,\xi_1 - (\tau - 10)\,\xi_2 - (\tau - 10)\,\xi_3 \end{bmatrix} d^3 \end{aligned}$$

The transfer matrix, M, of the observer-based controller calculated via state space techniques equals

$$M = (I_m + LM_2)^{-1}\,LM_1$$

where

$$M_1 = W + dV(I_{n-l} - dT)^{-1}\,U_1$$

$$M_2 = \ dV(I_{n-l} - dT)^{-1}\,U_2$$

In our case

$$M = \begin{bmatrix} 8 + 5d \\ (5\alpha - 1) - 5\alpha d \end{bmatrix} [1 - (5 + 5\alpha)\,d]^{-1}$$

which corresponds to $Q_1 P_1^{-1}$ in (22) on identifying $\tau = 10 - 5\alpha$.

Note the reduction of order in the controller: the deadbeat observer needs two dynamical elements but for the entire controller one is enough. This means that the observer/state feedback structure is not minimally realized; in fact, one finite mode of the observer is unobservable at the system input. The explanation is at hand: the observer reconstructs the entire state x_t while the overall controller reconstructs just Lx_t, a linear functional of the state.

An alternative method of designing observer-based deadbeat controllers for system (1) has been proposed. The procedure starts with the system description (11) and consists in solving the linear equation (18) in polynomial matrices. In this way the entire deadbeat controller, incorporating the observer plus state feedback, is obtained in a single step. This technique is believed to be computationally superior to the existing state space methods; a detailed discussion on the relevant algorithms can be found in Kučera [2] and Wolovich [7].

These results, when combined with those published earlier by the author in [3], lead to the following conclusions. The polynomial equation approach unifies (and simplifies) the theory as well as the design of deadbeat controllers: a simple equation like (18) is to be solved no matter whether the system's state is measurable or not. In the case of unmeasurable states, the approach advocated here provides often a more economical realization of the controller than deadbeat observers do.

The reader may wish to know the interpretation of the nonminimal solutions to equation (18). Well, each solution corresponds to an observer-based controller which drives the initial state to zero in a finite (not necessarily minimal) time. For instance, a full order ($=n$) observer for system (1) is described by

$$\begin{aligned} z_{t+1} &= (F - KH)\, z_t + K y_t + G u_t \\ \hat{x}_t &= z_t \end{aligned}$$

When this observer is used to implement the control law (9) in our Example, we obtain the overall controller transfer matrix

$$M = \begin{bmatrix} 13d + 8d^2 \\ (8\alpha - 1)\, d - 8\alpha d^2 \end{bmatrix} \left[1 + 2d - (8 + 8\alpha)\, d^2\right]^{-1}$$

This controller can be recovered from the general solution (21) by setting $t_1 = -8$, $t_2 = 1 + (16 - 8\alpha)\, d$. The resulting transients are of course suboptimal.

(Received January 21, 1980.)

REFERENCES

[1] J. Ackermann: Abtastregelung. Academic Press, New York 1972.

[2] V. Kučera: Discrete Linear Control — The Polynomial Equation Approach. Wiley, Chichester 1979.

[3] V. Kučera: Polynomial design of deadbeat control laws. Kybernetika *16* (1980), 2, 198–203.

[4] H. Kwakernaak and R. Sivan: Linear Optimal Control Systems. Wiley, New York 1972.

[5] C. T. Mullis: Time optimal discrete regulator gains. IEEE Trans. Automatic Control *AC-17* (1972), 3, 265–266.

[6] V. Strejc: State Space Theory of Discrete Linear Control. Wiley, Chichester, 1980.

[7] W. A. Wolovich: Linear Multivariable Systems. Springer, New York 1974.

Ing. Vladimír Kučera, DrSc., Ústav teorie informace a automatizace ČSAV (Institute of Information Theory and Automation — Czechoslovak Academy of Sciences), Pod vodárenskou věží 4, 182 08 Praha 8. Czechoslovakia.