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On the Smoothing of a Discrete Random Autoregressive Process

LUDVÍK PROUZA

A system described by a linear difference equation of the autoregressive type is given and a stationary white sequence is applied as its input. This article studies the problem of stabilizing the system with the aid of a feedback filter possesing the "simplest" structure. Some remarkable properties of such "minimum prototype" filter are shown.

1. INTRODUCTION

When solving some technical and economical problems (see e.g. [1] and the references cited therein), one has to deal with the following recurrence relation

$$\eta_{n+1} = \eta_n + \xi_n - \gamma_n , \tag{1}$$

where $\{\xi_n\}$ is a random sequence (usually stationary and white), $\{\eta_n\}$ will be called the smoothed sequence, and $\{\gamma_n\}$ the smoothing sequence. The terms of the latter are supposed to be formed linearly from the preceding terms of all sequences in (1) with a delay $m \geqq 0$ (m being a natural number). Symbolically

$$\gamma_n = L(\eta_{n-m}, \eta_{n-m-1}, \ldots, \eta_{n-m-q}, \xi_{n-m}, \xi_{n-m-1}, \ldots \\ \ldots, \gamma_{n-m}, \gamma_{n-m-1}, \ldots) . \tag{2}$$

The minus sign at γ_n in (1) has been elected for formal reasons, $q > 0$ is an arbitrary but fixed natural number.

It is desired that the variances in the sequences $\{\eta_n\}$ and $\{\gamma_n\}$ be both as small as possible.

This problem has been solved in [1] using methods connected immediately with (1). The same problem for the more general relation

$$\eta_{n+1} = A\eta_n + \xi_n - \gamma_n \tag{3}$$

has been solved in [2] by similar methods.

There seems to be little known about the solution of the problem in the case of the relation

$$\eta_{n+1} = A_0\eta_n + A_1\eta_{n-1} + \ldots + A_t\eta_{n-t} + \xi_n - \gamma_n, \tag{4}$$

where $A_0, A_1, \ldots, A_t$ are arbitrary real numbers and $t > 0$ is a natural number.

Some partial results interesting also for the interpretation of [2] will be shown in this article. The Z-transform will be used throughout and, for the sake of simplicity, the analysis will be firstly restricted on the "two-dimensional" case.

2. STABILIZATION WITHOUT DELAY. MINIMUM PROTOTYPE FILTER

Let us consider the relation

$$\eta_{n+1} = A_0\eta_n + A_1\eta_{n-1} + \xi_n - \gamma_n \tag{5}$$

and seek γ_n for $m = 0$ so that a system described by (5) is stable.

Theorem 2.1. *If the system described by (5) is to be stable, γ_n must be of the form*

$$\gamma_n = a_0\eta_n + a_1\eta_{n-1} + L(\eta_{n-2}, \ldots, \eta_{n-q}, \{\xi_k\}, \{\gamma_k\}), \tag{6}$$

where a_0, a_1 are generally distinct from zero.

Proof. It follows from (6) that the characteristic equation to (5) has the form

$$z^{q+1} + (a_0 - A_0)\, z^q + (a_1 - A_1)\, z^{q-1} + a_2 z^{q-2} + \ldots + a_q = 0, \tag{7}$$

where $a_2, a_3, \ldots, a_q$ have their origin in the expression L in (6). If in a stable system $a_0 = 0$, then (using the well known theorem of Vieta)

$$|A_0| \leqq |z_1| + |z_2| + \ldots + |z_{q+1}| < q + 1, \tag{8}$$

where $z_1, z_2, \ldots, z_{q+1}$ are roots of the characteristic equation. Since A_0 is arbitrary, this is impossible.

If in a stable system $a_1 = 0$, there would be again

$$|A_1| \leqq |z_1 z_2| + \ldots + |z_q z_{q+1}| < \binom{q+1}{2}. \tag{9}$$

Since A_1 is arbitrary, this is again impossible and the theorem is proven.

On the other hand it is obvious that even the simple form

$$\gamma_n = a_0\eta_n + a_1\eta_{n-1} \tag{10}$$

is sufficient to insure system stability if a_0, a_1 are properly adjusted.

The stabilizing filter satisfying (10) will be called *minimum prototype*. The diagram in Fig. 1. corresponds to (5) and (10). In Fig. 1, there is $X(z) = Z\{\xi_n\}$, $H(z) = Z\{\eta_n\}$, $\Gamma(z) = Z\{\gamma_n\}$ (assuming all sequences to possess Z-transforms). Further

(11)
$$Y(z) = \frac{z}{z^2 - A_0 z - A_1},$$
$$F(z) = \frac{a_0 z + a_1}{z}.$$

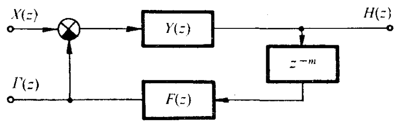

Fig. 1.

It is obvious that $F(z)$ has the minimum possible degree of the numerator and the denominator. The transfer functions of interest are

(12)
$$\frac{H(z)}{X(z)} = \frac{z}{z^2 + (a_0 - A_0) z + (a_1 - A_1)},$$

(13)
$$\frac{\Gamma(z)}{X(z)} = \frac{a_0 z + a_1}{z^2 + (a_0 - A_0) z + (a_1 - A_1)}.$$

3. STABILIZATION WITH DELAY. MINIMUM PROTOTYPE FILTER

Now, let us assume $m > 0$ in Fig. 1, and that the unknown transfer function $F(z)$ is to be found. The remaining symbols in the diagram have the same meaning as in section 2.

It is known that the open-loop delay will be reproduced in the closed-loop transfer function of any physically realizable system.

Thus, one may attempt (with respect to (13)) to meet the following postulate:

Postulate 3.1. Let the denominator of the transfer function $\Gamma(z)/X(z)$ be of the form

(14)
$$z^s[z^2 + (a_0 - A_0) z + (a_1 - A_1)],$$

where $s \geqq m$.

It is obvious that only the (generally multiple) root $z = 0$ will be added to the roots of the characteristic equation in this case, the stability remaining unaffected.

It will be shown that the postulate 3.1 can be easily met. Make

(15)
$$F(z) = \frac{P(z)}{Q(z)} = \frac{c_0 z^l + c_1 z^{l-1} + \ldots + c_l}{b_0 z^r + b_1 z^{r-1} + \ldots + b_r},$$

where $r \geqq l$ for obtain a physically realizable filter. $P(z)$ and $Q(z)$ are supposed throughout to be without common factors. Now there is

$$\frac{\Gamma(z)}{X(z)} = \frac{P(z)}{Q(z) \, . \, z^{m-1}(z^2 - A_0 z - A_1) + P(z)} \, . \tag{16}$$

From (14), (15), (16) there follows

$$\begin{aligned} (17) \quad & b_0 z^{r+m+1} + b_1 z^{r+m} + b_2 z^{r+m-1} + \ldots + b_r z^{m+1} - \\ & - A_0 b_0 z^{r+m} - A_0 b_1 z^{r+m-1} - \ldots - A_0 b_{r-1} z^{m+1} - A_0 b_r z^m - \\ & - A_1 b_0 z^{r+m-1} - \ldots - A_1 b_{r-2} z^{m+1} - A_1 b_{r-1} z^m - \\ & - A_1 b_r z^{m-1} + \\ & + c_0 z^l + c_1 z^{l-1} + \ldots + c^l = z^{s+2} + (a_0 - A_0) \, z^{s+1} + (a_1 - A_1) \, z^s \, . \end{aligned}$$

First of all, it appears from (17) that $s = r + m - 1$. Postulating again the degrees of the numerator and the denominator in (15) to a possible minimum, one gets

$$\begin{aligned} r &= l = m \, , \\ s &= 2m - 1 \, . \end{aligned} \tag{18}$$

The filter fulfilling (18) will be called again minimum prototype. For this filter there follows from (17):

$$b_0 = 1 \, , \tag{19}$$

$$b_1 = a_0 \tag{20}$$

$$b_2 = A_0 a_0 + a_1 \, , \tag{21}$$

$$b_j = A_0 b_{j-1} + A_1 b_{j-2} \quad \text{for} \quad j = 3, 4, \ldots \tag{22}$$

The relation (22) is a difference equation with the initial conditions (20) and (21). Further there follows from (17)

$$\begin{aligned} c_0 &= b_{m+1} \, , \\ c_1 &= A_1 b_m \, , \\ c_j &= 0 \quad \text{for} \quad j > 1 \, , \end{aligned} \tag{23}$$

where b_m, b_{m+1} are terms of the sequence defined in (19) to (22). For the minimum prototype filter one obtains

$$F(z) = \frac{c_0 z^m + c_1 z^{m-1}}{b_0 z^m + b_1 z^{m-1} + \ldots + b_m} \, , \tag{24}$$

$$\frac{\Gamma(z)}{X(z)} = \frac{c_0 z + c_1}{z^m(z^2 + \alpha_0 z + \alpha_1)}, \tag{25}$$

$$\frac{H(z)}{X(z)} = \frac{b_0 z^m + b_1 z^{m-1} + \ldots + b_m}{z^{m-1}(z^2 + \alpha_0 z + \alpha_1)}, \tag{26}$$

where $\alpha_0 = a_0 - A_0$, $\alpha_1 = a_1 - A_1$.

Let us now consider an arbitrary filter with rational transfer function fulfilling the postulate 3.1. Such filter has to fulfill (17) (and need not be the minimum prototype). From (16) there follows

$$1 - \frac{\Gamma(z)}{X(z)} = \frac{Q(z)\,(z^2 - A_0 z - A_1)}{z^r(z^2 + \alpha_0 z + \alpha_1)}. \tag{27}$$

$Q(z)$ has no common factors with the denominator in (27). Considering (14) and the denominator in (16), it is clear that $P(z)$ would also contain this factor, which contradicts the supposition enonced above.

Let the equation

$$z^2 - A_0 z - A_1 = 0 \tag{28}$$

have both roots lying outside or on the unit circle. One sees that these roots are also roots of the expression (27). Thus, a filter fulfilling the postulate 3.1 fulfills also a well known necessary condition for an arbitrary stabilizing filter (see e.g. [3]). This condition is therefore also satisfied by the minimum prototype filter.

4. OTHER PROPERTIES OF THE MINIMUM PROTOTYPE FILTER

Theorem 4.1. *Let a stationary white sequence be applied as the input of a system with the minimum prototype filter. Then*

$$\sigma_\eta^2 = \min \tag{29}$$

is reached for $\alpha_0 = 0$, $\alpha_1 = 0$, *i.e.* $a_0 = A_0$, $a_1 = A_1$.

Proof. The transfer function $H(z)/X(z)$ may be written as follows:

$$\frac{H(z)}{X(z)} = \frac{z}{z^2 - A_0 z - A_1} - \frac{c_0 z + c_1}{z^{m-1}(z^2 - A_0 z - A_1)(z^2 + \alpha_0 z + \alpha_1)}. \tag{30}$$

From this it can be seen that the initial terms of the expansion of $H(z)/X(z)$ to the term $w_{m+1} z^{-m-1}$ (included), are independent of α_0, α_1. Putting $\alpha_0 = 0$, $\alpha_1 = 0$, it follows from (26) that only these terms remain in the expansion. Since σ_η^2 is the sum of squares of the weight sequence $\{w_n\}$, the theorem is proven.

Thus, the system has the "finite-settling-time" property in this case.

Theorem 4.2. *Let a stationary white sequence be applied as the input of a system with the minimum prototype filter. Then*

$$\sigma_y^2 = \min \tag{31}$$

is reached for properly adjusted α_0, α_1.

Proof. Suppose for the sake of simplicity that the equation (28) has simple roots. Three cases will be distinguished below dependent on the position of the roots with respect to the unit circle.

a) Let both roots lie outside or on the unit circle. From the form of $Y(z)$ in Fig. 1, the existence of the delay of m sampling periods, and the physical realizability of the filter with the transfer function (15), there follows first of all that $\Gamma(z)/X(z)$ converges to 0 for $z \to \infty$, the rate of this convergence being at least of the order z^{-m-1}. Secondly, it is clear that the roots of (28) are also roots of (27), as stated above. Each stabilized system must fulfill these two conditions but there are no other necessary conditions.

Thus, one seeks to fulfill (31) under these supplementary conditions. When designing the filter, one may apply a theorem from [4] (see Theorem 18 on page 184). Since it is formulated in the variable $\zeta = z^{-1}$, this variable will be adopted here for the sake of convenience.

Both previously stated conditions and (31) may be reformulated so that a function $\Gamma(z)/X(z) = f(\zeta)$ is to be found that

$$\int_{c_1} |f(\zeta)|^2 \cdot |\mathrm{d}\zeta| = \min\,, \tag{32}$$

$$f(\beta_1) = f(\beta_2) = 1\,, \tag{33}$$

$$f(\zeta) \quad \text{contains the factor} \quad \zeta^{m+1}\,. \tag{34}$$

Here, β_1 and β_2 are the reciprocals of the roots of equation (28).

According to Walsh the problem solution is given by the formula

$$f(\zeta) = p(\zeta) - \frac{\Pi(\zeta)}{2\pi \mathrm{i}} \int_{c_1} \frac{p(t)}{\Pi(t)\,(t-\zeta)}\,\mathrm{d}t\,, \quad |\zeta| < 1\,, \tag{35}$$

where

$$\Pi(\zeta) = \frac{\zeta^{m+1}(\zeta - \beta_1)(\zeta - \beta_2)}{(1 - \bar{\beta}_1\zeta)(1 - \bar{\beta}_2\zeta)} \tag{36}$$

and $p(\zeta)$ is a polynomial fulfilling the conditions (33), (34). Clearly $p(\zeta)$ has the form $\zeta^{m+1}(g\zeta + h)$. In the proof, it is not necessary to know the coefficients g, h. From the well known theorem of residues there is

$$\frac{1}{2\pi i}\int_{c_1}\frac{p(t)}{\Pi(t)(t-\zeta)}\,dt = B + C + D\,, \tag{37}$$

where B, C, D are defined from the decomposition

$$\frac{(gt+h)(1-\bar{\beta}_1 t)(1-\bar{\beta}_2 t)}{(t-\beta_1)(t-\beta_2)(t-\zeta)} = A + \frac{B}{t-\beta_1} + \frac{C}{t-\beta_2} + \frac{D}{t-\zeta}\,. \tag{38}$$

Now, let the product of the expressions (36) and (37) be considered. The form of B, C, D is clear from (38). Now, it may be easily seen that precisely the factors $1 - \bar{\beta}_1\zeta$ and $1 - \bar{\beta}_2\zeta$ result in the denominator of $f(\zeta)$ from the products containing B and C. The numerator of $f(\zeta)$ will always contain (besides other factors) the factor ζ^{m+1} common to $p(\zeta)$ and $\Pi(\zeta)$.

This means in the variable z that $\Gamma(z)/X(z)$ fulfilling (31), (33), and (34), has the poles $\bar{\beta}_1$, $\bar{\beta}_2$, and thus also the poles β_1, β_2, that is the reciprocals of the roots of equation (28) which, however are roots of the equation

$$z^2 + \frac{A_0}{A_1}z - \frac{1}{A_1} = 0\,. \tag{39}$$

Returning back to expression (25), one sees that the minimum prototype filter fulfills (34) and, according to the reasoning after (28), also (33). To fulfill (39) too, it suffices to put $\alpha_0 = A_0/A_1$, $\alpha_1 = -1/A_1$.

b) Let one of the roots of equation (28) lie inside, and the other outside or on the unit circle. Then the condition which the stabilizing filter must necessarily meet (see [3]) concerns only the latter root. The former root (inside the unit circle) can be compensated including the respective factor in the transfer function numerator of the filter. The roots of (28) are here denoted A, B. Let $|A| \geqq 1$, $|B| < 1$. Since $z - B$ is a factor of the numerator of the filter transfer function $F(z)$, the diagram in Fig. 1. may be interpreted so that $Y(z) = 1/(z - A)$; the delay factor is z^{-m+1}, and the filter transfer function is $F^*(z)$ without the factor $z - B$.

For this "one-dimensional" case, the filter can be designed analogously as in (17) from the relation

$$\begin{aligned} b_0 z^{r+m} &+ b_1 z^{r+m-1} + \ldots + b_r z^m - \\ &- A b_0 z^{r+m-1} - \ldots - A b_{r-1} z^m - A b_r z^{m-1} + \\ &+ c_0 z^l + c_1 z^{l-1} + \ldots + c_l = z^{s+1} + (a - A)\, z^s\,. \end{aligned} \tag{40}$$

In this relation let

$$r = m\,, \quad s = 2m - 1\,, \quad l = r - 1 = m - 1\,. \tag{41}$$

Putting $l = m - 1$ (and not $l = m$) is necessary for the filter to be physically realizable after completing the factor $z - B$ in the numerator of $F^*(z)$. The resulting filter transfer function $F(z)$ will possess the minimum possible degrees of both the numerator and the denominator. From (40), there follows

(42)
$$\begin{aligned} b_0 &= 1\,, \\ b_1 &= a\,, \\ b_j &= Ab_{j-1} \quad \text{for} \quad j = 2, 3, \ldots, \\ c_0 &= Ab_m\,, \\ c_j &= 0 \quad \text{for} \quad j > 0\,. \end{aligned}$$

In $F(z)$, there is obviously

(43)
$$\begin{aligned} P(z) &= c_0 z^{m-1}(z - B) = c_0 z^m - c_0 B z^{m-1}\,, \\ Q(z) &= b_0 z^m + b_1 z^{m-1} + \ldots + b_m\,. \end{aligned}$$

The structure of this filter which can be obviously called the minimum prototype is identical with (24). Since the constraints concerned are the only necessary ones, it is possible to find the value a fulfilling (31) with the aid of the cited theorem from [4]. One obtains

(44)
$$\alpha = a - A = -1/A$$

(compare with α_1 in the preceding case). It is easy to see that $\alpha_0 = (-1/A) - B$, $\alpha_1 = B/A$.

c) Let both roots of equation (28) lie inside the unit circle. Then (31) can obviously be met putting $c_0 = 0$, $c_1 = 0$. From (20) to (23) there follows $a_0 = a_1 = 0$. This is natural, there is no filtering in this case.

Thus, theorem 4.2 is proven.

5. MINIMUM PROTOTYPE FILTERS IN THE "MULTI-DIMENSIONAL" CASE

The method used in section 3 can be used analogously to design the minimum prototype filter in the multi-dimensional case. By the same method as in section 4, the generalizations of the theorems 4.1 and 4.2 can be proven. Thus, for

(45)
$$Y(z) = \frac{z^{n-1}}{z^n - A_0 z^{n-1} - A_1 z^{n-2} - \ldots - A_{n-1}}$$

one gets $\sigma_\eta^2 = \min$ for

(46)
$$\alpha_0 = a_0 - A_0 = \alpha_1 = a_1 - A_1 = \ldots = \alpha_{n-1} = a_{n-1} - A_{n-1} = 0\,,$$

where $a_0, a_1, \ldots, a_{n-1}$ have the same meaning as in (6). Further, one gets $\sigma_\gamma^2 = \min$, in the case of all poles of (45) lying outside or on the unit circle, precisely if

$$(47) \qquad \alpha_0 = \frac{A_{n-2}}{A_{n-1}}, \quad \alpha_1 = \frac{A_{n-3}}{A_{n-1}}, \ldots, \alpha_{n-1} = -\frac{1}{A_{n-1}}.$$

If some of the poles of (45) lie inside the unit circle, one can proceed precisely like in the proof of the theorem 4.2.

6. CONCLUDING NOTES

For the one-dimensional case, precisely the filter from [2] is obtained as the minimum prototype. However, much more about this filter is proven in [2]. Namely, this filter "includes" not only the case of minimum σ_η^2 and σ_γ^2, but varying the "parameter" a, it defines in the σ_η^2, σ_γ^2 plane precisely the boundary of the set of all points corresponding to stabilized one-dimensional systems (see [1] and [2]).

It seems that this interesting property is no more characteristic for the minimum prototype filters in the multi-dimensional case. For the sake of simplicity, the two-dimensional case without delay will be studied heuristically. According to the general formulae (see [5], p. 165–170) one gets

$$(48) \qquad \sigma_\eta^2 = \frac{1 + \alpha_1}{(1 - \alpha_1)(1 + \alpha_1 - \alpha_0)(1 + \alpha_1 + \alpha_0)},$$

$$(49) \qquad \sigma_\gamma^2 = \frac{(1 + \alpha_1)(a_1^2 + a_0^2) - 2\alpha_0 a_0 a_1}{(1 - \alpha_1)(1 + \alpha_1 - \alpha_0)(1 + \alpha_1 + \alpha_0)}.$$

It is known that the denominator factors of (48) and (49) in the form

$$(50) \qquad \begin{aligned} 1 - \alpha_1 &> 0, \\ 1 + \alpha_1 - \alpha_0 &> 0, \\ 1 + \alpha_1 + \alpha_0 &> 0 \end{aligned}$$

represent precisely the stability conditions for the system. The stability domain is shown in Fig. 2. The mapping of the expressions (48), (49) can be easily obtained using a digital computer. A program for the computer MINSK 22 has been derived by ing. V. Borský. For the special case $A_0 = 5$, $A_1 = -6$, some σ_η^2 and σ_γ^2 constant level lines are sketched in Fig. 2. At the origin it is $\sigma_{\eta\min}^2 = 1$, at the point $*$ it is $\sigma_{\gamma\min}^2 = 35$. Now, consider the curve S which is defined as the locus of the points in which pairs of constant level lines (of both systems) touch togehter from the outside. Clearly, S joins the points $*$, 0. It is heuristically clear that the points on S have the following remarkable property: let p be a point (in the stability domain) not lying on S. Then, at least one point q on S can be found such that $\sigma_{\eta q}^2 \leqq \sigma_{\eta p}^2$ and $\sigma_{\gamma q}^2 \leqq$

$\leq \sigma_{\gamma p}^2$. Thus, the line S corresponds to the interval of the "parameter" a giving the "optimal-class" of filters in the sense of [1] and [2] in the one-dimensional case. S might be called the preference-line.

Now, a σ_η^2 constant level line will be considered. It is heuristically clear that there exists precisely one point on this line at which the relative minimum σ_γ^2 is reached, and precisely one point at which the relative maximum σ_γ^2 is reached.

Moving the point along the σ_η^2 constant level line, σ_γ^2 can be varied continuously from the minimum to the maximum. Thus, inner points of the set of possible pairs

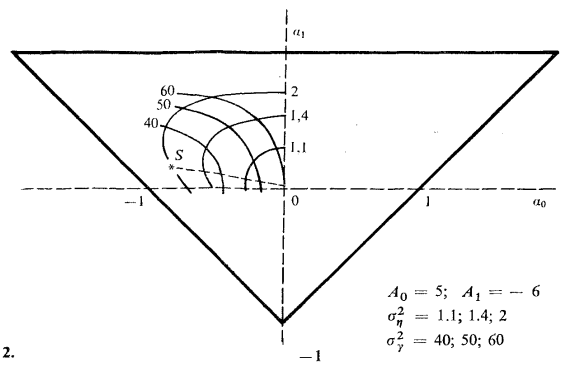

Fig. 2.

$\sigma_\eta^2, \sigma_\gamma^2$ (called the "smoothing capacity" in [1] and [2]) and not only its boundary are accessible by the minimum prototype filter. Thus, the hypothesis seems plausible that the minimum prototype filters form themselves a "complete class" (in the sense of [2]) in the multidimensional case.

It is easy to show heuristically that no continuous passage from the relative minimum to the relative maximum of σ_γ^2 is possible in the one-dimensional case. Thus, it is not surprising that additional "free" filter constants are necessary to reach the inner points of the set of all possible $\sigma_\eta^2, \sigma_\gamma^2$.

In [2] it has been proven that it is sufficient to add only one "free" constant in the numerator of the minimum prototype filter transfer function. Since the stability restraints from [3] are to be always satisfied, it is clear that the filters with "free" constants will be constructed from (40) choosing properly r, s, l. Choosing $r = l = m$, precisely the constant c_0 is "free" and one gets (with $\alpha = a - A$)

$$\frac{\Gamma(z)}{X(z)} = \frac{c_0 z + A(aA^{m-1} - c_0)}{z^m(z + \alpha)}. \tag{51}$$

For $m = 1$, putting $a - c_0 = b$, one gets

$$\frac{H(z)}{X(z)} = \frac{z + b}{z(z + \alpha)} \tag{52}$$

in accordance with (29) of [2] (irrespective of an additive constant.)

7. CONCLUSION

A considerable effort may yet be needed to obtain in the multi-dimensional case such "closed" results as in [2] in the one-dimensional one. However it is clear that the filter class denoted as complete in [2] is connected closely with the class of minimum prototype filters introduced in this article.

A digital computer analysis may be very helpful in obtaining clear insight and in solving concrete problems.

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VÝTAH

K vyhlazování diskretního náhodného autoregresního procesu

LUDVÍK PROUZA

Nechť je dána soustava, popsaná diferenční rovnicí

$$\eta_{n+1} = \sum_{i=0}^{t} A_i \eta_{n-i} + \xi_n - \gamma_n ,$$

kde $\{A_i\}$ jsou daná reálná čísla, $\{\xi_n\}$ je bílý diskrétní šum, $\{\eta_n\}$ je regulovaná (výstupní) posloupnost a $\{\gamma_n\}$ je regulující posloupnost s členy, vytvořenými lineární kombinací z předešlých členů posloupností $\{\xi_n\}$, $\{\eta_n\}$, $\{\gamma_n\}$ s nějakým zpožděním $m \geqq 0$.

V článku se konstruuje stabilizační filtr s výstupem $\{\gamma_n\}$, jehož vstupem je jen posloupnost $\{\eta_n\}$ a vyznačující se nejjednodušší strukturou („minimální" filtr). Dokazuje se, že vhodnou volbou konstant tohoto filtru lze vždy dosáhnout, aby bylo $\sigma_\eta^2 = \min$ nebo $\sigma_\gamma^2 = \min$. Heuristicky se odvozují některé další pozoruhodné vlastnosti takového filtru.

Dr. Ludvík Prouza, CSc., Ústav pro výzkum radiotechniky, Opočínek, p. Lány na Důlku.