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Semantic Evaluation of Prognostic Statements on the Base of Probabilistic Parameters

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The presented paper introduces some measures for the semantic evaluation of data used in prognostic procedures. The calculation of one of these measures, i.e. the calculation of the *G*-function is demonstrated by means of graphic form.

1. THE CONCEPT "PROGNOSTIC STATEMENT"

The interest in logical-semantic and semantic-information analysis of prognostic statements can be substantiated not only by the development of prognostics as such but also by the occurrence of prognostic statements in a number of empirical disciplines where it has proved inevitable to operate with statements on the future states of the systems under investigation. Neither is it possible to overlook the fact that the prediction of future states is one of the important cognitive aims of these disciplines.

The concept "prognostic statement" which will be analysed in this paper calls for a certain explication. First of all, it should be stressed that the grammatical future alone is no guarantee that a prognostic statement is involved. By prognostic statement we understand a statement made for the purpose of determining the future state of the system under investigation, future events within the given "univers de discours", etc.

From the point of view of form, three distinct types of prognostic statement may be distinguished:

I. *simple prognostic statements*, e.g. "Tomorrow it will rain", "In 1978, probes will be launched to the most distant planets of our solar system", etc.;

II. *conditional prognostic statements*, e.g. "If the air pressure today falls by another 5 mm, it will rain tomorrow", "If we get new co-workers next year, we shall be able to fulfil the given task", etc.;

III. *justified prognostic statements*, e.g. "Tomorrow it will rain because the air pressure today has fallen by 5 mm", "In the coming years, the demand for new furniture will double with regard to the present state because, in comparison with the present time, twice as many new flats will be completed".

It is, naturally, quite possible to imagine a combination of conditional and justified prognostic statements, e.g.: "If event A happens in time t, event B will occur after the time interval Δt, because so far the occurrence of an event of the type A has always been followed by an event of the type B."

Prognostic statements can further be classified also according to the degree of certainty with which a possible fact mentioned in a prognostic statement is expected. Intuitively it is quite obvious that the degree of certainty greatly varies in the following statements:

"At the end of this century, human civilization will be destroyed."

"Next month it will be foggy."

"On May 25, 1975, phenomena of this or that kind will be visible in the clear sky of the Northern hemisphere."

The degree of certainty with which we expect a possible fact referred to in the prognostic statement is, in current language, usually expressed in a qualitative manner, e.g. by the words "certainly", "likely", "very probably", etc. Neither is it impossible to determine the degree of certainty by using comparative expressions facilitating the ordering of possible facts according to the degree of certainty with which they may be expected.

The most advantageous form of determining the degree of certainty is, of course, quantitative determination, even though it is possible only in some cases; it is connected with a number of problems and difficulties, If the degree of certainty with which a possible fact may be expected can be determined quantitatively, the difference among the above-mentioned types of prognostic statements can be expressed with the help of the following schematic examples:

(I) With likelihood r $(0 \leqq r \leqq 1)$ we expect that in the time t, following after asserting this statement after the interval Δt, the system S will be in the state S_t.

(II) With the likelihood r $(0 \leqq r \leqq 1)$ we expect that, if in the time t following after asserting this statement after the interval Δt, an object occurs having the property A (or if we create objects with the property A etc.), we shall be able to ascertain the property B in the same object.

(III) With the likelihood r $(0 \leqq r \leqq 1)$ we expect that in the time t following after asserting this statement after the interval Δt an object with the property B will occur, because... (The expression following after the word "because" should justify the given prognostic statement, including its degree of certainty. This means that it is adequate to suppose that a nomological statement is involved, expressing certain scientific laws, hypotheses, empirical generalizations, etc.)

In literature we also find other attempts at a typology of prognostic statements. For instance, distinction is drawn between prognostic statements which can be rationally justified regardless of whether type I, II or III are concerned, and prognostic statements for which no rational justification is available. For denoting the latter, the term "prophecy" is sometimes used. Prophecies were e.g. the statements of the augurs and haruspices of Ancient Rome; many statements of contemporary publicists are close to prophecy, aiming rather at influencing present conditions and decisions than at a precise and reliable definition of future facts. The term "prophecy" is somethimes found in literature to describe prognostic statements concerning the systems beyond human influence. (Such an explication of the term "prophecy" can be found e.g. in K. Proper's work [12]). As distinct from this, prognostic statements concerning the systems which man is capable of influencing by his activity are characterized as "technical forecasts". However, such a distinction is inadequate because even some highly reliable and precise forecasts, e.g. in astronomy, have to be denoted as "prophecies", which is contrary to intuitive comprehension. This, however, is not meant to deny the usefulness of distinguishing between the so-called technical forecasts and forecasts relating to systems in whose development we are unable to interfere.

A further possibility is to differentiate prognostic statements according to the nature of the data they include. Naturally, we may refer to prognostic statements only in case they contain a purposeful definition of future facts, events, etc. The time in future can be determined with the help of data which are chronologically stable (e.g. "December 1, 1990) or pseudodata (e.g. "tomorrow", "by the end of next month", etc.) which are chronologically unstable.*

In the case of pseudodata, the chronological determination of the future fact is relativized with regard to the time when the prognostic statement is made. This means that the decisions of prognostic statements depend on whether also the time to which the relativization corresponds is known.

On the basis of the characteristics of the prognostic statements described so far it may be concluded that, in analysing prognostic statements, at least their following components must be respected: (a) the chronological determination of the future fact, (b) the degree of certainty with which we expect the future fact to occur, (c) the chronological determination of the time of asserting the prognostic statement. While in any prognostic statement the first component is always stated explicitly, be it in the form of data or pseudodata, the second and the third components need not always be explicitly stated.

With regard to the said components of prognostic statements, their logical analysis may be said to fall into two spheres of contemporary logic – i.e. the sphere of chronological logic** and the sphere of modal logic, in particular the sphere of epistemic

* On distinguishing data and pseudodata, see [15, p. 201].

** The term "chronological logic" is not yet firmly established. In Anglo-Saxon literature, we find the terms "tense logic" (A. N. Prior), "temporal modalities" (N. Rescher), "chronological logic" (N. Rescher), "calculus of 'before' and 'after'" (G. E. Anscombe, D. R. Luce), a.o. In Russian logical literature we find the term "logika vremeni" (A. Ivin).

modalities. For these reasons, prognostic statements may be considered as epistemological-chronological statement of of a special type. This basic characteristic of prognostic statements must be taken into account by any semantic analysis of these statements.

2. SEMANTIC PROBLEMS OF PROGNOSTIC STATEMENTS

The semantic decisions of prognostic statements depend on whether a *chronologically definite* or a *chronologically indefinite* statement is involved.* Chronologically definite statements are those chronological statements the semantic decisions of which (denoted by Rescher as their truth) do not depend on the time when the chronological statement is asserted, for instance:

"In 1348, the Prague university was founded."

"In 1974, the first trains of Prague underground will run."

"In 1990, the problem of food shortage in the world will be solved."

Chronologically indefinite are those chronological statements the semantic decisions of which depend on whether the time of asserting the statement can be determined. In other words, in chronologically indefinite statements it is necessary to take into account the component (c). Chronologically indefinite statements are for instance:

"Next year, a further 30 kilometres of the speedway between Prague and Brno will be completed."

"Last month, the number of motor accidents increased by twenty per cent."

"By the end of this year, the construction of the new hospital in our town will be finished."

Though in chronologically indefinite statements it is usual to operate with pseudodata and in chronologically definite statements with data, these two distinctions are not fully identical. It is possible to imagine a chronologically definite statement operating with pseudodata, e.g.:

"On October 3, 1973, the meteorologists stated that fog may be expected next week."

The problems of semantic decisions of prognostic statements are a relatively complex matter and cannot be reduced to decisions of current statements. Even if some authors refer to the truth of chronological statements, it is obvious that we can speak of truth stricto sensu only in the case of chronological statements relating to present or past facts. Since prognostic statements are supposed to relate to future facts, the choice of other characteristics than truth or falsity will be useful. These characteristics might be linked with the second component of prognostic statements, i.e. the degree of certainty with which we expect the future fact to occur. With a view to this second component, prognostic statements represent a specific kind of non-extensional con-

* On distinguishing between chronologically definite and chronologically indefinite statements see [15, p. 200].

text; this is in connection with the modal character of the expressions used to describe the degree of certainty. Therefore it is usual, in connection with prognostic statements, to speak of likelihood, credibility, reliability, etc.

The semantic analysis of prognostic statement cannot avoid the question of the extent to which the applicability of sense and denotation may be taken into account – i.e. the applicability of such semantic categories as logical semantics has been operating with ever since the days of Frege and other pioneers in this field. More easily solvable is the question of the sense of prognostic statements. If we consider sense – as is done currently in logical semantics – to be a category of a conceptual nature, i.e. an invariant which remains unchanged in any correct translation, then it is easy to prove that sense may be assigned to any meaningful prognostic statement we are able to understand. (It is of no importance whether, in this connection, we refer to a "prognostic proposition" or to other analogous categories of a conceptual nature.)

Though sense may be assigned to prognostic statements without great difficulty, this does not apply to the denotation of prognostic statements. Difficulties arise in particular if the problem of denotation of prognostic statements is conceived as the question of what "corresponds" to prognostic statements, what is "reflected" by them, what extra-linguitic objects are assignable to prognostic statements, etc. In other words, if entities are to be determined in some extra-linguistic universe which could be considered as donatations of prognostic statements, we find that the solution of this task is not at all unambiguous.

First of all, it is necessary to verify to what extent the well-known procedure of Tarski can be applied in determining the denotation of prognostic statements; according to this procedure e.g. the statement "the sun shining" is true if and only if the sun is shining, i.e. the fact that the sun is shining is provable.* On the assumption that facts provable in the extra-liquistic universe can be assigned to true statements, we may ask what facts can be assigned to prognostic statements. Because "facts which have not yet occurred" even if we expect them to occur cannot be considered as identical with facts which we consider to be legitimately assignable to current (indicative) statements, the above-mentioned procedure is evidently unsuitable. Moreover, because "facts which have not yet occurred" are actually "non-facts", a situation arises which is utterly contradictory to the mentioned procedure. Nor will these difficulties be overcome by considering the truth value as the denotation of the statements, which is in accordance with the line of development in logical semantics represented in particular by Frege, Carnap and Church. As was already pointed out, it is possible to consider the truth or falsity of some chronological statements but it is hardly possible to consider the truth or falsity of prognostic statements.

* This additional note to the well-known procedure of A. Tarski corresponds to the procedures used by A. A. Zinovjev [17]. In case this note is omitted it is necessary explicitly to determine the difference between object-language and metalanguage, or between two different modes of using a given text, e.g. by distinguishing the mode de dictu and de re, etc.

If – despite of the mentioned defficulties – it is desirable to consider the denotation of prognostic statements, the following solutions are possible:

(1) Something that could be characterized as the "states of mind" of the persons asserting prognostic statements may be assigned to these statements. Such a solution seems plausible especially if we require "something" in the actual world to correspond to the prognostic statements, i.e. hic et nunc. Though we may admit that what could be characterized as "states of mind" or something similar of the kind is of the nature of extra-linguistic entities presumable in the actual world, it is difficult to prove the presence of entities otherwise than by some – usually linguistic – behaviour. This also shows the absolute unreliability of such subjectivistic approach to the reconstruction of denotations of prognostic statements.

(2) That which could be characterized in the spirit of the Leibnizian tradition in semantics as "states of the possible world", might be assigned to prognostic statements. This form of semantic analysis was elaborated especially for modal logic. As J. Hintikka [8, p. 81] pointed out, "possible worlds" or " states of possible worlds" can be interpreted as states in various moments of time. This facilitates the interpretation of chronological logics as special cases of modal logics, which is current in a number of systems of chronological logics. In this connection Hintikka, who worked out a remarkable system of semantics for modal logics based on the mentioned traditions, pointed out the particular role of the relation of alternativeness. Just as in modal logics various states of "possible worlds" are certain alternatives of the actual world or of a differently conceived original world, also in chronological statements a relation having an analogical role should be taken into account. (J. Hintikka recommends the term "futurity relation".) It is obvious that the minimum demand made on this relation is the demand of transitivity.

This briefly outlined approach to the semantics of chronological statements is suitable wherever a system of simple chronological statements, conceived as a fragment or a special case of modal logic, is involved. However, because in empirical and experimental sciences, including prognostics, primary attention is paid to justified prognostic statements, the demand should be raised that whatever corresponds to Hintikka's "futurity relation" should have some further qualities. For this reason, a third solution – which of course might be considered as a certain modification of the second solution – is presented.

(3) "States of the possible world" that can be decided upon on the bais of available knowledge, i.e. knowledge available "hic et nunc", may be considered as denotations of prognostic statements. The term "deciding on the basis of available knowledge" can, of course, be interpreted in various ways. For instance, if available knowledge includes certain nomological statements, i.e. statements of scientific laws, hypotheses or empirical generalizations and statements of the respective empirical data on the basis of which prognostic statements can be deduced in the sense of Hempel's deductive-nomological model of explanation and prediction, then the term "deciding on states" may be interpreted as "deducing statements on states". However, since

the deductive-nomological model of explanation and prediction is merely one abstract model from among a whole range of possible procedures, which may also include probabilistic dependencies and statistical laws, it is better to keep to the more general term "deciding".

This conception of the denotation of prognostic statements is particularly suitable for situations involving rationally justified prognostic statements that are based on the knowledge of the respective scientific laws and empirical data concerning initial conditions. A prognostic statement conceived in this way could then be schematically expressed as follows: S_i, since S_t and S_e, where S_i is the prognostic statement, S_t is the nomological statement or the class of nomological statements (i.e. theory), and S_e is the statement of the respective empirical data concerning initial conditions.

Wherever the deductive nomological model of explanation and prediction is applicable, the mentioned scheme of the justified prognostic statement corresponds to the scheme of prediction, i.e.

$$S_t \,.\, S_e \rightarrow S_i \,.$$

S_i denotes the possible state to be decided upon on the basis of S_t and S_e. In other words, the denotation of S_i is relativized to the possibilities admitted by S_t and S_e.

Further, it is necessary to consider the extent to which an extra-linguistic entity corresponding to the truth value of current indicative statements cannot be assigned to rationally justified prognostic statements. Since – as has been mentioned above – the denotation of a prognostic statement is relativized to the possibilities admitted by the nomological statements and the statements of the respective empirical data, the semantic category corresponding to the truth value must be relativized with regard to what is presented by the justification (explicit or implicit) of the prognostic statement. For this reason, the concept "*nomological-empirical support*" will be introduced and an attempt made at indicating the quantitative measures of this concept. At the same time, we shall bear in mind that a semantic category is involved which, in the case of justified prognostic statements, can assume a position analogical to that of the truth value of indicative statements related to the actual world.

3. SOME COMMENTS ON CHRONOLOGICAL LOGICS

As has already been mentioned above, the problems of prognostic statements and particularly the semantic analysis of prognostic statements are linked with the wide sphere of chronological logics even if, of course, these two areas do not fully coincide. Moreover, it must not be forgotten that prognostic statements are connected also with other section of contemporaty logic. Thus, for instance, it is known that the need of the semantic characteristics of prognostic statements was one of the stimuli for criticizing two-value logics and Lukasiewicz's attempt at construing a three-value logic.

In contemporary chronological logics, particular attention is paid, on the one hand, to the concepts "before", "later", "at present" or to some modifications there of, and, on the other hand, to such concepts as "will be", "always will be", "has been", "always has been", etc. As for the first group of concepts which are usually conceived as predicates in languages representing the applications of modern logic on physical, biological or psychological systems, the systems worked out by R. Carnap for physics [2], [4] and by J. J. Wodger for biology must not be overlooked. Carnap e.g. elaborated axioms and theorems of the so-called $K - Z$ system where in K is the predicate of time coincidence and Z the predicate of time anteriority or posteriority. The pair or predicates $\langle K, Z \rangle$ represents a special case of a pair which, in the given universe, forms a quasi-series, facilitating thereby a time ordering of all the objects of the given universe.

Somewhat different are the systems concerning the second group of concepts, i.e. "has been", "always has been", "will be", "always will be", etc. As a rule, such systems are constructed so that these concepts are conceived as modal operators. The best-known systems of this kind are those of A. N. Prior [13], [14]; they are based on the modal systems of Lewis and Lukasiewicz, as well as on the ideas of Diodor of Krone concerning the concepts of "possibility" and "necessity". These axiomatically constructed systems actually define with more precision the meaning of the so-called weak and strong future ("always will be") and weak and strong past, and, on this basis, the meaning of further modal concepts.

A further way of constructing chronological logics has been indicated by N. Rescher [15]. Rescher starts from the concept of "chronological realization" which he regards as an operator with the statements. If p is a chronologically indifinite statement, then the operator for chronological realization R forms a new statement on the chronological realization of that to which p relates. "$R_t(p)$" then means that "p is realized in time t". The properties of the concept "chronological realization" are then presented by means of axioms in the language whose vocabulary forms the statements (variables) $p, q, r, \ldots$, the variables for data $t, t', \ldots$, etc., usual connectives, usual quantifiers over data variables, and finally the operator for chronological realization R. The properties of the concept "chronological realization" are then given by the following axioms:

$$R_t(\sim p) \equiv \sim R_t(p) , \tag{A1}$$

$$R_t(p \,.\, q) \equiv [R_t(p)] \,.\, [R_t(q)] , \tag{A2}$$

$$(\forall t)\, R_t(p) \rightarrow p , \tag{A3}$$

$$R_{t'}[(\forall t)\, R_t(p)] \equiv (\forall t)\, R_t(p) , \tag{A4}$$

$$R_{t'}[R_t(p)] \equiv R_t(p) . \tag{A5}$$

A different system of chronological logic os formed if (A5) is replaced by another axiom (A5*):

$$(\text{A5*}) \qquad R_{t'}[R_t(p)] \equiv R_{t'+t}(p)\,.$$

Both these systems of chronological logic represent variants of the systems offered by Prior's chronological approach to modal logic. As shown by Rescher, also the systems based on the concept of chronological realization may introduce the modal concepts of "possibility" (M) and "necessity" (N) which may then be defined as follows:

$$M(p) = (\exists t)\, R_t(p)\,,$$

$$N(p) = (\forall t)\, R_t(t)\ .$$

These or analogical approaches to chronological logic representing an interesting fragment of non-classical logic can specify the semantic properties of some terms occurring in prognostic statements, e.g. of the terms "will be", "will be realized in time t, that ...", etc. However, it is obvious that such approaches cannot present a semantic appreciation of prognostic statements as a whole. For this reason, a different starting-point should be chosen for construing the criteria of evaluating prognostic statements.

4. EVALUATION OF PROGNOSTIC STATEMENTS

The evaluation of prognostic statement may start with the question "Why will S_i be?"* The reply will the be: Because S_h and S_e, where S_h is a nomological statement or a class of nomological statements and S_e is a statement of empirical data concerning initial conditions. However, as has been pointed out by W. Stegmüller [16, p. 171], the why-question relating to future events may be conceived as having two forms: in the first case, we simply ask "Why will S_i be?"; in the second case we explicitly concentrate on the epistemical component of the prognostic statement, e.g. "Why do you think that S_i will be?", etc. As a matter of fact, in both cases a justification of the prognostic statement is required; in the first case, however, abstraction is made from the epistemical component "you consider", "you believe", "you are convinced", etc., whereas, in the second case, this component is explicitly stated. The distinction between the two mentioned forms of why-questions relating to prognostic statements, however, is merely a distinction of abstract types. Concrete procedures leading to rationally justified prognostic statements – such as are found in prognostics** – operate either by directly stating scientific laws, hypotheses, empiri-

* This is an epistemic why-question as distinct from explanation-seeking why-questions where S_i has the character of a true assertion.

** A survey of these prognostic methods and procedures is given e.g. by E. Jantsch [9].

cal generalizations and trends or by the judgements of competent experts, by the coincidence of these judgements, etc., without explicitly stating the actual basis for prediction. Thus, for instance, prognostic statements are solicited with the help of the so-called Delphi method or other analogical methods.*

Since however, it may be presumed that competent experts on whose judgement the prognostic statement is based are themselves capable of presenting a rational justification, we shall take for granted in our further considerations that prognostic statements can be evaluated on the basis of an explicit justification. In other words, the semantic evaluation of the prognostic statement S_i, relating to a possible future fact, is possible in view of S_h and S_e which are available hic et nunc. This also means that this semantic evaluation in always relativized to those epistemical means we can operate with at present. This moreover implies that such a semantic evaluation – relativized with regard to the present – can be modified if, in future, any change occurs in that which may be considered as the prediction basis for S_i.**

The semantic evaluation of this prediction basis involves, first of all, an evaluation of the nomological statement S_h with regard to prognostic statements or explananda. The concept "systematic, i.e. explanatory or predictive power" was introduced already by G. G. Hempel [6, p. 278] to denote such an evaluation of the nomological statement S_h with regard to statements characterized in this way; this author presented one of the first quantification variants of this concept.

The general conception of the concept "systematic power", based on Hintikka's concept "transmitted information" [7], was presented by Finnish logicians, expecially by J. Pietarinen and R. Tuomela [10], [11]. The intutitive starting-point is the following consideration: The fundamental informative task of nomological statements which form the basis of systematization procedure (i.e. scientific explanation, classification, prediction procedures, diagnosis, etc.) is to reduce our uncertainty or ignorance as regard certain fact or possible states. Hence, scientific systematization provides information the value of which lies in its capability of changing the initial entropic level or the initial uncertainty. If we are able to evaluate any change in the initial uncertainty quantitatively, then we can also evaluate the systematic power of the nomological statement that can lead to such a change. In other words, the systematic power of the nomological statement S_h, with regard to that to which the statement S_i (this could be e.g. a prognostic statement) relates, is the greater, the more it reduces the initial uncertainty associated with S_i, If we denote the measure of the initial uncertainty or ignorance associated with S_i as $U(S_i)$ and the uncertainty

* Thus the practice of contemporaty prognostics does not confirm Hempel's thesis that every adequate prediction is a potential explanation, and vice vera. This means that it is possible to formulate prognostic statements concerning future events without being able to explain these or other substantial events.

** The concept "prediction basis" corresponds to Hempel's concept "potential explanans" [6]. The evaluation of some of the difficulties we face in attempting to define this concept with more precision would exceed the scope of this study. For details, see [16, p. 708].

associated with S_i – provided that the nomological statement S_h is also given – as $U(S_i/S_h)$, then it is obvious that the measure of systematic power can be characterized by means of a function whose arguments are the initial uncertainty $U(S_i)$ and the uncertainty $U(S_i/S_h)$ which may be characterized as conditional uncertainty. (It is clear, or course, that also $U(S_i)$ may be conceived as conditional uncertainty on the assumption that S_h is a tautological statement, i.e. that $U(S_i) = U(S_i/S_t)$, where S_t is a tautology). Usually we take into consideration not only the nomological statement or a class of nomological statements S_h but also a class of empirical statements S_e concerning relevant initial conditions with respect to S_i. Then the concept of systematic power takes into account not only the role of S_h with respect to S_i, but the role of $S_h . S_e$ with respect to S_i.

When proceeding from the intuitive starting-point thus conceived it is possible to formulate requirements which must be met by any satisfactory quantitative measure of the systematic power of the nomological statement with regard to that to which the statement S_i relates. (It should be noted that no absolute measure of the systematic power S_h is referred to here, but a measure relativized to that to which S_i relates.) If we denote this measure as $E(S_h/S_i)$, it should meet the following requirements, as shown by J. Pietarinen [11]:

(R1) $$E(S_h/S_i) = f[U(S_i),\ U(S_i/S_h)]\,.$$

Moreover, it is reasonable to require that f be a linear function of both the arguments. As will be seen later, the requirement (R5) will make it possible to conceive f as a function which values are monotonously decreasing if the values of its second argument are increasing.

(R2) $$E(S_h/S_i) \gtreqless 0 \quad \text{iff} \quad U(S_i/S_h) \lesseqgtr 0\,.$$

It is further desirable that the systematic power of S_h with regard to S_i be maximum, wherever the conditional uncertainty $U(S_i/S_h)$ is a minimum uncertainty. This is in accordance with

(R3) $$E(S_h/S_i) = \max E \quad \text{iff} \quad U(S_i/S_h) = \min U\,.$$

It is reasonable to require that the minimum uncertainty be equal to zero, so that $\min U = 0$ might be added to (R3)

(R4) $$\max E = 1\,,$$

(R5) $$E(S_h/S_i) \geqq E(S_k/S_i) \quad \text{iff} \quad U(S_i/S_h) \leqq U(S_i/S_k)\,.$$

On the basis of these requirements it is possible to define the measure of the systematic power of the nomological statement S_h with regard to that to which S_i relates in the following way:

(D1) $$E(S_h/S_i) =_{\text{df}} \frac{U(S_i) - U(S_i/S_h)}{U(S_i)}\,.$$

Obviously, the measure of the systematic power characterized in this way assigns an equal systematic power to all the nomological statement reducing the initial uncertainty of that to which S_i relates – i.e. $U(S_i)$ – to the same level.

It is also desirable that this measure of the systematic power should be additive. This measure is additive for two nomological statement, S_h and S_k, as follows:

$$S(S_h \,.\, S_k/S_i) = E(S_h/S_i) + E(S_k/S_i)$$

iff

$$U(S_i/S_h) + U(S_i/S_k) - U(S_i/S_h \,.\, S_k) = U(S_i)\,.$$

The definition (D1) may be modified as follows: If we hold not only the nomological statement S_h, but also the empirical statement concerning certain initial conditions S_e to be the prediction basis for S_i, which is in agreement with Hempel's classical model which, in a deductive nomological instance, takes the form

$$S_h \,.\, S_e \underset{L}{\rightarrow} S_i\,,$$

then, if S_e is known, the systematic power S_h may be relativized to S_i.

The modified measure of the systematic power can be defined as follows:

(D2) $$E(S_h \,.\, S_e/S_i) =_{\mathrm{df}} \frac{U(S_i/S_e) - U(S_i/S_h \,.\, S_e)}{U(S_i/S_e)}\,.$$

Consequently, this modification presupposes three statements or classes of statements from which the ordered pair $\langle S_h, S_e\rangle$ forms the prediction basis and S_i is the prognostic statement.

If, for the sake of simplicity, we take into account only a pair of statement S_h and S_i, we find that, on the basis of (D1) and the mentioned requirements (R1)–(R5), no limitations are given for the minimum value $E(S_h/S_i)$, while max E, according to (R4), is always equal to 1. It is therefore necessary to formulate another requirement for min E. Here, two further possibilities are offered so that the requirement (R6) will appear in two alternatives:

($R6^{I}$) $$E(S_h/S_i) = \min E \quad \text{iff} \quad U(S_i/S_h) = \max U\,.$$

The second alternative for min E is the following:

($R6^{II}$) $$E(S_h/S_i) = \min E \quad \text{iff} \quad U(S_i/S_h) = U(S_i)\,.$$

This second alternative corresponds to the presumption that the nomological statement S_h is irrelevant with regard to what S_i relates to.

The fact that there are two alternatives for min E indicates that the term "uncertainty" hitherto employed is ambiguous and must be further specified. It is evident that the conditional uncertanty $U(S_i/S_h)$ is minimum, i.e. equal to zero, provided that S_h logically implies S_i. Then also $E(S_h/S_i) = \max E = 1$. Thus it holds good that – provided that $S_h \underset{L}{\rightarrow} S_i$ – $E(S_h/S_i) = 1$ and $U(S_i/S_h) = 0$.

The conditions for $\min E$ and thereby also the conditions for $\max U$ are more difficult to determine. Here it is necessary to take account both the above mentioned alternatives designated as I and II.

According to the first alternative $E(S_h/S_i)$ is minimum, if S_h L-implies the negation of S_i. Since S_h logically implies $\sim S_i$, the certainty with which we expect S_i to occur is minimum and the corresponding conditional uncertainty $U(S_i/S_h)$ is maximum, i.e.

$$\text{(I)} \qquad \text{if} \quad S_h \underset{L}{\rightarrow} \sim S_i\,, \quad \text{then} \quad E(S_h/S_i) = \min E\,.$$

The second alternative does not operate with the negation of S_i but with the negation of S_h. The measure of the systematic power of S_h with respect to S_i is minimum if S_i logically follows from the negation of S_h, i.e.

$$\text{(II)} \qquad \text{if} \quad \sim S_h \underset{L}{\rightarrow} S_i\,, \quad \text{then} \quad E(S_h/S_i) = \min E\,.$$

Both the alternatives, i.e. alternative (I) and (II), suppose different conceptions of the uncertainty of a statement. Alternative (I) which considers $E(S_h/S_i)$ as minimum where the conditional uncertainty $U(S_i/S_h)$ is maximum and, provided that the negation of S_i logically follows from S_h, interprets the measure of uncertainty of the prognostic statement S_i as the measure of unexpectedness of that to which S_i relates. The alternative (II) which considers $E(S_h/S_i)$ to be minimum where the conditional uncertainty S_i – if S_h is given – is equal to the initial uncertainty S_i, and, provided that S_i logically follows from the negation of S_h, interprets the measure of uncertainty of the prognostic statement S_i as the measure of the lack of knowledge of that to which S_i relates.

The measure of uncertainty, therefore, must satisfy the following requirements:

$$\text{(U1)} \qquad U(S_i/S_h) \geqq 0\,,$$

$$\text{(U2)} \qquad \text{if} \quad S_h \underset{L}{\rightarrow} S_i\,, \quad \text{then} \quad U(S_i/S_h) = 0\,,$$

$$\text{(U3)} \qquad \text{if} \quad S_h \quad \text{is a tautological statement, then} \quad U(S_i/S_h) = U(S_i)\,.$$

The intuitive sense of these postulates is quite evident. The fourth postulate is presented in two alternatives, according to whether the measure of uncertainty of the prognostic statement is conceived as the measure of unexpectedness (I) or as the measure of the lack of knowledge (II):

$$\text{(U4)}^{\text{I}} \qquad \text{if} \quad S_h \underset{L}{\rightarrow} \sim S_i\,, \quad \text{then} \quad U(S_i/S_h) \quad \text{is minimum}\,.$$

$$\text{(U4}^{\text{II}}\text{)} \qquad \text{if} \quad \sim S_h \underset{L}{\rightarrow} S_i\,, \quad \text{then} \quad U(S_i/S_h) = U(S_i)\,.$$

The concrete form of the quantification of the measure of uncertainty, whether interpreted according to (I) or according to (II), depends on circumstances of a progamatical nature. It is also possible to use traditional measure of semantic information i.e. either the information measure (inf) or the content measure (cont), the properties of which are described in the classical work by Carnap and Bar-Hillel [1]. Since the information or content measure of a given statement can also be characterized with the help of adequate probability measures (e.g. Carnap's m-function), both the conceptions of uncertainty can be expressed not only by means of information or content measure but also by means of the corresponding probability measure. If $p(i)$ is such a probability measure connected with statement S_i, $p(i/h)$ the conditional probability measure of S_i with respect to S_h, then the information or content measures can be characterized as follows:

$$\begin{aligned} \inf(S_i) &= -\log p(i)\,, \\ \mathrm{cont}(S_i) &= 1 - p(i)\,, \\ \inf(S_i/S_h) &= -\log p(i/h)\,, \\ \mathrm{cont}(S_i/S_h) &= 1 - p(i/h)\,. \end{aligned}$$

Hence, on the basis of alternative (I), the measure of uncertainty $U(S_i/S_h)$ may be quantified either as

$$\text{(1)} \qquad \inf(S_i/S_h)\,, \quad \text{i.e.} \quad -\log p(i/h)$$

or as

$$\text{(2)} \qquad \mathrm{cont}(S_i/S_h)\,, \quad \text{i.e.} \quad 1 - p(i/h)\,.$$

On the basis of alternative (II) we obtain either

$$\text{(3)} \qquad p(h \vee i) - p(i) = p(h) - p(h\,.\,i) = p(h)\,[1 - p(i/h)]$$

or

$$\text{(4)} \qquad \log \frac{p(h \vee i)}{p(i)}\,.$$

Some hesitations could be conected with the assignment of probability measure to statements of various kind, i.e. to nomological statements, prognostic statements and empirical statements concerning relevant initial conditions. It is therefore necessary to introduce the following convention: In spite of differences in construction or justification of the probability measure (which can be conceived as confirmation measure of nomological statements by all the evidence available so far) we shall suppose that this measure fulfil in all instances the axioms of probabilitity theory.

The four mentioned alternative measures of uncertainty* evidently do not exhaust all the possibilities: if other (usually pragmatical) means for determining the quality

* For a more detailed analysis of these four measures see [11].

of the result are available, we may construe other measures of uncertainty or shifts in the initial uncertainty corresponding to the intuitive principle: The shift in uncertainty is the greater, the higher is the quality of the result we are able to attain, or the more we can reduce the (average) risk connected with the final result.

By substituting, in (D1), one of the alternative (1)–(4) for the measures of uncertainty appearing in the definition of the systematic power of the nomological statement S_h with regard to that to which S_i relates, four different measures of systematic power are obtained. The choice of one of the alternatives depends on circumstances of a pragmatical nature. If, for instance, the value of the systematic power were to move between 0 and 1 and, moreover, if the values of all the applied measures of uncertainty were to have the same limits, then the alternative (3) should be chosen. Thus we obtain the quantitative determination of $E(S_h/S_i)$ suggested already by Hempel and Oppenheim [5] which can be expressed as follows:

$$\text{(D3)} \qquad E(S_h/S_i) =_{\mathrm{df}} \frac{1 - p(h \vee i)}{1 - p(i)} = 1 - \frac{p(h)\,[1 - p(i/h)]}{1 - p(i)} = p(\sim h/\sim i)\,.$$

This measure is additive for any two nomological statements which are logically disjunct, i.e. if $S_h \vee S_k$ is logically true, then

$$E(S_h \,.\, S_k/S_i) = E(S_h/S_i) + E(S_k/S_i)\,.$$

In connection with this condition for the additivity of the measure of the systematic power, certain consequences should be noted that are of importance for establishing *the optimum prediction basis*: from the above condition it is evident that for n nomological statements which are mutually exclusively disjunct it holds that

$$E(S_{h_1} \,.\, S_{h_2} \ldots S_{h_n}/S_i) = E(S_{h_1}/S_i) + E(S_{h_2}/S_i) + \ldots + E(S_{h_n}/S_i)\,.$$

If S_{theor} is a conjunction of nomological statements characterized in this way, it is expedient to look for all the components of this conjuctive class of nomological statements that are capable of increasing the systematic power with respect to that to which S_i relates. At the same time, it is possible to formulate the following *rule for reduction*:

All the nomological statements which do not increase the systematic power of S_{theor} with respect to S_i can be excluded from S_{theor}. In other words, we can exclude all the nomological statements of which it holds that $E(S_{h_j}/S_i) = 0$. This rule for reduction can also be formulated as follows: All the nomological statements for which it holds that $E(S_{h_j}/S_i) = 0$, may be characterized as systematically irrelevant nomological statements with respect to S_i. Hence, the rule for reduction has the following form:

All the nomological statements that are systematically irrelevant with respect to S_i can be exleded from S_{theor} as the prediction basis for S_i.

In establishing the optimum prediction basis for S_i, the requirement is worth formulating which is a kind of analogy to Carnap's requirement of total evidence for probabilistic and inductive logics: "In the application of inductive logic to a given knowledge situation, the total evidence available must be taken as a basis for determining the degree of confirmation" [3, p. 211]. If, for determining the degree of confirmation of a given nomological statement, it is expedient to formulate the requirenment of total evidence or some less strict variant of this requirement (e.g. the requirement of a maximum actually, i.e. hic et nunc available evidence), it is equally expedient to formulate the following requirement for establishing the optimum prediction basis:

[RI] *In determining an adequate prediction basis with respect to S_i, all knowledge that is not systematically irrelevant to S_i must be considered as its foundation.*

Following the above considerations concerning the prediction basis, some other concepts can be introduced, in particular that of "permissible reduction" for S_{theor} and that of "sufficient prediction basis". Supposing S_{theor} is a conjunctive class of nomological statements which are mutually logically disjunct, then the permissible reduction for S_{theor} with respect to S_i is conceived as the omission of those elements of S_{theor} which are only negligibly systematically relevant to S_i, i.e. in which

$$E(S_{h_j}/S_i) \geqq \varepsilon\,,$$

where ε is a conventionally agreed value not very different form 0.

Similarly we may introduce the concept of a "sufficient prediction basis" with respect to S_i: S_{theor} is a sufficient prediction basis with respect to S_i, if $E(S_{theor}/S_i) \geqq$ $\geqq \pi$, where π is a conventionally agreed value ensuring a sufficiently reliable level of predicting that to which S_i relates.

The requirement [RI] can be modified if we proceed from the modified measure of systematic power, i.e. from (D2). Hence, in establishing the prediction basis with respect to S_i, we must take into account not only S_{theor} which includes all the nomological statements that are systematically relevant with respect to S_i, but also all the statements concerning the empirically determined initial conditions that are systematically relevant with respect to S_i. Then the term "all knowledge which is not systematically irrelevant with respect to S_i" includes not only S_{theor} but also the total evidence available. As a matter of fact, the requirement for determining the optimum prediction basis conceived in this way is an inductive logical analogy of Laplace's famous conception of "inteligence absolue".

So far, the semantic informational evaluation of justified prognostic statements can be related mainly to that which has been characterized here as the prediction basis with respect to the prognostic statement S_i. However, another procedure may be chosen as the starting-point for evaluating S_i the intuitive starting-point of which is the following consideration: Since the role of S_h, or the role of $S_h \,.\, S_e$ as the prediction basis with respect to S_i is to reduce the uncertainty of that to which S_i relates, we may ask to what measure S_i is supported by disposing of S_h or of $S_h \,.\, S_e$. The measure to which S_i is supported by our having S_h or $S_h \,.\, S_e$ at our disposal can be characterized as a nomological support of S_i, or as a nomological – empirical support of S_i.

If $N(S_i/S_h)$ represents the nomological support of S_i on the basis of S_h, we may take into consideration the measure of the nomological support $N(S_i/S_h)$, or the measure of the nomological-empirical support $NE(S_i/S_h \,.\, S_e)$ which would fulfil the following requirements:

(r1) if $S_i \underset{L}{\equiv} S_j$ and $S_h \underset{L}{\equiv} S_k$, then $N(S_i/S_h) = N(S_j/S_k)$.

(r2) if $S_h \underset{L}{\rightarrow} S_i$, then $N(S_i/S_h) = \max N$.

This requirement can be complemented as follows: If it is desirable (in accordance with the manner of quantification hitherto employed) for the value $N(S_i/S_h)$ to range between 0 and 1, it may be added to (r2) that $\max N = 1$. In agreement with this, the following requirement in chosen:

(r3) if $S_h \underset{L}{\rightarrow} \sim S_i$, then $N(S_i/S_h) = \min N$,

(r4) $$0 \leqq N(S_i/S_h) \leqq 1 .$$

Analogical conditions may be formulated for $NE(S_i/S_h \,.\, S_e)$: these differ from (r1)–(r4) only by the fact that wherever there is the nomological statement S_h, we find conjugation $S_h \,.\, S_e$. It is therefore evident that these conditions are satisfied by a concept which is a complement of the concept cont (S_i/S_h) or of the concept cont $(S_i/S_h \,.\, S_e)$. On other words, the concept of "nomological support" or the concept of "nomological-empirical support" might also be characterized as the conditional certainty of S_i with regard to S_h, or to $S_h \,.\, S_e$. This also means that the concept of "nomological support" or that of "nomological-empirical support" can show the credibility or likelihood of the prognostic statement S_i with regard to S_h or to $S_h \,.\, S_e$ the more adequately, the more fully the requirement (RI) is met.

5. GAIN (OR LOSS) FUNCTION

The determination of a prognostic statement proceeding from the available prediction basis may also be considered as a decision-making process. If, in this process, decisions could be made with an absolute certainty as regards the reliability of all the data, no risk connected with this process would have to be considered. In reality, however, in a number of procedures at the determination of sufficiently reliable prognostic statements it is expedient to make allowance for a certain risk. For this reason, we may ecamine the applicability of some categories of the decision theory.

We are primarily concerned with what we gain by having opted for the prognostic statement S_i and not for its negation, i.e. $\sim S_i$. *The expected gain* will be the greater, the higher is the measure of the nomological support of S_i on the basis of S_h or on the basis of $S_h \,.\, S_e$, and the greater, the higher is the measure of the systematic power S_h or $S_h \,.\, S_e$ with regard to S_i. At the same time we can say that the expected gain will be the smaller, the higher is the measure of the nomological support of $\sim S_i$ on the basis of S_h or $S_h \,.\, S_e$, and the smaller, the higher is the systematic power of S_h or $S_h \,.\, S_e$ with respect to $\sim S_i$.

If the concept of "expected gain" is considered as a quantitative concept. then we are bound to respect also the possibility of a neagative gain, i.e. of a loss. For this reason it is desirable that, in the quantification of this concept, we would be able to operate also with negative values. Therefore, the following definition of the measure of the expected gain (or of the expected loss, if negative values are involved) is at hand; let us denote is as $G(S_i/S_h)$ or $G(S_i/S_h \,.\, S_e)$:

(D4) $$G(S_i/S_h) =_{\mathrm{df}} N(S_i/S_h)\, E(S_h/S_i) - N(\sim S_i/S_h)\, E(S_h/\sim S_i)\,,$$

or

$$G(S_i/S_h \,.\, S_e) =_{\mathrm{df}} NE(S_i/S_h \,.\, S_e)\, E(S_h/S_e \,.\, S_i) -$$
$$- NE(\sim S_i/S_h \,.\, S_e)\, E(S_h \,.\, S_e/\sim S_i)\,.$$

If this measure of the expected gain is required to attain values ranging from -1 to $+1$, the complement of cont (S_i/S_h) or the complement of cont $(S_i/S_h \,.\, S_e)$ should be chosen for determining $N(S_i/S_h)$ or $N(S_i/S_h \,.\, S_e)$, i.e. the likelihood of the prognostic statement S_i with regard to the respective prediction basis. If likelihood is determined on the basis of the conditional probability $p(i/h)$ or $p(i/h \,.\, e)$, the following determination of the measure of the expected gain is obtained:

$$G(S_i/S_h) = p(i/h)\, p(\sim h/\sim i) - p(\sim i/h)\, p(\sim h/i)\,, \quad \text{or}$$
$$= p(i/h \,.\, e)\, p(\sim(h \,.\, e)/\sim i) - p(\sim i/h \,.\, e)\, p(\sim(h \,.\, e)/i)\,.$$

Let us now examine the following extreme cases. If it holds that

$$S_h \,.\, S_e \underset{L}{\rightarrow} S_i\,,$$

i.e. in a reliably deductive-nomological case, we obtain

$$NE(S_i/S_h \,.\, S_e) = 1\,,$$
$$E(S_h \,.\, S_e/S_i) = 1\,,$$
$$NE(\sim S_i/S_h \,.\, S_i) = 0\,.$$

Regardless of that, in this case, it holds that

$$E(S_h \,.\, S_e/\sim S_i) \geqq 0\,,$$

we arrive at

$$G(S_i/S_h \,.\, S_e) = 1\,.$$

If we opt for S_i and if it holds that $S_h \,.\, S_e \underset{L}{\rightarrow} S_i$, the expected gain is max G, i.e. it is equal to 1.

However, if we opt for $\sim S_i$ and if it holds that $S_h \,.\, S_e \underset{L}{\rightarrow} S_i$, we obtain

$$G(\sim S_i/S_h \,.\, S_e) = NE(\sim S_i/S_h \,.\, S_e)\, E(S_h \,.\, S_e/\sim S_i) -$$
$$- NE(S_i/S_h \,.\, S_e)\, E(S_h \,.\, S_e/S_i)\,.$$

Since the same values are obtained in the reverse order, our loss is a maximum loss, i.e.

$$G(\sim S_i/S_h \,.\, S_e) = \min G = -1\,.$$

If $S_h \,.\, S_e \underset{L}{\rightarrow} S_i$ does not hold, i.e. the deductive-nomological model cannot be used to determine the prognostic statement, e.g. if we operate only with statistical laws or, in other words, if that to which S_i relates can be deduced only on the basis of probabilitical dependencies, G may attain values between -1 and 1.

The expected gain (or expected loss) can be considered as a function of three initial data:

(1) the (a priori) probability measure assigned to the fact or event denoted by S_i, i.e. $p(i)$;

(2) the measure of nomological-empirical support of S_i provided $S_h \,.\, S_e$ are given, i.e. $NE(S_i/S_h \,.\, S_e)$ or $p(i/h \,.\, e)$;

(3) a probabilistic measure assigned to $S_h \,.\, S_e$. A probabilistic measure assigned to nomologiced statements can we corrider as the measure of confirmation by all the evidence anailable so ar. A probabilistic measure assigned to an empirical statement S_e may be considered as probability measure assigned to the fact or event denoted by S_e.

If we denote $p(i)$ as A, $p(h \,.\, e)$ as B and $p(i/h \,.\, e)$ as C, following conditions for limits of the values of A, B and C can be proved:

If A and B are given, then:

(a) if $B \geqq A$ and $A + B \geqq 1$, then

$$\frac{A}{B} \geqq B \geqq 0\,,$$

(b) if $B \geqq A$ and $A + B \geqq 1$, then

$$\frac{A}{B} \geqq C \geqq \frac{A + B - 1}{B}\,,$$

(c) if $A \geqq B$ and $A + B \leqq 1$, then

$$1 \geqq C \geqq 0\,,$$

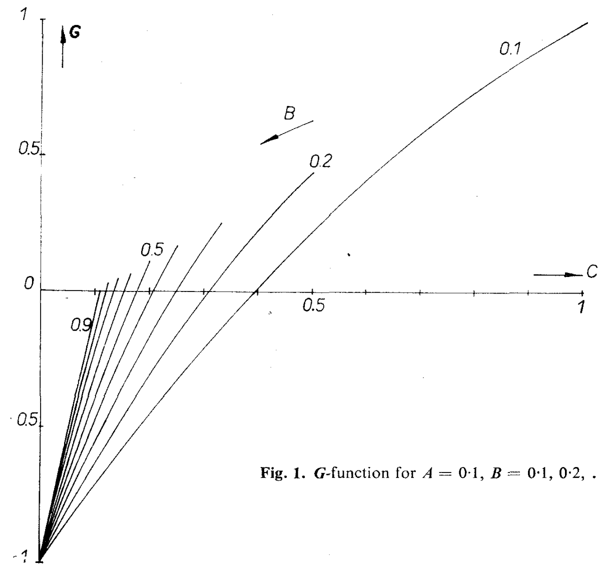

Fig. 1. ***G*-function for $A = 0{\cdot}1$, $B = 0{\cdot}1, 0{\cdot}2, \ldots, 0{\cdot}9$.**

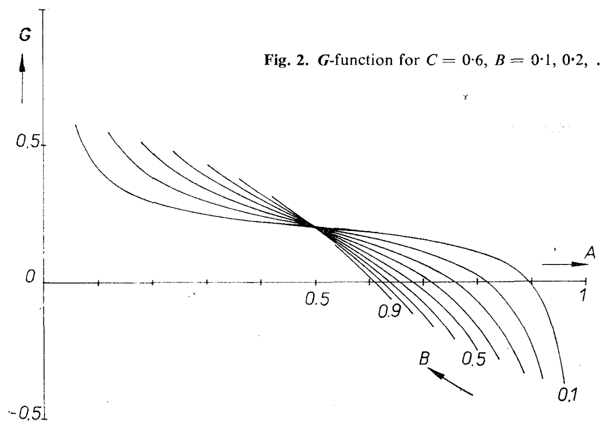

Fig. 2. ***G*-function for $C = 0{\cdot}6$, $B = 0{\cdot}1, 0{\cdot}2, \ldots, 0{\cdot}9$.**

(d) if $A \geqq B$ and $A + B \geqq 1$, then

$$1 \geqq C \geqq \frac{A + B - 1}{B} .$$

If B and C are given, then

$$1 + BC - B \geqq A \leqq BC .$$

If A and C are given, it holds that

(a) if $A \geqq C$, then

$$\frac{1 - A}{1 - C} \geqq B \geqq 0 ,$$

(b) if $C \geqq A$, then

$$\frac{A}{C} \geqq B \geqq 0 ,$$

Since

$$\begin{aligned} G(S_i/S_h \,.\, S_e) &= NE(S_i/S_h \,.\, S_e)\, E(S_h \,.\, S_e/S_i) - \\ &\quad - NE(\sim S_i/S_h \,.\, S_e)\, E(S_h \,.\, S_e/\sim S_i) \end{aligned}$$

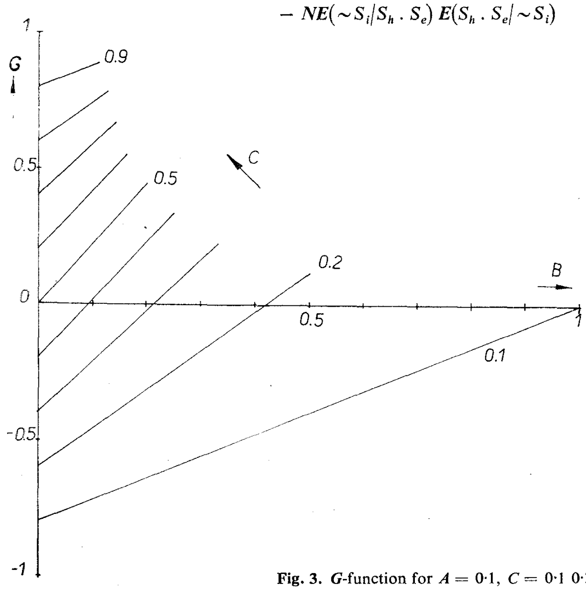

Fig. 3. *G*-function for $A = 0{\cdot}1$, $C = 0{\cdot}1$ $0{\cdot}2, \ldots, 0{\cdot}9$.

and

$$NE(S_i/S_h \,.\, S_e) = C\,,$$

$$E(S_h \,.\, S_e/S_i) = 1 - \frac{B(1 - C)}{1 - A}\,,$$

$$NE(\sim S_i/S_h \,.\, S_e) = 1 - C\,,$$

$$E(S_h \,.\, S_e/\sim S_i) = 1 - \frac{BC}{A}\,,$$

the expected gain (or loss) equals

$$G(S_i/S_h \,.\, S_e) = \frac{C - AC - BC + BC^2}{1 - A} - \frac{A - AC - BC + BC^2}{A}$$

It is also possible to demonstrate the ***G***-function by means of graphic forms. We will show only a few examples of these graphic forms with indicated conditions.

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