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Solving of Heat Shock on a Hybrid System*)

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The paper deals with solving of heat conduction by a large diameter cylindrical wall described by means of a diffusion equation.

INTRODUCTION

The paper deals with the question of heat conduction by a cylindrical wall, the conduction being described by a parabolic partial differential equation of heat conduction in an idealized rod with a boundary condition of the third kind (Robin's problem). The entire transfer phenomenon is studied through the implementation of the classical CSDT method (continuous space – discrete time). The start of the transitory phenomenon is studied by the decomposition method according to Silvey and Barker. The resulting temperature courses can be used for the investigation of heat stress in materials.

1. FORMULATION OF THE TASK

In dealing with the problem of heat conduction by a cylindrical wall of large diameter (300 mm) on account of the ratio of the wall thickness (10 mm) and the cylinder diameter the diffusion equation in form (1) can be used for the mathematical description. The influence of the curvature radius of the cylindrical surface and of the exterior surface of the cylinder can be neglected. The solution gives results sufficiently acurate for technical practice.

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The phenomenon in question is described by the heat conduction equation (diffusion equation) in the form

$$\frac{\partial \vartheta(x, t)}{\partial t} = a \frac{\partial^2 \vartheta(x, t)}{\partial x^2} \tag{1}$$

where: ϑ – the temperature in the place x at the time t,
x – the space variable,
t – the time,
a – the factor of heat conduction.

Initial condition:

$$\vartheta(x, 0) = f(x)$$

is a straight line dropping in the direction of axis x. Its value on the exterior edge of the cylinder must satisfy the second boundary condition. At temperature $f(0) = 100\ °C$ the temperature in point L equals $f(L) = 75\ °C$.

Boundary Conditions

The first boundary condition determining the temperature inside the cylinder is

$$\vartheta(0, t) = \mu_1(t)$$

For $\mu_1(t)$ the following holds good:
in the interval $0 < t \leqq 2\text{s}$

$$\mu_1(0) = 100 \text{ deg} ; \quad \frac{d\mu_1}{dt} = 100 \text{ deg/s} ;$$

for $t > 2\text{s}$

$$\mu_1(t) = 300 \text{ deg} .$$

The other boundary condition $\mu_2(t)$ describing the temperature conditions at the limit solid matter – air is given in the form

$$\left. \frac{d\vartheta(x, t)}{dx} \right|_{x=L} = - \frac{\alpha}{\lambda} [\vartheta(L, t) - \vartheta_0] \tag{2}$$

where λ – the factor of the heat conductivity of the material,
α – the specific cooling capacity of the environment,
ϑ_0 – the temperature of the environment.

Given values of the constants:

$$a = 0{,}167\ \mathrm{mm^2/s} \qquad \alpha = 7\ \ \mathrm{W/m^2\,deg}$$
$$L = 10\ \mathrm{mm} \qquad \lambda = 0{,}3\ \mathrm{W/m\,deg}$$
$$\vartheta_0 = 20\ \mathrm{deg}$$

2. SELECTION OF THE METHOD OF SOLUTION

Considering the requirements of the task in question, especially taking account of the boundary condition of the third kind (Robin's problem) and the knowledge of the heat profile in the direction of axis x, the CSDT method appears to be the most suitable. This method described in literature [5] and [6] and also in some works of the authors [2], [3] and [4] requires the utilisation of a hybrid computer. For dealing with this task we made use of the hybrid computer system AP 3M – RC 1000/22 – GIER. On account of the unstable program circuit diagram it is not possible with the above CSDT method to choose Δt lower than 50 seconds. To obtain heat courses at lower intervals ($\Delta t = 1$ second) one of the CSDT methods should be used which do away with the instability of the program circuit diagram. In our case we used the decomposition method according to Silvey and Barker [10].

3. THE CSDT METHOD – CONSTRUCTION OF EQUATIONS

By applying the CSDT method directly to the equation (1) we get

$$\frac{\mathrm{d}^2\vartheta_i(x)}{\mathrm{d}x^2} = \frac{1}{a\,.\,\Delta t}\left[\vartheta_i(x) - \vartheta_{i-1}(x)\right]. \tag{3}$$

In order that the speed of convergence of the iterating process necessary for ensuring the fulfilment of the other boundary condition be maximal we must, along with equation (3), deal also with the sensitivity equation which we get through a partial derivation of the equation (1) according to the $\dot{\vartheta}_0$ parameter where

$$\dot{\vartheta}_0 = \left.\frac{\mathrm{d}\vartheta}{\mathrm{d}x}\right|_{x=0}.$$

This method is given more in detail in [1], [7], [8], [9]. We shall give here only the last relation

$$\frac{\partial^2 w}{\partial x^2} = \frac{1}{a}\,.\,\frac{\partial w}{\partial t}, \tag{4}$$

where

$$w = \frac{\partial\vartheta}{\partial\dot{\vartheta}_0}.$$

After arrangement into the difference-differential relation and applying the CSDT method we get

(5)
$$\frac{d^2w}{dx^2} = \frac{1}{a \cdot \Delta t}(w_i - w_{i-1}) .$$

For the initial conditions of w_i the following will hold

(6)
$$\frac{\partial \dot{\vartheta}_{i0}}{\partial \dot{\vartheta}_{i0}} = \dot{w}_{i0} = 1 , \quad \frac{\partial \vartheta_{i0}}{\partial \dot{\vartheta}_{i0}} = w_{i0} = 0 .$$

The deviation in point $x = L$ is defined

(7)
$$\dot{\vartheta}_i + \frac{\alpha}{\lambda}[\vartheta_i(L) - \vartheta_0] = \varepsilon ,$$

where the value $\vartheta_i(L)$ is the correct value of the boundary condition derived from the equation (2). The initial value of temperature derivation according to the space variable for the individual iteration steps is obtained from the relation

(8)
$$\dot{\vartheta}_{i0}^{(k+1)} = \dot{\vartheta}_{i0}^{(k)} + \delta\dot{\vartheta}_{i0}^{(k)}$$

where k is the number of iterations.
The increase of the initial condition is calculated:

(9)
$$\delta\dot{\vartheta}_{i0} = - \frac{\sum_{i=1}^{H} g_i \varepsilon_i \frac{\partial \varepsilon_i}{\partial \dot{\vartheta}_{i0}}}{\sum_{i=1}^{H} g_i \frac{\partial \varepsilon_i}{\partial \dot{\vartheta}_{i0}}}$$

Program circuit diagram is on Fig. 1.

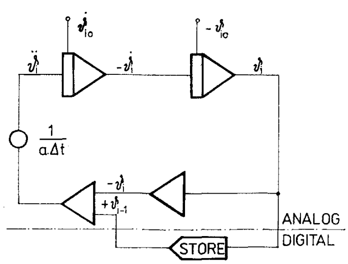

Fig. 1.

4. DECOMPOSITION METHOD ACCORDING TO SILVEY AND BARKER. CONSTRUCTION OF EQUATIONS

In clarifying the method we may start from the equation (3) which is arranged as

$$\frac{d^2\vartheta_i}{dx^2} - \frac{\vartheta_i}{a\,.\,\Delta t} = -\frac{\vartheta_{i-1}}{a\,.\,\Delta t}\,. \tag{10}$$

The left-hand side of the equation (10) is distributed into form

$$\frac{d^2\vartheta_i}{dx^2} - \frac{\vartheta_i}{a\,.\,\Delta t} = \left(\frac{d}{dx} + \frac{1}{\sqrt{(a\,.\,\Delta t)}}\right) . \left(\frac{d\vartheta_i}{dx} - \frac{\vartheta_i}{\sqrt{(a\,.\,\Delta t)}}\right). \tag{11}$$

Transferring equation (11) into equation (10) we get

$$\left(\frac{d}{dx} + \frac{1}{\sqrt{(a\,.\,\Delta t)}}\right) . \left(\frac{d\vartheta_i}{dx} - \frac{\vartheta_i}{\sqrt{(a\,.\,\Delta t)}}\right) = -\frac{\vartheta_{i-1}}{a\,.\,\Delta t}\,. \tag{12}$$

We include into the calculation the auxiliary variable u_i which we define

$$\frac{d\vartheta_i}{dx} - \frac{\vartheta_i}{\sqrt{(a\,.\,\Delta t)}} = \frac{u_i}{\sqrt{(a\,.\,\Delta t)}}\,. \tag{13}$$

Introducing u_i into equation (12) we get

$$\frac{du_i}{dx} + \frac{u_i}{\sqrt{(a\,.\,\Delta t)}} = -\frac{\vartheta_{i-1}}{\sqrt{(a\,.\,\Delta t)}}\,. \tag{14}$$

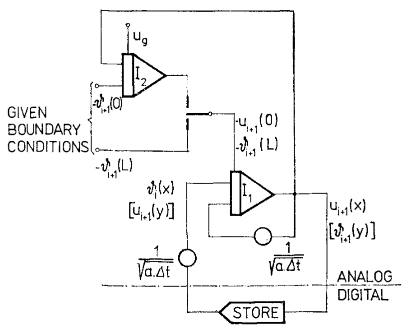

Fig. 2.

Equation (13) is unstable. By introducing $y = L - x$, $dy = -dx$ we get from equation (13) a stable equation in a change of the space direction of integration

$$\frac{d\vartheta_i}{dy} + \frac{\vartheta_i}{\sqrt{(a \cdot \Delta t)}} = -\frac{u_i}{\sqrt{(a \cdot \Delta t)}} . \tag{15}$$

The solution obtained by this method has a much more complex algorithm than the CSDT method.

Program circuit diagram is on Fig. 2.

5. RESULTS

The results are curves representing the course of the temperature of the material as a function of the space variable in the individual time intervals $\Delta t = 1$ s and $\Delta t = 50$ s.

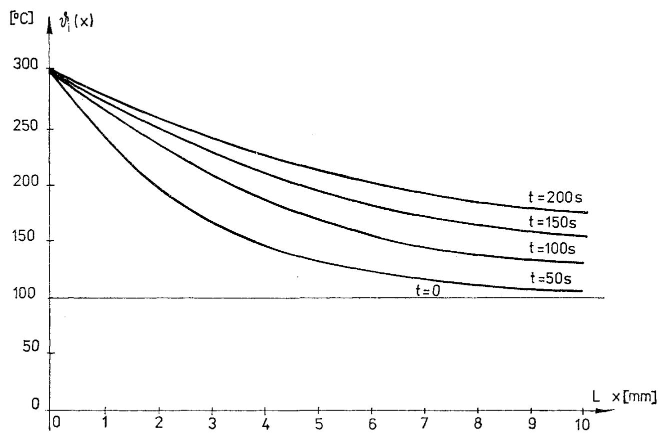

Fig. 3.

The courses of temperatures are on Fig. 3 — Classical CSDT methods, $\Delta t = 50$ s; Fig. 4 — Decomposition method $\Delta t = 1$ s, classical CSDT method $\Delta t = 50$ s.

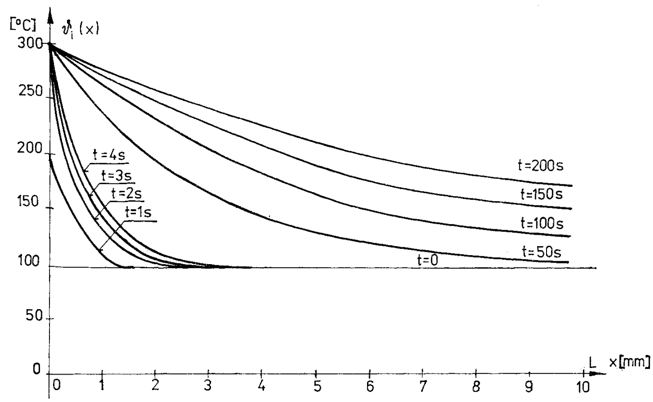

Fig. 4.

6. CONCLUSION

Obtaining the courses of temperatures in relation to the space variable is very advantageous for the further processing of the results. From the courses it is possible to solve the mechanical strain in the material. For the investigation of settled states it is suitable to use the classical CSDT method ($\Delta t = 50$ s). With this method it is not possible, on account of the unstability of the program circuit diagram, to select any small Δt. That is why for the investigation of quick transferance phenomena in this task the decomposition method according to Silvey and Barker was used [10] this method doing away with the instability of the program circuit diagram. In the decomposition method the temperature courses in the material are solved for a time interval of $\Delta t = 1$ s.

The utilization of two methods in dealing with the above problem was conditioned by the necessity of knowing the entire transferrance phenomenon which lasts relatively long and by need of investigating the courses of temperature at the beginning of the phenomenon at the so-called heat shock.

This problem could be dealt with the decomposition method alone, according to Silvey and Barker. This method is, however, much more complicated, being there a greater number of steps in the accessible hybrid system, it is also less exact and much more exacting as to time than the classical CSDT method. That is why from an overal aspect (economy, precision) it is suitable to combine both the above methods.

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