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Adaptive Control with Finite Settling Time

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The parameters of a discrete compensating filter at the plant input are adjusted automatically by means of a modified gradient method in order to fulfil the conditions for the finite settling time of the control process.

INTRODUCTION

This paper deals with a discrete selfadjusting filter forming a convenient control signal from a given command signal in order to achieve the desired state of the controlled variable in a finite time period. The problem consists in modelling the gradient of the modified mean square error in a transfer function form.

Because of the open-loop control used in our case, the plant transfer function need not to be known (the plant identification being unnecessary).

PRINCIPLE OF THE ADAPTIVE CIRCUIT

Let us have a stable linear plant $P(z)$ – including an actuator and a sampling device – whose parameters vary sufficiently slowly so that they can be considered as constant. We shall suppose that only the order of the plant is known. In order to reach the desired state of the controlled variable under optimal conditions, a convenient signal on the plant input must be formed which is adaptive with regard to the changes of the plant parameters.

This input signal can easily be derived by means of a modified gradient method.

The error φ between the command signal and the plant output ought to be zero after a finite number of sampling intervals. In order to fulfil this requirement by means of an automatic device employing the mean square minimization, the given number of sampling periods after the start of the command signal must be skipped (by switching off the error signal within this time). Only the values after this period are taken into account for the minimization.

Of course, we suppose that the command signal is shaped from steps the duration of which is substantially longer than the given number of periods to be skipped.

Minimizing the mean square error $\overline{\varphi^2(a_0, a_1, a_2, \dots)}$, the parameters a_i of the correcting filter must fulfil the condition

$$\frac{\partial \overline{\varphi^2}}{\partial a_i} = 0 . \tag{1}$$

The above condition can be expressed as

$$\overline{\varphi \frac{\partial \varphi}{\partial a_i}} = 0 . \tag{2}$$

Making use of (2), the equations of the parameter adjusting loops may be written (the gradient eqs.)

$$\frac{\mathrm{d}a_i}{\mathrm{d}t} = - \lambda_i \overline{\varphi \frac{\partial \varphi}{\partial a_i}} \quad (i = 0, 1, 2, \dots, m) \quad \lambda_i > 0 . \tag{3}$$

Instead of the mean value $\overline{\varphi^2}$ direct the value φ may be used without loss of validity of Eq. (2):

$$\frac{\mathrm{d}a_i}{\mathrm{d}t} = - \lambda_i \varphi \frac{\partial \varphi}{\partial a_i} . \tag{4}$$

It can readily be proved. Integrating Eq. (4) yields

$$a_i(T) - a_i(k\tau) = \lambda_i \int_{k\tau}^{T} \varphi \frac{\partial \varphi}{\partial a_i} \, \mathrm{d}t \tag{5}$$

($k\tau$ = the skipped interval, k = order of plant).

Dividing by $T - k\tau$ Eq. (5) becomes

$$\frac{a_i(T) - a_i(k\tau)}{T - k\tau} = - \lambda_i \frac{1}{T - k\tau} \int_{k\tau}^{T} \varphi \frac{\partial \varphi}{\partial a_i} \, \mathrm{d}t . \tag{6}$$

For sufficiently large values of T one obtains

$$\lim_{T \to \infty} \frac{a_i(T) - a_i(k\tau)}{T - k\tau} \to 0 , \tag{7}$$

so that

$$\lim_{T \to \infty} \frac{1}{T - k\tau} \int_{k\tau}^{T} \varphi \frac{\partial \varphi}{\partial a_i} \, \mathrm{d}t \to 0 . \tag{8}$$

In the steady state (where $a_i \to$ const., the value $1/(T - k\tau) \int_{k\tau}^{T} \varphi(\partial \varphi / \partial a_i) \, \mathrm{d}t$ also being constant) the expression (8) may be written as

$$\lim_{T\to\infty} \frac{\partial}{\partial a_i} \left\{ \frac{1}{T - k\tau} \int_{k\tau}^{T} \varphi^2 \, \mathrm{d}t \right\} \to 0$$

and then,

$$\frac{\partial}{\partial a_i} \overline{\varphi^2} \to 0 \tag{9}$$

what corresponds to the condition (1).

Thus, the validity of Eq. (4) is proved.

To obtain the necessary derivatives with respect to the parameters a_0, a_1, a_2, etc., a modelling technique is used. The derivatives are computed analytically from the transfer functions of the circuit and modelled.

As mentioned above, in our case the transfer function of the plant need not to be known because its model is performed through the plant itself.

The model of the circuit is shown in Fig. 1a and 1b. The skipping of the necessary number of sampling periods is carried out by means of the switch S.

In accordance with Fig. 1a and 1b, the error function is (in Z-transform notation):

$$\Phi(z) = W(z) \,.\, C(z, a_0, a_1, a_2 \ldots) \,.\, P(z) - W(z) \tag{10}$$

where $W(z)$ – the command signal, $P(z)$ – the plant transfer function, $C(z, a_0, a_1, a_2, \ldots)$ – the transfer function of the correcting filter with parameters $a_0, a_1, a_2, \ldots$ to be determined, the relation among them being

$$C(z, a_0, a_1, a_2, \ldots) = a_0 + a_1 z^{-1} + a_2 z^{-2} + \ldots. \tag{11}$$

Differentiating Eq. (10) with respect to the parameters a_i yields

$$\frac{\partial \Phi}{\partial a_i} = W \frac{\partial C}{\partial a_i} P \quad (i = 0, 1, 2, \ldots)\,. \tag{12}$$

It is evident that this relation cannot be realized directly on account of the necessary sequence of transfers; for our control we need the sequence $W - C - P$.

Fortunately, the commutative properties of the transfer functions enable us to write and model (with the desired sequence $W - C - P$):

$$\frac{\partial \Phi}{\partial a_i} = W \frac{\partial C}{\partial a_i} P = WCPC^{-1} \frac{\partial C}{\partial a_i} \tag{13}$$

as represented in Fig. 1a or 1b.

The derivatives $\partial C / \partial a_i$ are obtained very easy from (11):

$$\frac{\partial C}{\partial a_0} = 1\,, \quad \frac{\partial C}{\partial a_1} = z^{-1}\,, \quad \frac{\partial C}{\partial a_2} = z^{-2}\,, \quad \text{etc.} \tag{14}$$

The complete arrangement of the adaptive circuit is represented in an analogue form in Fig. 1b. The circuit is not too complicated but the necessary number of multipliers is not too pleasant, since contemporary analogue devices do not abound in them.

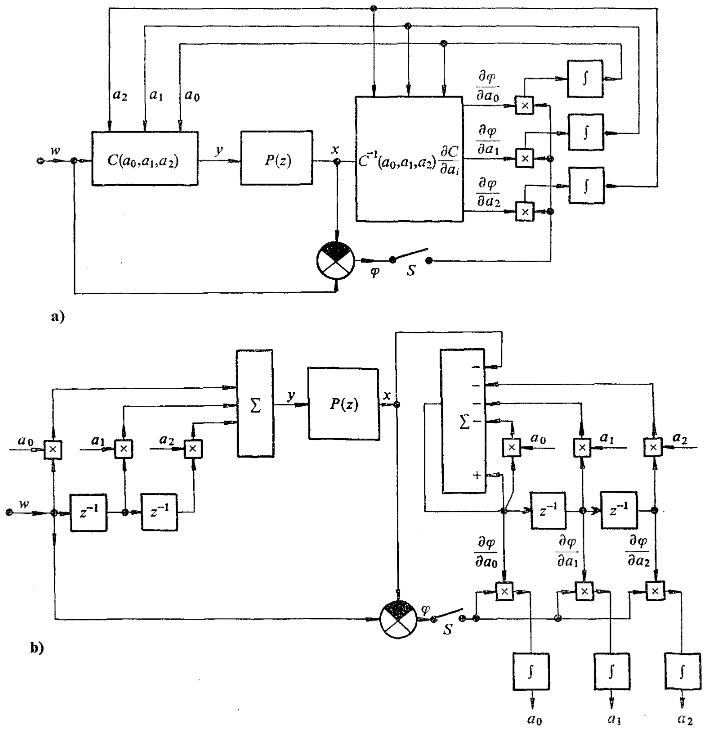

Fig. 1. The arrangement of the corrector C and plant P for adaptive control.

Fortunately, the multipliers for multiplying the error by its derivatives need not to be precise only the drift ought to be negligible. (Even multiplying by the sign might be sufficient what corresponds to the algorithm for minimizing $|\overline{\varphi}|$ instead of $\overline{\varphi}^2$.) However, there arises another problem due to the inverse transfer function C^{-1}. If any zero of the function $C(z)$ lies outside the unit circle in the complex plane, then the inverse function C^{-1} becomes unstable. Thus, the whole system becomes unstable, regardless of the compensation of C^{-1} by C.

It is true that $CC^{-1} = 1$ but with respect to the input signal. With regard to any inner disturbance, the compensation is not valid and the system is unstable.

Nevertheless, in our case, the stability is preserved because of the conditions for the finite settling of the control process.

The resultant transfer function $C(z)\ P(z)$ must be a *polynomial of finite order*.

From this condition follows that the polynomial $C(z)$ must be divisible by the denominator of $P(z)$. This denominator has all zeros inside the unit circle (the plant was supposed stable). Consequently, the polynomial $C(z)$ must have the same zeros. If $C(z)$ is of the same order as the denominator of $P(z)$, there cannot be any other unstable zeros and therefore also $C^{-1}(z)$ is stable. In addition to it, the selfadjusting circuit tries to stabilize the whole system even if the conditions for the finite settling time are not yet (or cannot be) fulfilled.

Thus, the coefficients of $C(z)$ are adjusted in order to stabilize $C^{-1}(z)$. (It must be emphasized again that this is valid for slow changes of the adjusted parameters only).

For time-invariant plants a simpler device may be considered which serves for evaluation of the corrector parameters only (once for all – without participating in the proper control; see Fig. 2a and Fig. 2b).

As well as in the previous case, the sequence of the transfer functions of the corrector and plant can be interchanged to facilitate the realization. At the same time, also the troubles with stability are avoided, since it can readily be proved that the adaptation is stable even for fast changes of the parameters adjusted. For the arrangement in Fig. 2a we have:

$$\Phi(z) = -T(z) \,.\, P(z)\, C(z, a_0, a_1, a_2, \ldots) + T(z)\,, \tag{15}$$

$$\frac{\partial \Phi}{\partial a_i} = -T \,.\, P \frac{\partial C}{\partial a_i} \quad (i = 0, 1, 2, \ldots)\,. \tag{16}$$

Corresponding to Fig. 2b:

$$\begin{aligned} \Phi &= -\ T \,.\, P[a_0 + a_1 z^{-1} + a_2 z^{-2} + \ldots] + T = \\ &= -\ X \,.\, [a_0 + a_1 z^{-1} + a_2 z^{-2} + \ldots] + T\,, \end{aligned} \tag{17}$$

$$\frac{\partial \Phi}{\partial a_i} = -\ TPz^{-i} = -\ Xz^{-i} \quad (i = 0, 1, 2, \ldots)\,. \tag{18}$$

Then in the time domain:

$$\begin{gathered} \varphi[n\tau] = -\ a_0 x[n\tau] - a_1 x[(n-1)\,\tau] - a_2 y[(n-2)\,\tau] - \ldots + T[n\tau]\,, \\ \frac{\partial \varphi[n\tau]}{\partial a_i} = -\ x[(n-i)\,\tau] \end{gathered} \tag{19}$$

(τ is the sampling interval).

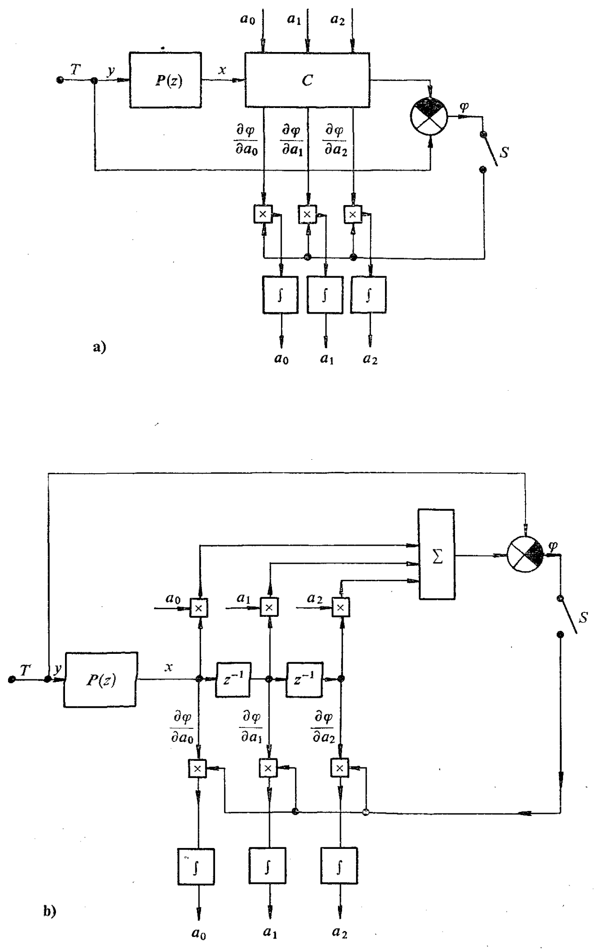

Fig. 2. The arrangement for determination of corrector parameters only.

 The equations of adaptation run:

$$\frac{da_i}{dt} = \lambda_i \, . \, [T - a_0 x[n\tau] - a_1 x[(n-1)\tau] - a_2 x[(n-2)\tau] - \ldots] \, . \, x[(n-i)\tau]$$
(20)

($i = 0, 1, 2, 3, \ldots, m$; m = order of the plant).

It is seen that the derivatives $\partial\varphi/\partial a_i$ are independent of a_i because φ is a linear function of a_i.

Since also the plant output signal is independent of a_i, the derivatives are correct even for rapid changes of a_i.

Then, the Eqs. (20) are of *first order only and linear* with regard to the parameters a_i.

The stability of these equations can be proved very easy by means of the primary equations from which they originated:

$$\frac{da_i}{dt} = -\lambda_i \frac{\partial}{\partial a_i} \varphi^2 \,, \quad \lambda_i > 0 \quad (i = 0, 1, 2, \ldots, m) \tag{21}$$

(see also Eqs. 4).

Suppose a Liapunov function

$$V = \varphi^2 \tag{22}$$

then

$$\frac{dV}{dt} = \sum_{i=1}^{m} \frac{\partial \varphi^2}{\partial a_i} \frac{da_i}{dt} \tag{23}$$

Making use of Eq. (21), we obtain

$$\frac{dV}{dt} = -\sum_{i=0}^{m} \lambda_i \left[\frac{\partial \varphi^2}{\partial a_i}\right]^2 = -\sum_{i=0}^{m} \frac{1}{\lambda_i} \left[\frac{da_i}{dt}\right]^2 . \tag{24}$$

dV/dt is negative definite, therefore the Eq. (21) are stable, being simultaneously a convenient Liapunov function. Thus, the Eqs. (20) are also stable. Because they are linear in respect of the parameters a_i, the steady state solution will be unique, for, in the steady state, the problem leads to linear algebraic equations. These equations are composed of exponential terms as coefficients at the unknown variables a_i. The problem can be regarded as solving this set of algebraic equations by means of the gradient method on an analogue computer. As to stability, the difference between the former and the latter circuit consists in the position of the adjusted parameters in the system.

In the former case, the order of the adaptive loops depends on the order of the plant and corrector too, while in the latter system, no additional lags deteriorate the selfadjustment dynamics.

Therefore, the stability of the latter system is guaranteed, while the stability of the former one is conditioned by sufficiently slow adjustment of the parameters.

The latter method can also be used without the skipping mentioned above.

In this case, the device determines the optimal parameters for minimizing the mean square error (the conditions for finite time settling being not fulfilled).

However, there might arise another important question: whether the parameters actually converge to the values fulfilling the condition of the finite settling time? Fortunately, the answer is affirmative.

As mentioned above, the error φ is a linear function of the parameters. Consequently, also grad $\overline{\varphi^2}$ is linear. Therefore the equation

$$\operatorname{grad} \overline{\varphi^2} = 0$$

has a unique steady-state solution what corresponds to a unique minimum of $\overline{\varphi^2}$.

The solution depends naturally on the given conditions (input signal, noise, some constraints etc.) but if there are conditions for fulfilling

$$\varphi^2 \equiv 0 \quad (t \geqq k\tau)$$

at all, then this is the correct and sole minimum (a "smaller minimum" cannot exist). Thus, the parameters tend to this minimum.

CONCLUSION

In the case of an open-loop control circuit, there is possible to make an adaptive correcting filter for finite settling time of the control process, without plant identification, supposing that only the order of the plant is known. The realization of the device, though possible in an analogue form, is more convenient for a digital computer.

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Adaptivní řízení s konečnou dobou nastavení

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Popisuje se způsob řízení pomocí adaptivního diskrétního korektoru, zařazeného před soustavou, jehož parametry se automaticky nastavují tak, aby byly splněny podmínky pro kritérium konečného počtu kroků. K nastavení parametrů se využívá speciálně upravené gradientové metody.

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