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The Representation of a Wave in a Development Matrix of a Subsystem

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A concept of a loop, a branch and a wave (wave of activities, wave of contacts, wave of informations) in a subsystem is introduced, if the subsystem considered is constituted of elements. The loop and the branch, further a loop wave and a branch wave, are represented by means of certain matrices. The corresponding loop matrix, branch matrix, loop-wave matrix and branch-wave matrix make — in a certain manner — a description of the subsystem and a "visualization" of its development possible.

1. INTRODUCTION

When investigating the development of a subsystem, we often have to know whether a wave of a certain kind propagates in the subsystem, for example, whether a "green wave" propagates in a traffic subsystem.

Let any subsystem of a system constituted of elements be considered. This subsystem can possess certain properties, e.g. those given below by Definition 1. The configuration of the elements can be represented by a picture in many cases. An example of a subsystem constituted of 9 elements (numbered by figures 1, ..., 9) is depicted in Fig. 1.

But that example is too simple regarding subsystems beeing usually solved and those which ought to be solved in technical cybernetics, biological cybernetics, etc. A geometric picture in a plane can get more and more difficult with the number of elements and with the number of interelementar relations. One of the further possible means is a matrix representation of the subsystem considered and matrix representation of a development of the subsystem (see [1], [2]). The subsystem being represented by a square matrix, the development of that subsystem can be expressed by a three-dimensional matrix (development matrix), as introduced in [1].

One of the concepts occuring often in a development of a subsystem is the concept of a wave. The aim of the present paper is to introduce the representation of a wave when applying the above mentioned development matrix. First the matrix representation of a subsystem and of its development will be recalled [1]. The concept of a loop and of a branch will be introduced when the loop and the branch are constituted of elements. Then the concept of a wave of contacts in a subsystem will be recalled

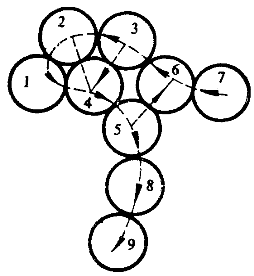

Fig. 1. An example of a subsystem constituted of elements.

if the subsystem considered is constituted of elements as well [2]. Analogically, the wave of activity and the wave of information are introduced.

Recalling the concepts of activity matrix, information matrix, activity-development matrix and information-development matrix [2], the concepts of contact matrix,

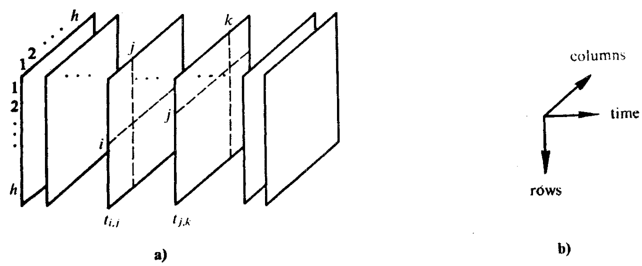

Fig. 2. One of the possible arrangements of a three-dimensional development matrix.

contact-development matrix, activity (contact, information) submatrix, activity-development (contact-development, information-development) submatrix are introduced. The concept of loop (branch) matrix allows to identify or recognize, to a certain extent of course, the structure and configuration of a subsystem. On the other hand the familiarity with the arrangement of a loop-wave (branch-wave) matrix makes possible, again to a certain extent, to recognize the form of the development of the subsystem considered. Several examples illustrate the application of the above mentioned concepts.

2. WAVE (OF CONTACTS, OF ACTIVITY, OF INFORMATION). LOOP WAVE. BRANCH WAVE

First several properties indispensable for our further dealing with subsystems will be digested out of definition of a system introduced in [1]. The restriction results in Definitions 1 and 2.

Definition 1. Let a *subsystem* of a system constituted of elements be given. The elements can carry out their *activities* either in one or in more *quality types.* The single activities of any but the same quality type can be classified with respect to their relative priority class (*relative priorities of activities*). If an arbitrary element is *in contact* with another arbitrary element, then the contact has only one of two possible instantaneous *transit directions* for each quality type. Any two elements possess one contact at most for a certain but the same quality type of activity. An arbitrary element can influence any other element through a contact only. The contact is a total property of a certain pair of elements. Any subsystem can have or/and pursue one or more *aims* (*goals*). The aims of the single subsystems can be classified with respect to their relative priorities (*relative priorities of goals*). In general, some of the activities are interpreted as *signals* to *transfer informations.* The information content can be evaluated (or classified) with respect to one or more goals.

Definition 2. Let a set of elements out of whatever but the same state of a system be given:

(i) If a sequence of the elements can be found so that the elements of each order (in the sequence) is in contact with the element of the "one less" order or with the element of the "one more" order (including both of them),
(ii) if the sequence proceeds in the transit directions of the single contacts,

then the elements of the sequence considered constitute a *path.*

The configuration of the elements constituting a path can be very various. We shall focus our attention to two types of those configurations given by

Definition 3. Let a path constituted of elements be given. If the path encounters each of its elements once and only once, the path is called a *branch.* If the path encounters one and only one of its elements twice, but it encounters all of its other elements once and only once, then the path is called a *loop.*

Remark 1. That concept of a branch and of a loop is in accordance with those applied in the technical and mathematical literature usually.

Several examples of loops and branches are sketched in Fig. 1 and Fig. 3–7. The simplest loop is constituted of three elements (Fig. 3a), because any two elements

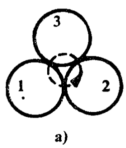

Fig. 3. a) The simplest loop is any (oriented) three-element loop. b) The simplest branch is any (oriented) two-element branch.

(in contact) cannot satisfy the properties required by Definitions 2 and 3. On the other hand, two elements will do for the simplest branch (Fig. 3b). Two possible directions have to be considered in each contact. Thus, if, for example, a quintuple

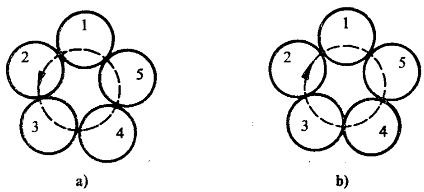

Fig. 4. Two possible orientations of a loop of five elements.

of elements forms a loop, two orientations of the loop are possible (Fig. 4a, b). For the same reason, if, for example, a quadruple of elements is considered to constitute a branch, two possible orientations of the branch are possible (Fig. 5a, b).

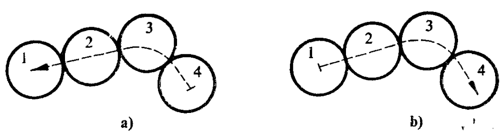

Fig. 5. Two possible orientations of a branch of four elements.

Any two loops can, but need not, have one or more common elements. Several examples of such loops are given in Fig. 6. Two or more loops can constitute a further loop. In Fig. 7, three loops constitute a fourth one. Several loops along with several

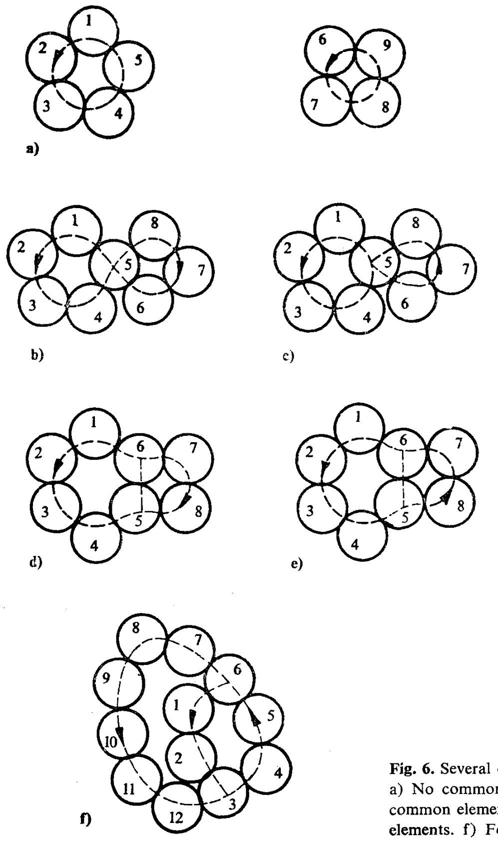

Fig. 6. Several examples of two loops: a) No common element. b), c) One common element. d), e) Two common elements. f) Four common elements.

branches can be seen in the above mentioned Fig. 1, where an example of a subsystem constituted of elements is sketched.

Now the definition of a wave of contacts will be recalled [2].

Definition 4. Let a set of h elements be given. Let $\tau_{i,i+1}$ be the time interval of the contact between the elements e_i and e_{i+1} with transit direction from e_i to e_{i+1}, if $i = 1, 2, \ldots, h-1$. Let $\Delta\tau_{i,i+1}$ be any part (any subinterval) of the time interval $\tau_{i,i+1}$:

$$(1) \qquad \Delta\tau_{i,i+1} \subset \tau_{i,i+1} .$$

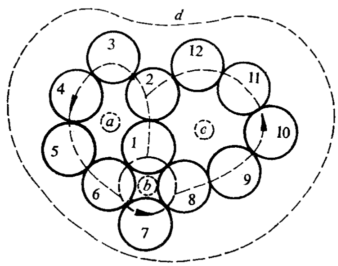

Fig. 7. Three loops (a, b, c) constituting a fourth (d) one.

Let $\alpha_{i,i+1}$ be the initial instant and $\beta_{i,i+1}$ be the terminal instant (if any exist) of the subinterval $\Delta\tau_{i,i+1}$:

$$(2) \qquad \Delta\tau_{i,i+1} = \langle \alpha_{i,i+1}; \beta_{i,i+1} \rangle .$$

If

(i) a location of the subintervals $\Delta\tau_{1,2}, \Delta\tau_{2,3}, \ldots, \Delta\tau_{h-1,h}$ can be found on the time axis so that the ordering of the mentioned subintervals follows the ordering of the instants in time sequence, coincidences included, that is

$$(3) \qquad \alpha_{j,j+1} \leqq \alpha_{j+1,j+2}, \quad j = 1, 2, \ldots, h-2,$$

(ii) the differences

$$\alpha_{j+1,j+2} - \alpha_{j,j+1}$$

and the magnitudes

$$|\Delta\tau_{i,i+1}|$$

of the single subintervals are within certain upper and lower limits for the single values i and j,

then the contacts form a wave (*a wave of contacts*) on the elements $e_1, \ldots, e_h$, or in other words, the contacts form a wave between the elements e_1 and e_h, or a wave of contacts propagates from element e_1 to element e_h, with respect to the mentioned limits which can be time dependent. In this case the element e_1 is called a source of a wave and the element e_h is called the destination of the wave.

Remark 2. The wave of contacts (e.g. in Fig. 8) can propagate from element e_h away to a further element, or/and can be propagated towards the element e_1 from another one.

When applying Definitions 3 and 4 we can introduce

Definition 5. Let a wave of contacts be given. If the wave of contacts encounters each of its elements once and only once, the wave is called a *branch wave of contacts.* If the wave of contacts encounters one and only one of its elements twice (e.g. $e_1 \equiv$

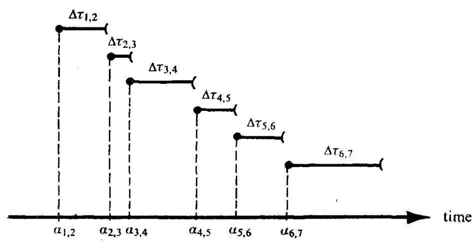

Fig. 8. An example of the location of subintervals $\Delta\tau$ of a wave of contacts on the time axis.

$\equiv e_{h+1}$), but it encounters all its other elements once and only once, then the wave is called a *loop wave of contacts.*

Considering the concept of wave of contacts (Definition 4) and the interelementar activities and informations (Definition 1), Definitions 6 and 7 can be introduced.

Definition 6. Let a certain wave of contacts propagate on and only on a set of h elements. Let any quality type (m) of activity be chosen. Let $\vartheta_{i,i+1}$ be a time interval of the activity of the chosen quality type exerted by the element e_i on the element e_{i+1}, where $i = 1, 2, \ldots, h - 1$. Let $\Delta\tau_{i,i+1}$ be the time interval within which the contact between the elements e_i and e_{i+1} takes part on the above mentioned wave of contacts, where

$$\vartheta_{i,i+1} \subset \Delta\tau_{i,i+1} \, .$$

Then the mentioned single activities form a *wave of activity* (*activity wave*) of the said quality type on the given elements.

Definition 7. Let a certain wave of contacts propagate on and only on a set of h elements. Let $\delta_{i,i+1}$ be a time interval of the information transfer from element e_i to element e_{i+1}, where $i = 1, 2, \ldots, h - 1$. Let $\Delta\tau_{i,i+1}$ be the time interval within which the contact between the elements e_i and e_{i+1} takes part on the above mentioned wave of contacts, where

$$\delta_{i,i+1} \subset \Delta\tau_{i,i+1} .$$

Then the single mentioned information transfers form a *wave of information* (*information wave*) on the given elements.

Remark 3. An information transmission is a special case of an information wave if the single transfers relate partly at least to a certain common information content.

Applying Definitions 6 (7) and 5, Definition 8 (and 9) can be introduced.

Definition 8. Let a wave of activity (a wave of information) be considered. Let the corresponding wave of contacts be a loop wave. Then the wave of activity is a *loop wave of activity.* (Then the wave of information is a *loop wave of information.*)

Definition 9. Let a wave of activity (a wave of information) be considered. Let the corresponding wave of contacts be a branch wave. Then the wave of activity is a *branch wave of activity.* (Then the wave of information is a *branch wave of information.*)

3. ACTIVITY-DEVELOPMENT (CONTACT-DEVELOPMENT, INFORMATION-DEVELOPMENT) MATRIX

In the further text first the concepts of four matrices (Definitions 10–13) are recalled.

Definition 10. Let n elements be given. Let $a_{i,k,t}$ be the probability of activity exerted by the i-th element on the k-th element. Then the matrix $\mathbf{A}_t$ with components $a_{i,k,t}$ in the i-th row and the k-th column is an *activity matrix* related to instant t. (See [1], [2].)

Definition 11. Let n elements be given. Let $\Gamma_{i,k,t}$ be the content of information (e.g. classification of activities, formulation of goals, classification of goals, etc.) related with the pair of elements e_i and e_k, if the element e_i is a source of information. Then the matrix Γ_t with components $\Gamma_{i,k,t}$ in the i-th row and the k-th column is an *information matrix* with respect to instant t ([1], [2]).

Definition 12. Let a time sequence of activity matrices be given, the single activity matrices of which relate one by one to the single but all and only all instants out of any given time interval. Then the time sequence of those activity matrices constitute a three-dimensional *activity-development matrix* ($\mathbf{D}$) related to the time interval. The third dimension of that matrix is the time axis ([1], [2]).

Definition 13. Let a time sequence of information matrices be given, the single information matrices of which relate one by one to the single but all and only all instants out of any given time interval. Then the time sequence of those information matrices constitute a three-dimensional *information-development matrix* ($\mathbf{I}$) related to the time interval. The third dimension of that matrix is the time axis [2].

Now several further matrices will be introduced (Definitions 14–19).

Definition 14. Let n elements be given. Let $f_{i,k,t}$ be the probability that there is a contact between the elements e_i and e_k with transit direction from the element e_i towards the element e_k at instant t. Then the matrix $\mathbf{F}_t$ with components $f_{i,k,t}$ in the i-th row and the k-th column is a *contact matrix* with respect to instant t.

Remark 4. The contact matrix is a special case of the configuration matrix (Configuration matrix see [1], [2]).

Definition 15. Let a time sequence of contact matrices be given, the single contact matrices of which relate one by one to the single but all and only all instants out of any given time interval. Then the time sequence of those contact matrices constitutes a three-dimensional *contact-development matrix* ($\mathbf{G}$) related to the time interval. The third dimension of that matrix is the time axis.

Definition 16. Let n elements be given. Let the activity matrix of those elements related to instant t be given. Let any combination of the h-th class out of the given n elements be chosen ($h \leqq n$). Then the corresponding submatrix (of the activity matrix) related to the chosen elements is an *activity submatrix.*

Definition 17. Let n elements be given. Let the contact matrix of those elements related to instant t be given. Let any combination of the h-th class out of the given n elements be chosen ($h \leqq n$). Then the corresponding submatrix (of the contact matrix) related to the chosen elements is a *contact submatrix.*

Definition 18. Let n elements be given. Let the information matrix of those elements related to instant t be given. Let any combination of the h-th class out of the given n elements be chosen ($h \leqq n$). Then the corresponding submatrix (of the information matrix) related to the chosen elements is an *information submatrix.*

Definition 19. Let any combination of the h-th class out of the given n elements be chosen. Let a time sequence of the corresponding activity submatrices (contact submatrices, information submatrices, respectively) be given, the single activity (contact, information) submatrices of which one by one relate to the single but all and only all instants out of any given time interval. Then the time sequence of those activity (contact, information) submatrices, respectively, constitute a three-dimensional *activity-development submatrix* (*contact-development submatrix, information-development submatrix*, respectively) related to the time interval. The third dimension of those submatrices is the time axis.

Example 1. Due to Definition 16, the activity submatrix is a proper or improper part of the activity matrix and originates from a certain partitioning of the activity matrix. Let the subsystem sketched in Fig. 1 be considered. Let two combinations of elements, e.g. e_3, e_4, e_5, e_6 and e_5, e_8, e_9, be chosen. Then the corresponding activity matrix is

$$\begin{array}{c} \\ e_1\\ e_2\\ e_3\\ e_4\\ e_5\\ e_6\\ e_7\\ e_8\\ e_9 \end{array}\begin{array}{c} \begin{array}{ccccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 \end{array}\\ \left[\begin{array}{cc|cc|c|c|c|cc} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} & a_{1,5} & a_{1,6} & a_{1,7} & a_{1,8} & a_{1,9}\\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} & a_{2,5} & a_{2,6} & a_{2,7} & a_{2,8} & a_{2,9}\\ \hline a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} & a_{3,5} & a_{3,6} & a_{3,7} & a_{3,8} & a_{3,9}\\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} & a_{4,5} & a_{4,6} & a_{4,7} & a_{4,8} & a_{4,9}\\ \hline a_{5,1} & a_{5,2} & a_{5,3} & a_{5,4} & a_{5,5} & a_{5,6} & a_{5,7} & a_{5,8} & a_{5,9}\\ \hline a_{6,1} & a_{6,2} & a_{6,3} & a_{6,4} & a_{6,5} & a_{6,6} & a_{6,7} & a_{6,8} & a_{6,9}\\ \hline a_{7,1} & a_{7,2} & a_{7,3} & a_{7,4} & a_{7,5} & a_{7,6} & a_{7,7} & a_{7,8} & a_{7,9}\\ \hline a_{8,1} & a_{8,2} & a_{8,3} & a_{8,4} & a_{8,5} & a_{8,6} & a_{8,7} & a_{8,8} & a_{8,9}\\ a_{9,1} & a_{9,2} & a_{9,3} & a_{9,4} & a_{9,5} & a_{9,6} & a_{9,7} & a_{9,8} & a_{9,9} \end{array}\right] \end{array}$$

and the corresponding activity submatrices are

$$\begin{array}{c} \\ e_3\\ e_4\\ e_5\\ e_6 \end{array}\begin{array}{c} \begin{array}{cccc} e_3 & e_4 & e_5 & e_6 \end{array}\\ \begin{bmatrix} a_{3,3} & a_{3,4} & a_{3,5} & a_{3,6}\\ a_{4,3} & a_{4,4} & a_{4,5} & a_{4,6}\\ a_{5,3} & a_{5,4} & a_{5,5} & a_{5,6}\\ a_{6,3} & a_{6,4} & a_{6,5} & a_{6,6} \end{bmatrix} \end{array}, \quad \begin{array}{c} \\ e_5\\ e_8\\ e_9 \end{array}\begin{array}{c} \begin{array}{ccc} e_5 & e_8 & e_9 \end{array}\\ \begin{bmatrix} a_{5,5} & a_{5,8} & a_{5,9}\\ a_{8,5} & a_{8,8} & a_{8,9}\\ a_{9,5} & a_{9,8} & a_{9,9} \end{bmatrix} \end{array}$$

respectively. In general, some or none or all of the entries can be zeros.

Example 2. Let the subsystem sketched in Fig. 1 be considered at instants t, $t + 1$, $t + 2$. Then the activity-development matrix (the time axis is arranged in the vertical direction) is

$$\begin{array}{c} \\ e_1\\ \vdots\\ e_9 \end{array}\begin{array}{c} \begin{array}{cc} e_1 & \cdots\ e_9 \end{array}\\ \begin{bmatrix} a_{1,1,t} & \cdots & a_{1,9,t}\\ \cdots & \cdots & \cdots\\ a_{9,1,t} & \cdots & a_{9,9,t} \end{bmatrix} \end{array},$$

$$\begin{array}{c} \\ e_1\\ \vdots\\ e_9 \end{array}\begin{array}{c} \begin{array}{cc} e_1 & \cdots\ e_9 \end{array}\\ \begin{bmatrix} a_{1,1,t+1} & \cdots & a_{1,9,t+1}\\ \cdots & \cdots & \cdots\\ a_{9,1,t+1} & \cdots & a_{9,9,t+1} \end{bmatrix} \end{array},$$

$$\begin{array}{c} \\ e_1\\ \vdots\\ e_9 \end{array}\begin{array}{c} \begin{array}{cc} e_1 & \cdots\ e_9 \end{array}\\ \begin{bmatrix} a_{1,1,t+2} & \cdots & a_{1,9,t+2}\\ \cdots & \cdots & \cdots\\ a_{9,1,t+2} & \cdots & a_{9,9,t+2} \end{bmatrix} \end{array}$$

and the activity-development submatrices, corresponding to combinations of elements $e_3, \ldots, e_6$ and e_5, e_8, e_9, are

$$\begin{array}{c} \\ e_3 \\ \vdots \\ e_6 \end{array}\begin{array}{c} \begin{array}{ccc} e_3 & & \cdots e_6 \end{array} \\ \begin{bmatrix} a_{3,3,t} & \cdots & a_{3,6,t} \\ \cdots & \cdots & \cdots \\ a_{6,3,t} & \cdots & a_{6,6,t} \end{bmatrix} \end{array},$$

$$\begin{array}{c} \\ e_3 \\ \vdots \\ e_6 \end{array}\begin{array}{c} \begin{array}{ccc} e_3 & & \cdots e_6 \end{array} \\ \begin{bmatrix} a_{3,3,t+1} & \cdots & a_{3,6,t+1} \\ \cdots & \cdots & \cdots \\ a_{6,3,t+1} & \cdots & a_{6,6,t+1} \end{bmatrix} \end{array},$$

$$\begin{array}{c} \\ e_3 \\ \vdots \\ e_6 \end{array}\begin{array}{c} \begin{array}{ccc} e_3 & & \cdots e_6 \end{array} \\ \begin{bmatrix} a_{3,3,t+2} & \cdots & a_{3,6,t+2} \\ \cdots & \cdots & \cdots \\ a_{6,3,t+2} & \cdots & a_{6,6,t+2} \end{bmatrix} \end{array},$$

and

$$\begin{array}{c} \\ e_5 \\ e_8 \\ e_9 \end{array}\begin{array}{c} \begin{array}{ccc} e_5 & e_8 & e_9 \end{array} \\ \begin{bmatrix} a_{5,5,t} & a_{5,8,t} & a_{5,9,t} \\ a_{8,5,t} & a_{8,8,t} & a_{8,9,t} \\ a_{9,5,t} & a_{9,8,t} & a_{9,9,t} \end{bmatrix} \end{array},$$

$$\begin{array}{c} \\ e_5 \\ e_8 \\ e_9 \end{array}\begin{array}{c} \begin{array}{ccc} e_5 & e_8 & e_9 \end{array} \\ \begin{bmatrix} a_{5,5,t+1} & a_{5,8,t+1} & a_{5,9,t+1} \\ a_{8,5,t+1} & a_{8,8,t+1} & a_{8,9,t+1} \\ a_{9,5,t+1} & a_{9,8,t+1} & a_{9,9,t+1} \end{bmatrix} \end{array},$$

$$\begin{array}{c} \\ e_5 \\ e_8 \\ e_9 \end{array}\begin{array}{c} \begin{array}{ccc} e_5 & e_8 & e_9 \end{array} \\ \begin{bmatrix} a_{5,5,t+2} & a_{5,8,t+2} & a_{5,9,t+2} \\ a_{8,5,t+2} & a_{8,8,t+2} & a_{8,9,t+2} \\ a_{9,5,t+2} & a_{9,8,t+2} & a_{9,9,t+2} \end{bmatrix} \end{array},$$

respectively.

4. LOOP MATRIX, BRANCH MATRIX, LOOP-WAVE MATRIX, BRANCH-WAVE MATRIX

It will be proved that a loop or a branch can be represented by a square matrix constructed in a certain manner, and that a loop wave or a branch wave can be represented by a three-dimensional development matrix.

Theorem 1. *Let a contact (activity, information) submatrix of order* $(h \times h)$ $(h = 3, 4, \ldots, n)$ *be given. If any matrix of the same order can be constructed*

(i) *by selecting one and only one non-zero valued entry out of each row and each column of the mentioned submatrix,*

(ii) *by putting all other entries of the constructed matrix to be zeros,*

then the constructed matrix represents a loop.

Proof. There exists a non-zero entry $b_{j,k,t}$ $(i, j, k = 1, 2, \ldots, h;\ i \neq j;\ j \neq k;$ t arbitrary) to each non-zero entry $b_{i,j,t}$ (where $b_{i,j,t}$ equals $a_{i,j,t}$ or $f_{i,j,t}$ or $\Gamma_{i,j,t}$).

Theorem 2. *Let a contact (activity, information) submatrix of order* $(h \times h)$ $(h = 2, 3, \ldots, n)$ *be given. If any matrix of the same order can be constructed*

(i) *by selecting one and only one non-zero valued entry out of each row and each column of the mentioned submatrix, except one and only one row and column,*

(ii) *by putting all other entries of the constructed matrix to be zeros,*

then the constructed matrix represents a branch.

Proof. The mentioned exception acts as an absence of contact between certain pair of the chosen elements, the pair given by the only one row and column of the constructed matrix.

Definition 20. Any matrix (Theorems 1, 2) which represents a loop (branch) is a *loop (branch) matrix.*

Example 3. Let the subsystem sketched in Fig. 1 be considered and let the combination of elements $e_3, \ldots, e_6$ be chosen. The following matrix can be constructed by selecting certain entries (according to Theorem 1) of the entries of the activity submatrix corresponding to the chosen elements (Example 1):

$$\begin{array}{c} \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{array} \begin{array}{c} \begin{array}{cccc} e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 0 & a_{3,4} & 0 & 0 \\ 0 & 0 & a_{4,5} & 0 \\ 0 & 0 & 0 & a_{5,6} \\ a_{6,3} & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

It can be proved that the constructed matrix is a loop matrix (Theorem 1, Definition 20).

Example 4. Let the subsystem sketched in Fig. 1 be considered and let the combination of elements e_5, e_8, e_9 be chosen. When applying the two steps, (i) and (ii), introduced in Theorem 2, then the following matrix can be constructed:

$$\begin{array}{c} \\ e_5 \\ e_8 \\ e_9 \end{array} \begin{array}{c} \begin{array}{ccc} e_5 & e_8 & e_9 \end{array} \\ \begin{bmatrix} 0 & a_{5,8} & 0 \\ 0 & 0 & a_{8,9} \\ 0 & 0 & 0 \end{bmatrix} \end{array}.$$

It can be seen that the constructed matrix is a branch matrix (Definition 20).

Example 5. Let two possible orientations of a loop constituted of five elements be considered (Fig. 4). Then the corresponding loop matrices of activities are

$$\begin{array}{c} \\ 1\\2\\3\\4\\5 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & a_{1,2} & 0 & 0 & 0 \\ 0 & 0 & a_{2,3} & 0 & 0 \\ 0 & 0 & 0 & a_{3,4} & 0 \\ 0 & 0 & 0 & 0 & a_{4,5} \\ a_{5,1} & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} ; \quad \begin{array}{c} \\ 1\\2\\3\\4\\5 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & a_{1,5} \\ a_{2,1} & 0 & 0 & 0 & 0 \\ 0 & a_{3,2} & 0 & 0 & 0 \\ 0 & 0 & a_{4,3} & 0 & 0 \\ 0 & 0 & 0 & a_{5,4} & 0 \end{bmatrix} \end{array} .$$

Theorem 3. *Let a three-dimensional contact-development (activity-development, information-development, respectively) submatrix be given within a time interval.*

(i) *If any matrix of the same order as the mentioned submatrix can be constructed*

(i,i) *by selecting one and only one non-zero valued entry out (of all rows and all columns) of and only of each of the single contact (activity, information) submatrices related to the single considered instants of time,*

(i,ii) *by putting all other entries of the constructed matrix to be zeros,*

(ii) *if in the constructed matrix to each* $b_{i,j,t(i,j)}$ *(where* $b_{i,j,t(i,j)}$ *means* $a_{i,j,t(i,j)}$ *or* $f_{i,j,t(i,j)}$ *or* $\Gamma_{i,j,t(i,j)}$*, respectively;* $i, j = 1, 2, \ldots, h$; $i \neq j$; $t_{(i,j)}$ *arbitrary out of the considered instants of time) there exists* $b_{j,k,t(j,k)}$ $(j, k = 1, 2, \ldots, h;$ $j \neq k;$ $t_{(i,j)} \leqq t_{(j,k)})$,

then the constructed matrix represents a loop wave.

Proof follows from the conditions (i) and (ii). Each element acts on its successor in the considered loop wave and is acted by its predecessor in the same loop wave.

Theorem 4. *Let a three-dimensional contact development (activity-development, information-development, respectively) submatrix be given within a time interval.*

(i) *If any matrix of the same order as the mentioned submatrix can be constructed*

(i,i) *by selecting one and only one non-zero valued entry out (of all rows and all columns) of and only of each of the single contact (activity, information) submatrices related to the single considered instants of time, except one and only one row and column,*

(i,ii) *by putting all other entries of the constructed matrix to be zeros,*

(ii) *if in the constructed matrix to each* $b_{i,j,t(i,j)}$ *(where* $b_{i,j,t(i,j)}$ *means* $a_{i,j,t(i,j)}$ *or* $f_{i,j,t(i,j)}$ *or* $\Gamma_{i,j,t(i,j)}$*, respectively;* $i, j = 1, 2, \ldots, h$; $i \neq j$; $t_{(i,j)}$ *arbitrary out of the considered instants of time) there exists* $b_{j,k,t(j,k)}$ $(j, k = 1, 2, \ldots, h;$ $j \neq k;$ $t_{(i,j)} \leqq t_{(j,k)})$, *except one and only one* $b_{i,j,t(i,j)}$,

then the constructed matrix represents a branch wave.

Proof follows from conditions (i) and (ii).

Definition 21. Any matrix which represents a loop wave (branch wave) is a *loop-wave* (*branch-wave*) *matrix* or, briefly, a *wave matrix*.

Corollary 1. *Let a loop-wave matrix be given. Then there exists a certain loop matrix which is a special case of the loop-wave matrix.*

Proof. The case $t_{(i,j)} = t_{(j,k)}$ is the matter.

Corollary 2. *Let a branch-wave matrix be given. Then there exists a certain branch matrix which is a special case of the branch-wave matrix.*

Proof is the same as for Corollary 1.

Corollary 3. *The concept of wave matrix is invariant with respect to the order of the chosen elements in the construction of the matrix.*

Proof. Due to Theorem 3 and 4 and Definition 21, the chosen elements of a considered subsystem

(i) can be taken in any order, and
(ii) can be permuted.

Remark 4. The wave matrix allows to track certain changes in the investigated subsystem. That matrix makes possible a look into the subsystem from a certain point of view. That matrix facilitates a visualization of the development of the subsystem. For example a decision can be made from the contact-development matrix or information-development matrix, whether a "green — wave" propagates in a traffic subsystem or not.

Example 6. Let the subsystem sketched in Fig. 1 be considered at instants t, $t+1$, $t+2$, $t+3$, $t+4$, $t+5$. Let the combination of elements e_3, e_4, e_5, e_6 be chosen. Then the following three-dimensional matrix can be constructed when applying the procedure introduced in Theorem 3:

$$\begin{array}{c} \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{array} \begin{array}{c} \begin{array}{cccc} e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & a_{4,5,t} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array},$$

$$\begin{array}{c} \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{array} \begin{array}{c} \begin{array}{cccc} e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{5,6,t+1} \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array},$$

$$\begin{array}{c} \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{array} \begin{array}{c} \begin{array}{cccc} e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{5,6,t+2} \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array},$$

$$\begin{array}{c|cccc|} & e_3 & e_4 & e_5 & e_6 \\ \hline e_3 & 0 & 0 & 0 & 0 \\ e_4 & 0 & 0 & 0 & 0 \\ e_5 & 0 & 0 & 0 & a_{5,6,t+3} \\ e_6 & 0 & 0 & 0 & 0 \end{array},$$

$$\begin{array}{c|cccc|} & e_3 & e_4 & e_5 & e_6 \\ \hline e_3 & 0 & 0 & 0 & 0 \\ e_4 & 0 & 0 & 0 & 0 \\ e_5 & 0 & 0 & 0 & 0 \\ e_6 & a_{6,3,t+4} & 0 & 0 & 0 \end{array},$$

$$\begin{array}{c|cccc|} & e_3 & e_4 & e_5 & e_6 \\ \hline e_3 & 0 & a_{3,4,t+5} & 0 & 0 \\ e_4 & 0 & 0 & 0 & 0 \\ e_5 & 0 & 0 & 0 & 0 \\ e_6 & 0 & 0 & 0 & 0 \end{array}.$$

It can be seen that the constructed matrix is a loop-wave matrix.

Example 7. Let the subsystem sketched in Fig. 1 be considered at instants t, $t+1$, $t+2$, $t+3$, $t+4$. Let the combination of elements e_5, e_8, e_9 be chosen. Then the following three-dimensional matrix can be constructed, when using the procedure introduced in Theorem 4:

$$\begin{array}{c|ccc|} & e_5 & e_8 & e_9 \\ \hline e_5 & 0 & a_{5,8,t} & 0 \\ e_8 & 0 & 0 & 0 \\ e_9 & 0 & 0 & 0 \end{array},$$

$$\begin{array}{c|ccc|} & e_5 & e_8 & e_9 \\ \hline e_5 & 0 & a_{5,8,t+1} & 0 \\ e_8 & 0 & 0 & 0 \\ e_9 & 0 & 0 & 0 \end{array},$$

$$\begin{array}{c|ccc|} & e_5 & e_8 & e_9 \\ \hline e_5 & 0 & a_{5,8,t+2} & 0 \\ e_8 & 0 & 0 & 0 \\ e_9 & 0 & 0 & 0 \end{array},$$

$$\begin{array}{c|ccc|} & e_5 & e_8 & e_9 \\ \hline e_5 & 0 & 0 & 0 \\ e_8 & 0 & 0 & a_{8,9,t+3} \\ e_9 & 0 & 0 & 0 \end{array},$$

$$\begin{array}{c|ccc|} & e_5 & e_8 & e_9 \\ \hline e_5 & 0 & 0 & 0 \\ e_8 & 0 & 0 & a_{8,9,t+4} \\ e_9 & 0 & 0 & 0 \end{array}.$$

It can be seen that the constructed matrix is a branch-wave matrix.

Remark 5. Let any loop-wave matrix be given. Let its single square matrices (of order $(h \times h)$) with their entries be projected by a parallel projection in the direction of the time axis on an auxiliary plane, which is parallel with the planes of the single mentioned square matrices. The incidences of the single nonzero entries on the auxiliary projection plane form a loop matrix (all other entries being zeros). Hence the name "loop" wave. The path of the wave forms a loop.

Remark 6. Let any branch-wave matrix be given. Let its single square matrices (of order $(h \times h)$) with their entries be projected by a parallel projection in the direction of the time axis on an auxiliary plane which is parallel with the planes of the single mentioned square matrices. The incidences of the single nonzero entries on the auxiliary projection plane form a branch matrix (all other entries being zeros). Hence the name "branch" wave. The path of the wave forms a branch.

5. CONCLUSION

A representation of a wave (wave of activities, wave of contacts, wave of informations) has been introduced applying a matrix form. The corresponding three-dimensional matrix can be helpful in an evaluation of the subsystem considered. The possibility of an application of the wave matrix depends on the considered quality types of activities in the activity-development matrix and/or on the information content in the information-development matrix. For example a decision can be made, whether a "green wave" propagates in a traffic subsystem, or not. The wave matrix can replace a geometric representation of a wave.

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Zobrazení vlny v matici vývoje podsystému

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Je uveden pojem smyčky, větve a vlny (vlny aktivit, vlny kontaktů, vlny informací) v podsystému, jestliže uvažovaný podsystém je tvořen elementy. Smyčka a větev, dále smyčková vlna a větvová vlna jsou vyjádřeny pomocí jistých matic. Příslušná smyčková matice (matice smyčky), větvová matice (matice větve), matice smyčkové vlny a matice větvové vlny svým způsobem umožňují popis podsystému a „zviditelnění" vývoje toho podsystému.

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