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Stabilization and Control of Some Microbial Populations

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The cultivation process of some microorganisms is inherently oscillatory. However, it has been shown that it is possible to stabilize this process by a simple proportional control. An aperiodical character can be achieved and the settling time can be substantially reduced. The problem was solved on an analog computer.

1. INTRODUCTION

In the nature we often find biological processes with an expressive periodical character, e.g. such as increasing and dying out of animal species, particularly of certain microorganisms.

Because of a long period and bad damping of the oscillatory process, the industrial cultivation of some microorganisms is very difficult.

The properties of such processes have been analysed by academician J. Kožešník in his paper [1]. It has been shown that the mathematical analysis of these properties is always laborious. The differential equations describing the process are nonlinear but – despite the oscillatory character – always stable (i.e. in the current mathematical sense). Consequently, the application of a usual stability analysis together with linearisation – in order to find the conditions for a good transient behaviour – is often quite unsatisfactory.

Thus, an analog computer is the most effective means for solving this problem (notwithstanding certain difficulties with inaccuracy, as will be shown later). Using the results of the paper [1], we shall show the way of improving the system dynamics by automatic control.

It was found (rather intuitively) that the use of a simple proportional controller is likely optimal for this purpose.

Unfortunately, we did not succeed in finding a satisfactory mathematical proof of this assertion, because of the difficulties mentioned above. Therefore we had to demonstrate it experimentally by simulating the problem on an analog computer.

In accordance with [1], let us analyse the system described by following nonlinear differential equations:

$$\frac{\mathrm{d}C_S}{\mathrm{d}t} = -\frac{4C_S C_V}{0{\cdot}2 + C_S} + \frac{C_S^* - C_S}{\Theta}, \tag{1abc}$$

$$\frac{\mathrm{d}C_V}{\mathrm{d}t} = \frac{C_S C_V}{0{\cdot}2 + C_S} - C_V C_T - \frac{C_V}{\Theta},$$

$$\frac{\mathrm{d}C_T}{\mathrm{d}t} = 0{\cdot}002\,\frac{C_S C_V}{0{\cdot}2 + C_S} + 0{\cdot}3 C_T C_V - \frac{C_T}{\Theta}.$$

Remark. This equations correspond to Eqs. (16abc) of [1] with concrete values of constants substituted from p. 199. They describe a continuous cultivation in a single (lumped) vessel with constant inlet and outlet flow.

The symbols used in Eqs. (1abc) denote:

C_V – concentration of the microorganisms to be cultivated,
C_S – concentration of the substrate (food for the cultivated matter),
C_S^* – concentration of the substrate at the inlet to the cultivation vessel,
C_T – concentration of the inhibitor (another species of microorganisms devouring that cultivated one),
$1/\Theta$ – flow rate through the vessel (both the inlet and outlet flow are supposed to be equal. The quantity Θ can be regarded as a time-lag of filling the vessel).

First of all, let us take notice of some special properties of the above relations.

It is evident that the cultivation cannot be started if the initial value of C_V is zero. Consequently, the inhibitor would die out as well, even if starting itself with any non-zero initial value.

Further, it must be pointed out that any *nonoscillatory* steady state can be achieved only with certain constraints. It depends not only on the initial conditions but also on the flow rate $1/\Theta$.

The behaviour of the system can become most sensitive to parameter fluctuations or other disturbances. For illustration, a numerical calculation of the steady-state values must always be carried out with a considerable accuracy (at least 5 decimal digits), otherwise it would fail.

These unfavourable properties must be taken into account when Eqs. (1abc) are simulated on an analog device.

In our case, Eqs. (1abc) were transformed by introducing new variables with different scale factors:

$$C_S = 20x\,, \quad C_S^* = 20x^*\,,$$
$$C_V = 20y\,,$$
$$C_T = 2z\,,$$
$$\frac{1}{\Theta} = 2u\,,$$
$$t = \tfrac{1}{6}\tau\,.$$

In that way, following equations were obtained:

(2abc)
$$\frac{dx}{d\tau} = -0{\cdot}666\,\frac{xy}{0{\cdot}01 + x} + 0{\cdot}333(x^* - x)\,u\,,$$
$$\frac{dy}{d\tau} = 0{\cdot}166\,\frac{xy}{0{\cdot}01 + x} - 0{\cdot}333yz - 0{\cdot}333uy\,,$$
$$\frac{dz}{d\tau} = 0{\cdot}003\,\frac{xy}{0{\cdot}01 + x} + yz - 0{\cdot}333uz\,.$$

The nonlinear term can be expressed as follows

(3)
$$r = \frac{xy}{0{\cdot}01 + x} = y - \frac{0{\cdot}01y}{0{\cdot}01 + x}\,.$$

This decomposition is advantageous because the term r can be generated by no more than one servodivider (the adjustment of which, however, must be done with maximum precision).

Rearranging Eqs. (2abc) and using Eq. (3) we get

(4abc)
$$\frac{dx}{d\tau} = -0{\cdot}333ux + 0{\cdot}333ux^* - 0{\cdot}666r\,,$$
$$\frac{dy}{d\tau} = -0{\cdot}333y(u + z) + 0{\cdot}166r\,,$$
$$\frac{dz}{d\tau} = -z(0{,}333u - y) + 0{\cdot}003r\,.$$

Eqs. (4abc) and Eq. (3) were simulated as shown in Fig. 1.

Since the quantity u is now variable because of being controlled, three diode multipliers were applied.

Using a simple proportional feedback to control x by means of u, i.e.

$$(5) \qquad u = (w - x) A$$

we can write following equations of the system (A being the controller gain):

$$(6abc) \qquad \frac{dx}{d\tau} = -0{,}333x[Ax^* + (w - x) A] + 0{\cdot}333Awx^* - 0{\cdot}666r\,,$$

$$\frac{dy}{d\tau} = -0{\cdot}333y[(w - x) A + z] + 0{\cdot}166r\,,$$

$$\frac{dz}{dt} = -z[0{\cdot}333(w - x) A - y] + 0{\cdot}003r\,.$$

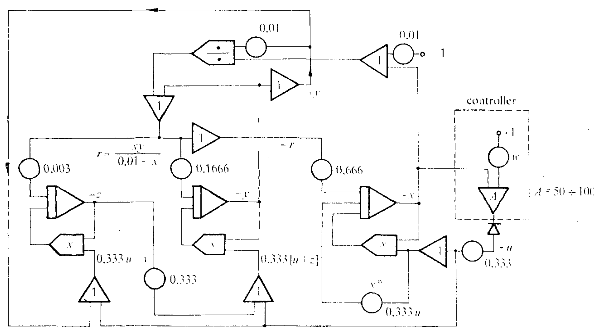

Fig. 1.

Having compared the Eqs. (4abc) with Eqs. (6abc), we can say:

a) As for the variable x, it is evident that its dynamic behaviour will be considerably improved by control. The velocity of its motion increases, the coupling with the other variables decreases (the term r, which represents the coupling, become relatively small) provided that the gain A is sufficiently high. Consequently, the other variables can now be regarded as disturbances. These mathematical considerations correspond to the physical ones as well.

b) As for the variables y or z, respectively, the influence of control cannot be judged as easily as in the previous case. Nevertheless, it can be seen that the variable flow $u = (w - x) A$ makes the system more stable than if u = const.

In order to prove this assertion, we can write the Eq. (6b) in the form (having used Eq. (3)):

$$\frac{dy}{d\tau} = -0{\cdot}333 \left[(w - x)\, A + z - 0{\cdot}166 \frac{x}{0{\cdot}01 + x} \right]. \tag{7}$$

The steady state can be maintained only if the term in parentheses is zero. Further it is apparent from Eq. (4a) or Eq. (6a) together with Eq. (3) that any growth of y must cause a corresponding decrease of x. However, the decrease of x makes the term in parentheses in Eq. (7) positive and therefore y must finally decrease, too. This stabilizing effect is much higher in the controlled system than in the uncontrolled one.

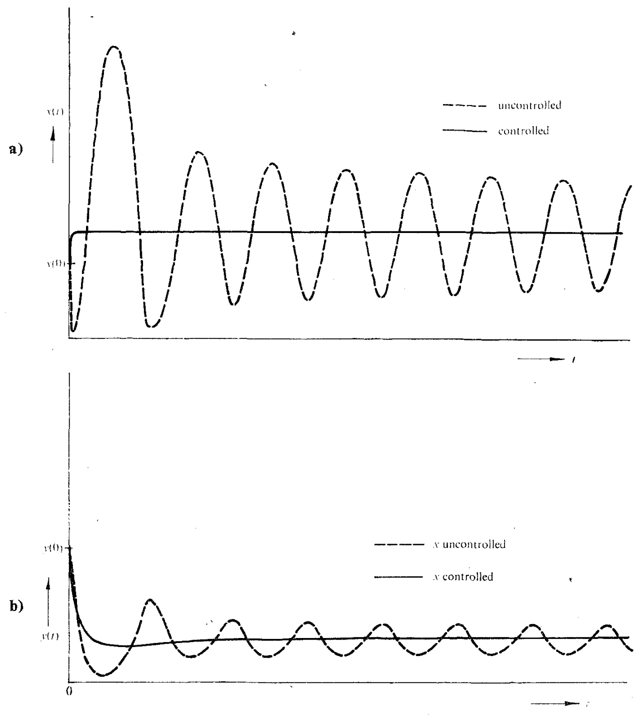

Fig. 2.

In the uncontrolled system, the only stabilizing term was that nonlinear one – namely $x/(0{\cdot}01 + x)$ – while in the controlled system, we have, in addition, the term Ax whose influence is much greater (see Eq. (7) again).

Similar analysis can also be applied to the variable z with similar results.

However, all these considerations are rather qualitative and inaccurate. Without any physical imagination they were uncertain and hardly convincing. That's why some experiments had to be carried out on the computer.

4. EXPERIMENTAL RESULTS

In order to compare the transient response of the controlled system with that of the uncontrolled one, following procedure was applied:

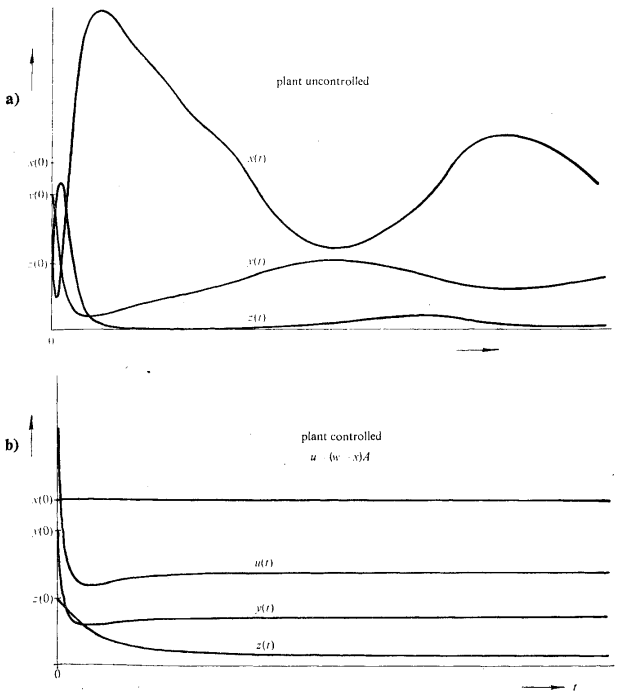

Fig. 3.

Firstly, the plant under control was tested and then, having been started with the same initial conditions $x(0)$, $y(0)$, $z(0)$ and with the same steady flow $u(\infty)$, the experiment was repeated without control.

Many tests were made with the pure proportional control described above. In addition, another control was examined, with y as the controlled variable (according to the control law $u = -(w - y)A$). Finally, both methods were combined, (the control law being $u = (w - x + ky)A$).

In all cases, the tests were quite satisfactory. It has been found that other types of controller, e.g. I, PI, PID, give always inferior results.

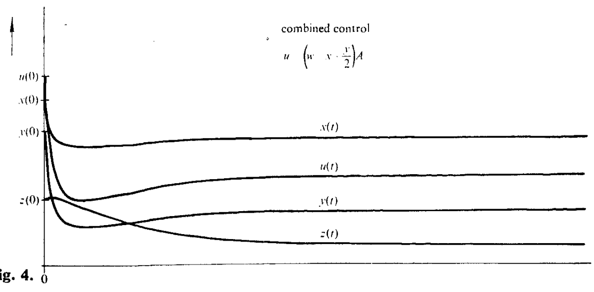

Fig. 4.

It must be pointed out that we did not search any conditions for the optimal production. Nevertheless, under any reasonable conditions for cultivation, the stabilization was excellent.

Some experiments were made under wrong conditions, too (near the state of wash-out or starvation, respectively). In these cases, the transient process was good only for a short time, thereafter it broke down.

From numerous experiments three examples were selected to demonstrate the typical effect of the control. The most illustrative case is shown in Figs. 2a and 2b. It is evident that not only the directly controlled variable x but also the variables y and z are stabilized.

A similar but more detailed case can be seen in Figs. 3a and 3b.

In Fig. 3a, the relation among all uncontrolled variables is recorded, in Fig. 3b, we have the same case but with x being controlled. In addition, the flow rate u is attached so as to judge its influence.

Finally, the above mentioned combination of controlling both the variables x a y (the control law being $u = A(w - x + \frac{1}{2}y)$) is shown in Fig. 4.

Comparing the results from Fig. 3b and Fig. 4, we can say that the control of x is quite sufficient. Consequently, the combination is supposed to be unnecessary, but it could ever be used if it were advantageous for any reason.

5. CONCLUSION

It has been shown that certain microbial cultivation processes, which are inherently oscillatory, can be stabilized and controlled by simple means without any difficulties. In our case, a pure proportional controller proved to be optimal for this purpose. Perfect transient responses have been achieved, however, some tests of actual plants would be necessary to confront our theoretical results with the practical ones. Especially all transport lags must be taken into account and made as small as possible.

REFERENCE

[1] Kožešník, J.: Contribution to Dynamics of Models of Producting Microbial Cultures. Acta Technica (1970), 3, 189—224.

VÝTAH

Stabilita a řízení některých populací mikroorganismů

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Proces rozmnožování některých mikroorganismů má výrazně cyklický charakter, který je při kultivaci zdrojem značných potíží. V článku jsou ukázány možnosti jednoduchého řízení tohoto procesu, kterým lze dosáhnout aperiodického průběhu i podstatně rychlejšího ustálení. Problém byl řešen na analogovém počítači.

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