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Sampled-Data Controls and the Bilinear Transformation

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By the use of the bilinear transformation, the methods devised for the study of continuous control systems may be applied to sampled-data systems. A simple method facilitating considerably the practical performing of the transformation is presented here.

1. INTRODUCTION

Recently, two methods facilitating the substitution

$$z = \frac{s+1}{s-1} \tag{1}$$

in the polynomial

$$F(z) = a_n z^n + a_{n-1} z^{n-1} + \ldots + a_1 z + a_0 \tag{2}$$

have been published in [1], [2]. However, special coefficients for each degree of (2) must be computed in advance in the method described in [1], special (although simple) matrices are needed in the method of [2]. In this article, a very simple method without these inconveniences will be derived.

2. PERFORMING THE BILINEAR TRANSFORMATION

We will make the substitution (1) in two stages

$$z - 1 = \frac{2}{w}, \tag{3}$$

$$w + 1 = s. \tag{4}$$

First of all, the coefficients b_k $(k = 0, 1, \ldots, n)$ in the equation

$$F(z) = b_n(z - 1)^n + \ldots + b_1(z - 1) + b_0 \tag{5}$$

are needed. By successive differentiation on both sides of (5), one gets

$$b_k = F^{(k)}(1)/k! \,. \tag{6}$$

Then, we substitute from (3) in (5). One gets

$$F(z) = \frac{1}{w^n} \cdot \left(b_0 w^n + 2b_1 w^{n-1} + \ldots + 2^n b_n\right) = G(w)/w^n \,. \tag{7}$$

Then, we substitute from (4) in $G(w)$.

$$G(w) = c_n s^n + c_{n-1} s^{n-1} + \ldots + c_1 s + c_0 \,. \tag{8}$$

By successive differentiation on both sides of (8), one gets

$$c_k = G^{(k)}(-1)/k! \,. \tag{9}$$

Finally, one gets

$$F(z) = \frac{1}{(s - 1)^n} \cdot \left(c_n s^n + \ldots + c_1 s + c_0\right) . \tag{10}$$

The coefficients in (6) and (9) are easily computed by the known method of Horner schemes.

Thus, the whole procedure can be summarized as follows:

1. Find the coefficients in (6) by the first sequence of Horner schemes.
2. Form the coefficients of $G(w)$ in (7).
3. Find the coefficients in (9) by the second sequence of Horner schemes.

3. ILLUSTRATIVE EXAMPLE

Let (as in [2])

$$F(z) = z^5 + 3z^4 + 4z^3 + 5z^2 + 2z + 4 \,,$$

The steps 1. and 2. of the procedure are

	1	3	4	5	2	4		
		1	4	8	13	15		
1	1	4	8	13	15	19	. 1	19
		1	5	13	26			
1	1	5	13	26	41		. 2	82
		1	6	19				
1	1	6	19	45			. 4	180
		1	7					
1	1	7	26				. 8	208
		1						
1	1	8					.16	128
	1						.32	32

The step 3. of the procedure is

	19	82	180	208	128	32	
		−19	−63	−117	−91	−37	
−1	19	63	117	91	37	−5	
		−19	−44	−73	−18		
−1	19	44	73	18	19		
		−19	−25	−48			
−1	19	25	48	−30			
		−19	−6				
−1	19	6	42				
		−19					
−1	19	−13					
	19						

Thus

$$F(z) = \frac{1}{(s-1)^5} \cdot (19s^5 - 13s^4 + 42s^3 - 30s^2 + 19s - 5)\,.$$

(The last two coefficients are incorrect in [2]. Moreover the term $f_{3,7} = 80$ of the matrix F_1 in [2] and all terms depending theoren are also incorrect. This term should be 60.)

4. CONCLUDING REMARKS

Comparing the described method with the method of [1] (which is less laborious than that of [2]) one sees in the preceding example that 30 additions or subtractions and 6 multiplications are needed here, whereas 30 additions or subtractions and 12 multiplications are needed in [1].

Since no special expressions must be prepared in advance to perform the computations, the described method seems to be useful.

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REFERENCES

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VÝTAH

Impulsní regulace a bilineární transformace

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S pomocí bilineární transformace může být k vyšetřování impulsních regulací použito metod, navržených pro regulace spojité. V článku se navrhuje jednoduchá metoda, která podstatně usnadňuje praktické provedení transformace.

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